
Thermal design analysis of compact heat exchanger using nanofluids

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Abstract: Compact heat exchangers have been widely used in various applications in thermal fluid systems including automotive thermal fluid systems. Radiators for engine cooling systems, evaporators and condensers for HVAC systems, oil coolers and inter coolers are typical examples that can be found in ground vehicles. Recent development of nanotechnology brings out a new heat transfer coolant called 'nanofluids'. These fluids exhibit larger thermal properties than conventional coolants (water, ethylene glycol, engine oil etc.) due to the presence of suspended nanosized particles in them such as Al_2O_3 , Cu, CuO, TiO_2 etc. In this paper, a theoretical analysis was carried with the ϵ – NTU rating method by using $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid as coolant on flat tube plain fin compact heat exchanger and different characteristics are graphically presented.

Keywords: automobile radiator; compact heat exchanger; ϵ – NTU method; nanofluid.

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1 Introduction

The automobile industry continuously faces challenges to obtain best automobile design in aspects of performance, fuel consumption, safety etc. The thermal performance of an automobile radiator plays an important role in the performance of automobiles cooling system and all other associated systems. The air cooled heat exchanger found in a radiators, AC condenser, and evaporator etc. has an important role in its weight and also in the design of its front end module, which also has a strong impact on the car aerodynamic behaviour. To improve the heat transfer from a surface, it is common to apply turbulence promoters, roughness elements to the surface. In recent years, a growing and intense attention has been turned to the study of new concept compact heat exchangers, as they represent a good solution in terms of dimensions and efficiency for industrial applications compared to traditional ones. Compact heat exchangers usually setup in a cross flow arrangement are characterised by extended surfaces with large surface area/volume ratios ($> 700 \text{ m}^2/\text{m}^3$) that can often arrangements (Rohsenow et al., 1998). A variety of increased heat transfer surfaces are used: plain, wavy, offset strip, perforated and louvered fins (Kays and London, 1984).

Charyulu et al. (1999) presented a numerical model based on the ϵ – NTU method of a radiator in diesel engine type TBD 232 V-12 and gave radiator characteristics for different fin and tube materials. Carluccio et al. (2005) presented a numerical analysis of an air-oil radiator made of aluminum alloy using CFD and compared the results with experimental data. Witry et al. (2005) presented the thermal performance of automotive aluminum plate radiators using CFD and found to have higher heat transfer levels; lesser pressure drop, smaller size and coolant flows velocities decrease because of impingement and erosion/corrosion of the plate. Mahmoudi (2007) conducted experimental and theoretical analysis on copper-based automobile radiators. He has developed a 2D CFD model and found that the inlet and outlet parameter are important for design of the

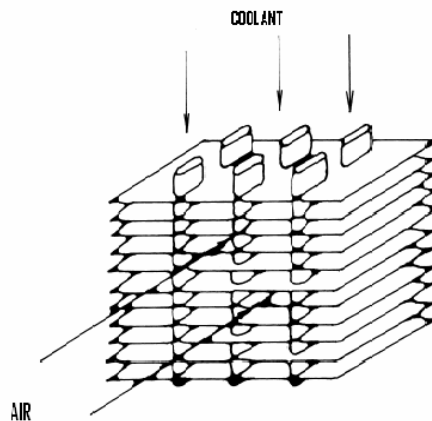
radiator. Oliet et al. (2007) proposed a numerical model using the ε – NTU method and CFD. They presented a detailed knowledge base of parametric study on design of automotive radiations.

In radiators, which are vital component in the control of the engine temperature in automobiles, a liquid (commonly water – glycol mixture) is to be cooled by air. The liquid flows in flat tubes while the air flows in channels setup by fin surfaces. With recent developments, nanotechnology has been widely used in traditional industries because materials with grain size of nanometers posses unique optical, electrical and thermal properties etc. Recently, nanoparticles can be dispersed in conventional heat transfer fluids such as water, ethylene glycol and engine oil to produce a new class of high efficient heat exchange fluids called nanofluids (Choi, 1995). Many experimental (Eastman et al., 2001; Das et al., 2003; Xuan et al., 2003; Ding and Wen, 2004) and theoretical (Ravi et al., 2005; Maiga et al., 2006) analyses were carried out and found that these new heat exchanger coolants are excellent. Vasu et al. (2007a, 2007b, 2008) has developed thermophysical correlations to calculate thermal conductivity, viscosity, Nusselt number in turbulent and laminar flows of different nanofluids ($\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$, $\text{Cu} + \text{H}_2\text{O}$ etc) and found that these fluids posses very high thermal properties compared to conventional coolants. In this paper, numerical analysis was carried out with the ε – NTU method by using $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid as coolant to compact heat exchanger and different characteristics are graphically presented.

2 Compact heat exchanger geometry

As illustrated in Figure 1, the compact heat exchanger major sub components of the core are coolant tubes and fins. Flat tubes are more popular for automotive applications due to their lower profile drag compared with round tubes. The directions of the coolant and air flows cross each other as shown in Figure 1. Therefore, ultimate design object of the heat exchanger is to maximise the heat rejection rate while minimising the flow resistance. Due to many parameters, the numerical ε – NTU method can be very useful for this analysis.

Figure 1 Structure of a typical compact heat exchanger core



3 Problem formulation

The considered radiator (Charyulu et al., 1999) mounted on the present turbo-charged diesel engine of type TBD 232V-12 cross flow compact exchanger with unmixed fluids in Figure1 consists of 644 tubes made of brass and 346 continuous fins made of copper. The following geometrical factors operating conditions are described in the following Tables 1–2.

Table 1 Fluid parameters and normal operating conditions

S. no.	Description	Air	Coolant
1	Fluid mass rate	8–20 kg/s	6000–10000 kg/hr
2	Fluid inlet temperature	20–55°C	70–95°C
3	Fluid temperature rise / drop	28°C	6°C
4	Core width	0.6 m	
5	Core height	0.5 m	
6	Core depth	0.4 m	
7	Tube size	1.872 cm*.245cm	

Table 2 Surface core geometry of flat tubes, continuous fins

S. no.	Description	Air side	Coolant side
1	Fin pitch	4.46 fin/cm	
2	Fin metal thickness	0.01 cm	
3	Hydraulic diameter, D_h	0.351cm	0.373cm
4	Min free flow area / Frontal area, σ	0.780	0.129
5	Total heat transfer area / Total volume, α	886 m ² /m ³	138 m ² /m ³
6	Fin area / Total area, β	0.845	

Source: Surface 11.32-0.737-SR, Kays and London (1984)

4 Equations used for calculations

Air side

- 1 Heat transfer coefficient (Charyulu et al., 1999), h_a

$$h_a = \frac{j_a G_a C_{p_a}}{(\text{Pr}_a)^{2/3}} \quad (1)$$

where

$$j_a = \frac{0.174}{(\text{Re}_a)^{0.383}}$$

$$G_a = \frac{W}{A_{fr} \sigma_a} \quad (2)$$

$$\text{Re}_a = \frac{G_a D_{h,a}}{\mu_a} \quad (3)$$

- 2 Fin efficiency of plate fin can be calculated as

$$\eta = \frac{\tanh mL}{mL} \text{ where } m = \sqrt{\frac{2h_a}{k t}} \quad (4)$$

The area-weighted fin efficiency is determined by

$$\eta' = \lambda \eta + 1 - \lambda \text{ where } \lambda = \frac{A_f}{A} \quad (5)$$

- 3 Pressure drop for fin side

$$\Delta P = \frac{G_a}{2\rho_{i,a}} \left[\left(1 + \sigma_a^2 \right) \left(\frac{\rho_{i,a}}{\rho_{i,a}} - 1 \right) + f \frac{A}{A_{\min}} \frac{\rho_{i,a}}{\rho_m} \right] \quad (6)$$

where

$$\frac{1}{\rho_m} = \frac{1}{2} \left(\frac{1}{\rho_{i,a}} + \frac{1}{\rho_{o,a}} \right)$$

- 4 Friction factor f is given by

$$f = \frac{0.3778}{\text{Re}_a^{0.3565}} \quad (7)$$

- 5 Air heat capacity rate, C_a

$$C_a = m_a C_{p_a} \quad (8)$$

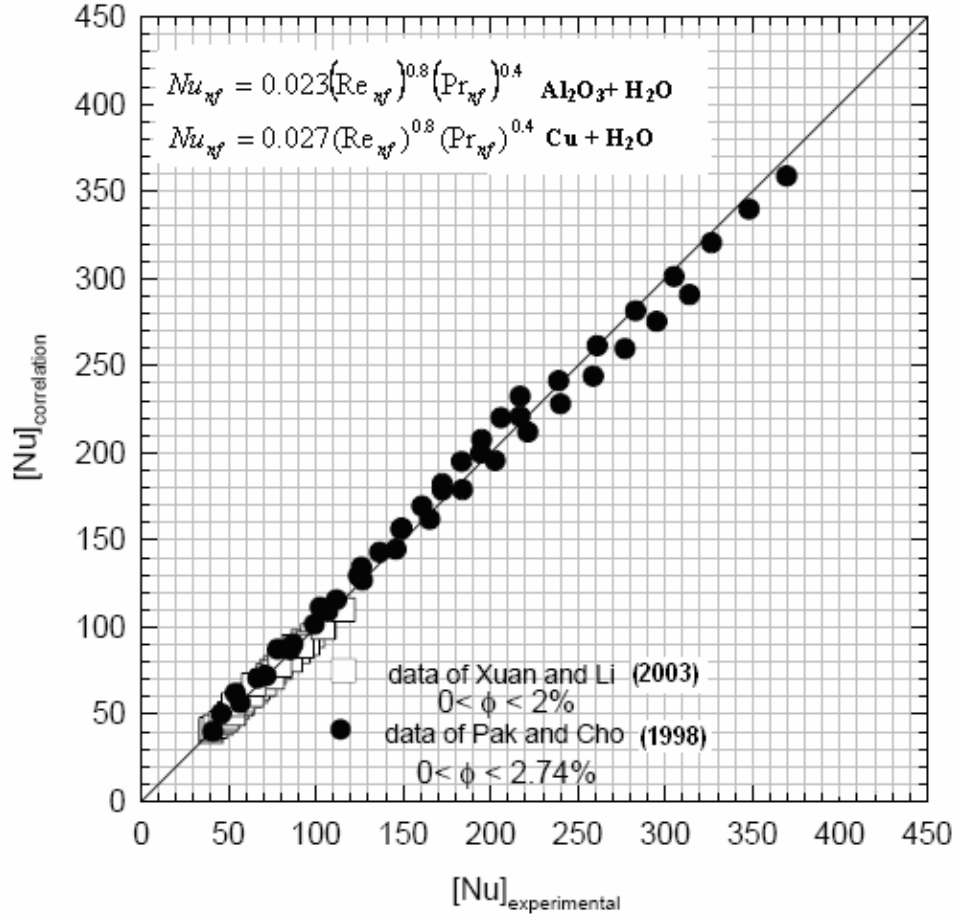
Nanofluid as coolant side

- 1 Heat transfer coefficient of the $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid in turbulent flow has been developed in previous studies (Vasu et al. 2007b; 2008). The comparison is shown in Figure 2, which is found to be in good agreement with the experimental data with standard deviation of 6.4% and average deviation of 5%.

$$h_{nf} = \frac{Nu_{nf} K_{nf}}{D_{h,nf}} \quad (9)$$

where

$$Nu_{nf} = 0.023(Re_{nf})^{0.8}(Pr_{nf})^{0.4} \text{ for Al}_2\text{O}_3 + \text{H}_2\text{O}$$

Figure 2 Comparison of Nu correlation with experimental data

Source: Xuan et al. (2003) and Pak and Cho (1998)

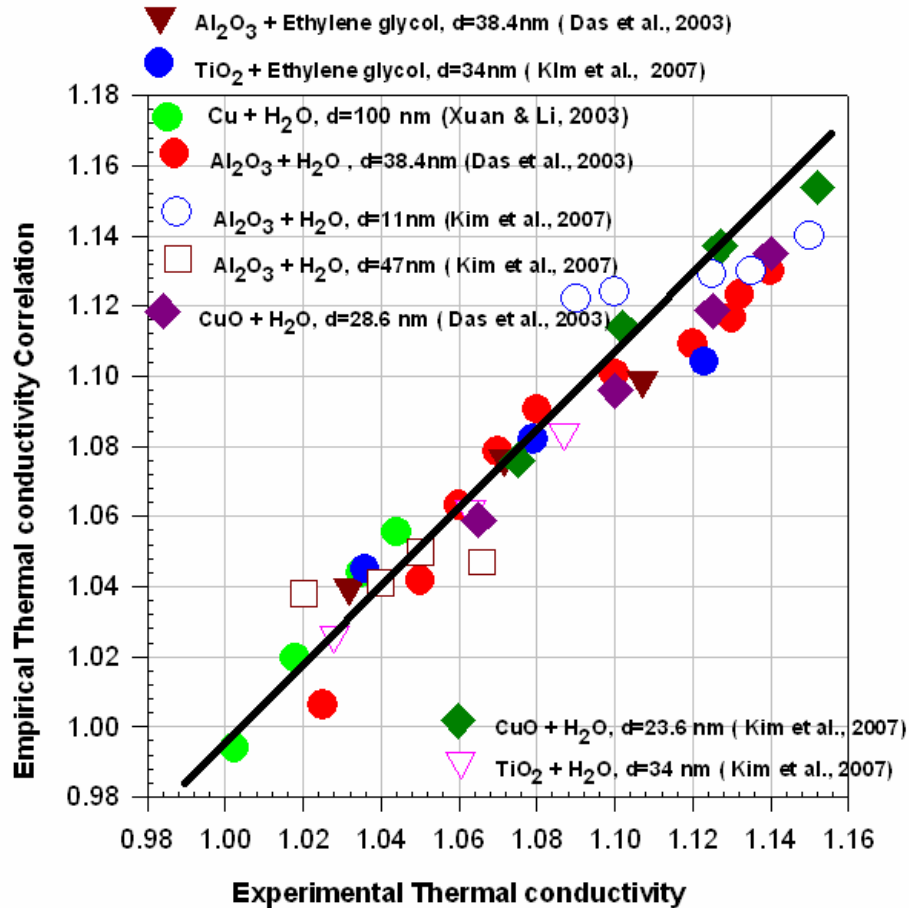
$$Re_{nf} = \frac{u_{\max} D_{h,nf}}{\nu_{nf}} \quad (10)$$

$$Pr_{nf} = \frac{\mu_{nf} C_{p,nf}}{k_{nf}} \quad (11)$$

$$\frac{k_{nf}}{k_f} = Re_m^{0.175} \phi^{0.05} \left(\frac{k_p}{k_f} \right)^{0.2324} \text{ for Al}_2\text{O}_3 + \text{H}_2\text{O} \quad (12)$$

Equation (12) is used to calculate the thermal conductivity for nanofluids (Vasu et al., 2007b) which is found to be in good agreement with the experimental data as shown in Figure 3 with a standard deviation of 5% and average deviation of 4%.

Figure 3 Comparison of present thermal conductivity correlation with experimental data (see online version for colours)



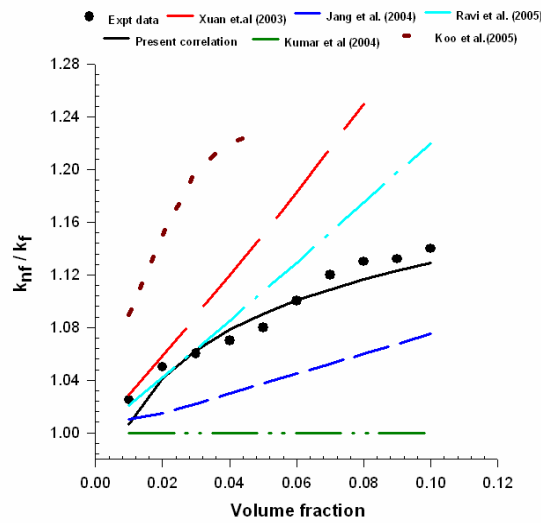
Source: Das et al. (2003), Xuan and Li (2003) and Kim et al. (2007).

4.1 Validation with other theoretical models

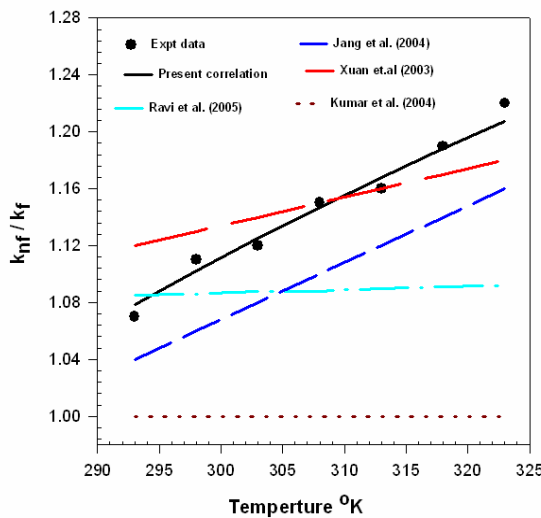
Figures 4 (a) and (b) show the prediction based on the present model (solid line) in comparison with the published models for the case of 38.4 nm Al_2O_3 at 1 vol% in water. The symbols show the corresponding experimental data (Das et al., 2003). Xuan et al. (2003) and Koo and Kleinstreuer (2005) excessively overestimate and their model shows the limitation of the simple modification of Maxwell's model to apply for nanofluids. Jang and Choi (2004) model shows underestimated values with experimental data. This is believed attributing to their incorrect postulation in determining the Nusselt number as

previously pointed out. The model by Kumar et al. (2004) wrongly postulates the mean free path of the base fluid and completely fails to predict the nanofluid thermal conductivity. The model by Ravi et al. (2005) shows good agreement with the experimental data as shown in Figure 4(b). However, with temperature variation, their model breaks down showing excessive underestimation with experimental data as shown in Figure 4(a).

Figure 4 Comparison of the present model with published models (see online version for colours)



(a)



(b)

Source: Das et al. (2003)

These models inherently lack the dependency of the material properties of nanoparticle other than incorporating their sizes and concentrations. Jang and Choi (2004) also show large discrepancies, possibly because of the same reason of incompressive parametric dependency. Xuan et al. (2003) does not show agreeable temperature dependency, and Kumar et al. (2004) model does not show any physically meaningful representation. However, the present model of equation (12) shows fairly good agreement comprehensively for both nanofluids and for all the tested conditions of temperatures and volume concentrations.

The viscosity, density and specific heat of nanofluids are calculated by using the following equations (13–15).

$$\mu_{nf} = \mu_f(1 + 39.11\phi + 533.9\phi^2) \quad (13)$$

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p \quad (14)$$

$$Cp_{nf} = \frac{(1 - \phi)\rho_f Cp_f + \phi\rho_p Cp_p}{\rho_{nf}} \quad (15)$$

2 Pressure drop is given as

$$\Delta P_c = \frac{2G_{nf}f_{nf}H}{\rho_{nf}D_{h,nf}} \quad (16)$$

where

$$f_{nf} = 0.079(\text{Re}_{nf})^{-0.25} \quad (17)$$

3 Coolant heat capacity rate, C_{nf}

$$C_{nf} = m_{nf}Cp_{nf} \quad (18)$$

The heat exchanger effectiveness for cross flow unmixed fluids, ε is given as (Kays and London, 1984)

$$\varepsilon = 1 - \exp\left[\frac{1}{C^*}NTU^{0.22} \exp(-C^*NTU^{0.78} - 1)\right] \quad (19)$$

where

$$C^* = \frac{C_a}{C_{nf}} \quad NTU = \frac{U_a A_a}{C_a} \quad (20)$$

Overall heat transfer coefficient, based on air side is given as

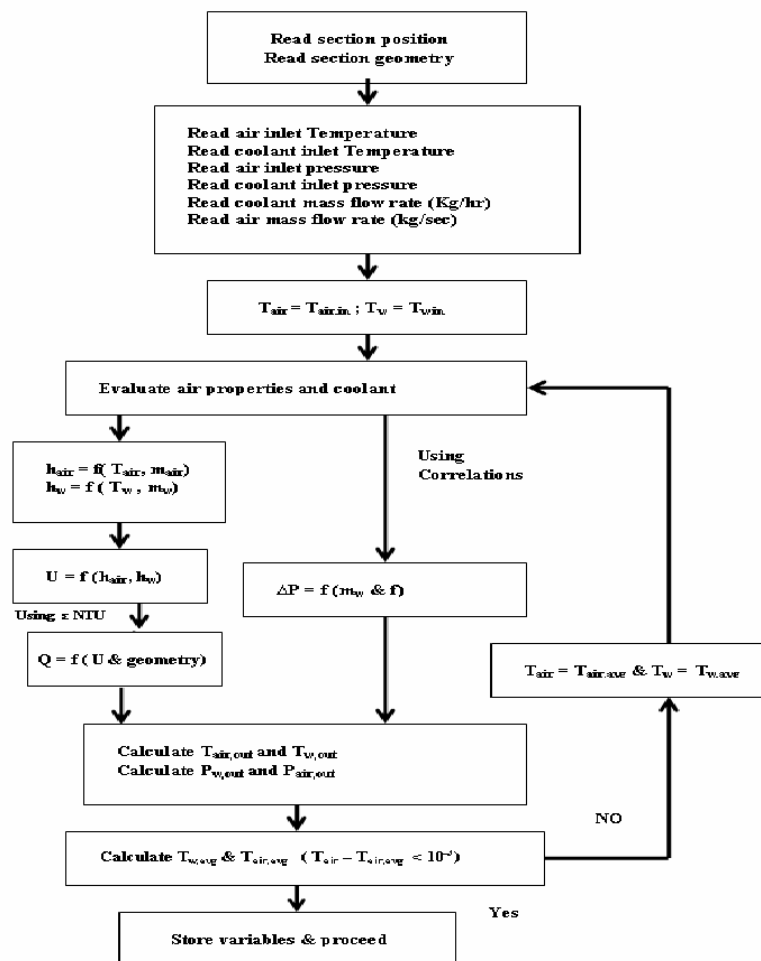
$$\frac{1}{U_a} = \frac{1}{\eta' h_a} + \frac{1}{\left(\frac{\alpha_{nf}}{\alpha_a}\right) h_{nf}} \quad (21)$$

Total heat transfer rate

$$Q = \varepsilon C_{\min} (T_{c,in} - T_{a,in}) \quad (22)$$

For implementing the analysis, a computer program in MATLAB is developed for the compact heat exchanger. This program is useful in estimating the fluid properties at operating temperatures, surface core geometry of cross flow heat exchanger, heat transfer coefficients, pressure drops, overall heat transfer coefficients and heat transfer rate. The flowchart of the numerical analysis is shown in Figure 5.

Figure 5 Schematic scheme of the numerical method

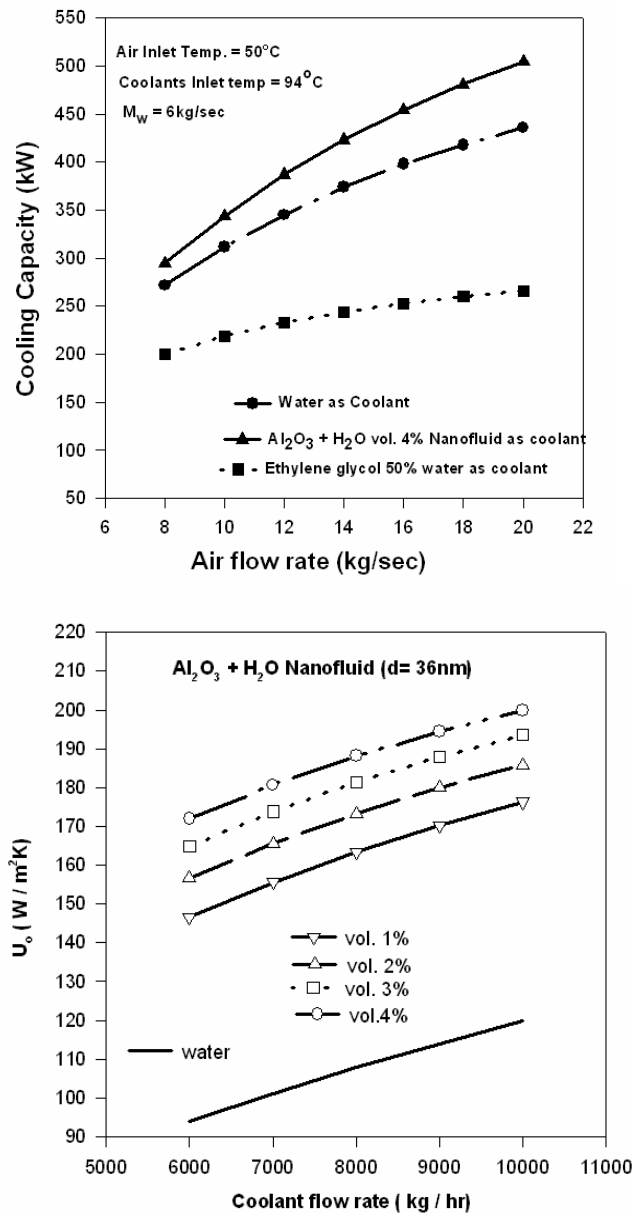


Source: Shah et al. (2001)

5 Results and discussions

Figure 6 indicates that nanofluids possess higher heat transfer characteristics than conventional coolants such as water and 50% ethylene glycol. The overall heat transfer coefficient is very high for nanofluids compared to water and increases with increase of the volume fraction of nanoparticles.

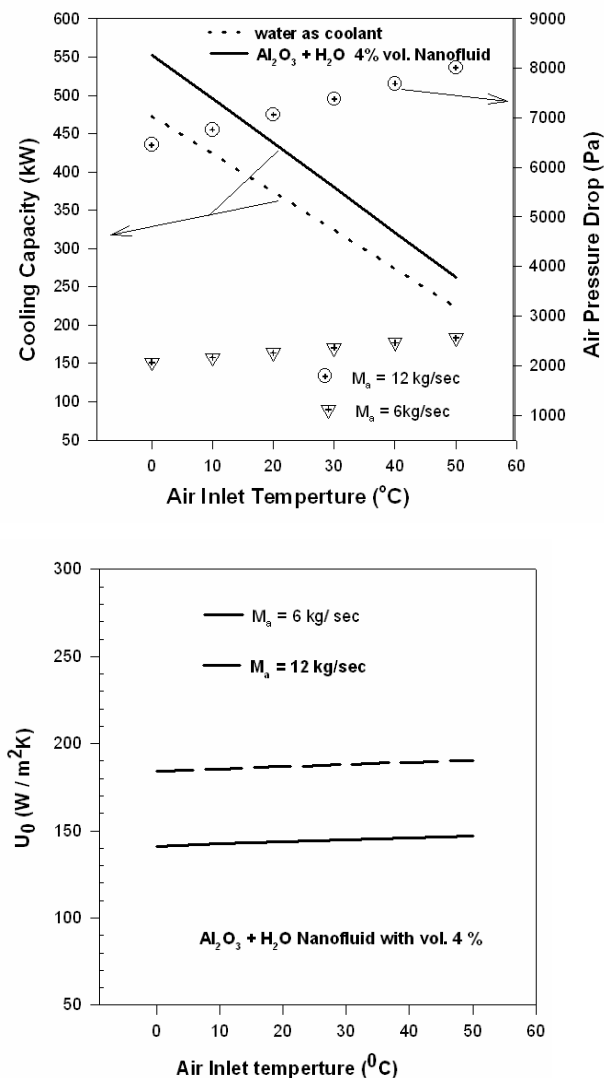
Figure 6 Comparison of nanofluid as coolant with conventional coolant (water)



5.1 Effect of air inlet temperature

As one of the most important factor in an automotive radiator system, the air inlet temperature is analysed and shown in Figure 7 for the two limiting air flows (12 kg/sec and 6 kg/sec) for a range of temperature from 0°C to 50°C. As expected, the heat transfer rate clearly decreases with air inlet temperature rise, as the cooling temperature difference is being reduced. It is interesting to point out the $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluids have higher cooling capacity compared to water as coolant. There is a small influence of air inlet temperature on the overall heat transfer coefficient whereas the air pressure drop reveals moderate.

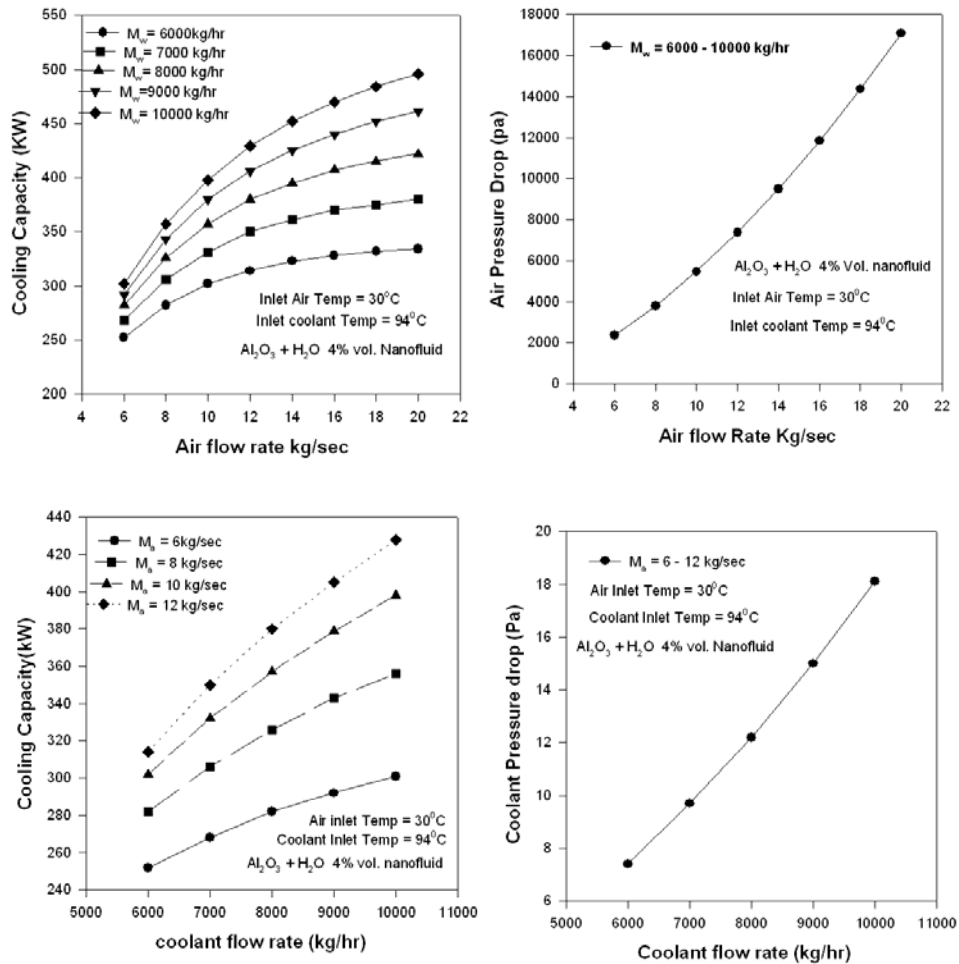
Figure 7 Air inlet temperature influence on the thermal and fluid dynamic performance of compact heat exchanger



5.2 Effect of air and coolant mass flow rate

The cooling capacity of the radiator is strongly dependent on both fluids mass flow rate. Figure 8 shows the behaviour of the selected radiator over a wide range, while maintaining the geometry and temperature levels at the normal situation. It is observed that the cooling capacity is increasing with both air and coolant flow rates. The cooling capacity is more with air flow rate due to higher thermal resistance. The pressure drop also increases quadratically with both air and coolant mass flow rates and is almost the same for all flow rates of air (6–12 kg/sec) and coolant (6000–10000 kg/hr). It is interesting to point out that the cooling capacity and overall heat transfer coefficient of the radiator is very high with mass flow rates of the air and coolant when $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluids is used as coolant as shown in Figure 6, but the pressure drop are higher when compared with conventional coolants.

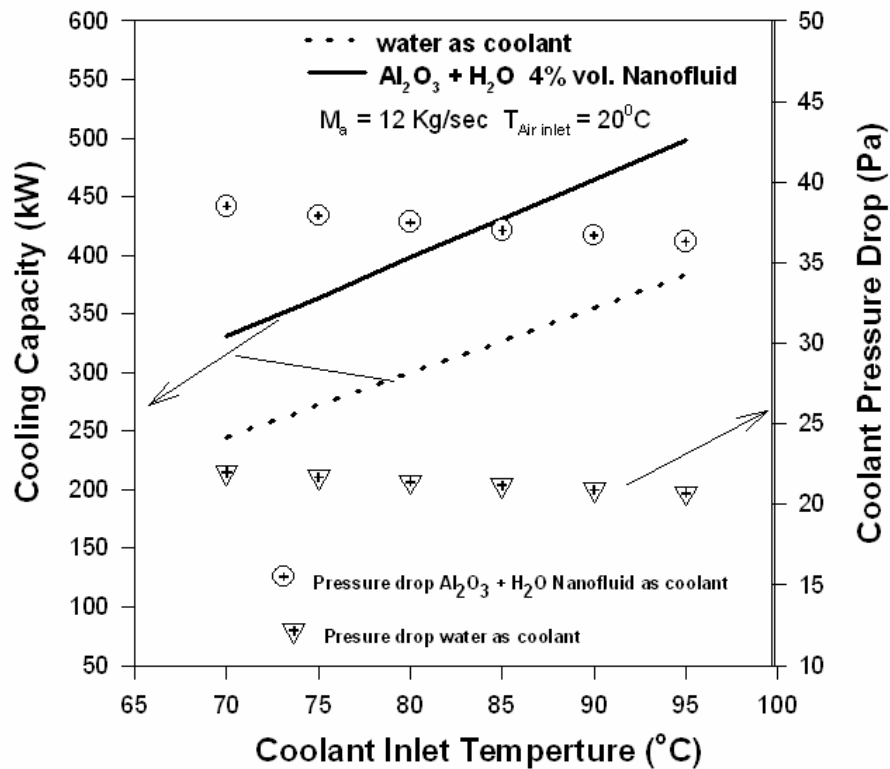
Figure 8 Air and coolant flow influence on the thermal and fluid dynamic performance of compact heat exchanger



5.3 Effect of coolant inlet temperature

Another characteristic of radiator is the coolant inlet temperature. It is observed from Figure 9 that with increase of the coolant inlet temperature, the cooling capacity is increased. It is also observed that the cooling capacity $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid is very high compared to water as coolant. However, the pressure drop is nearly double that of water.

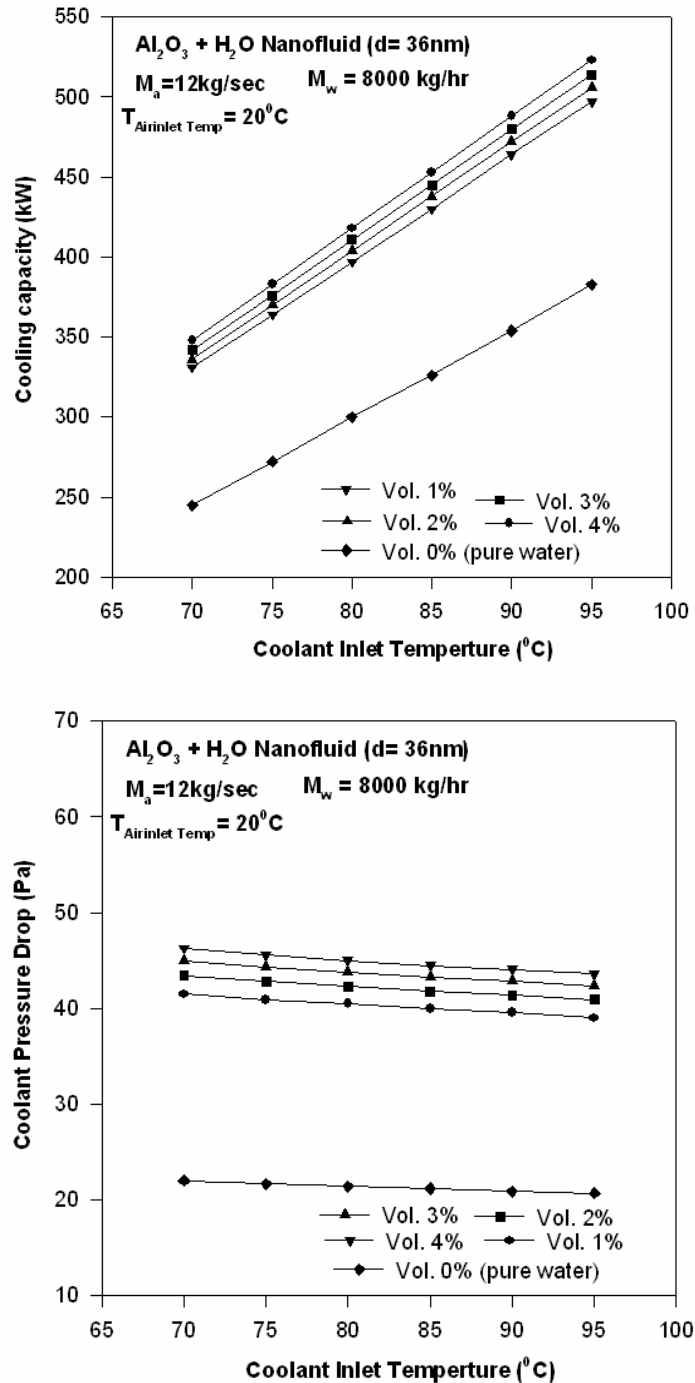
Figure 9 Coolant inlet temperature influence on the thermal and fluid dynamic performance of compact heat exchanger



5.4 Effect of nanoparticle volume fraction

Figure 10 indicates that with increase of the volume fraction of the nanoparticle concentration, the cooling capacity increases in moderate manner and the pressure drop decreases with coolant inlet temperature, but cooling capacity is very high when compared with 0% volume fraction (pure water).

Figure 10 Coolant inlet temperature and volume fraction of nanoparticle influence on the thermal and fluid dynamic performance of compact heat exchanger



6 Conclusions

- A detailed study of the parametric studies on compact heat exchanger is done by using the ε – NTU numerical method and $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid as coolant.
- A detailed flow chart of the numerical method and correlations used for $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid are presented.
- Comparing the study of $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid as coolant with conventional coolants, it is observed that the cooling capacity of the $\text{Al}_2\text{O}_3 + \text{H}_2\text{O}$ nanofluid is very high.
- Different factors for compact heat exchanger are graphically presented.

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Nomenclature

A_{fr}	frontal area of the exchanger, m ²
A_{fin}	surface area of fin exposed to heat transfer, m ²
A	total heat transfer surface area, m ²
C	specific heat, kJ/kg K
d	diameter, nm
k	thermal conductivity, W/m K
t	thickness, nm
T	temperature, K
v	velocity, m/s
h	heat transfer coefficient, W/m ² °C
j	Colburn factor
G	mass velocity, kg/m ² s
Pr	Prandtl number
Re	Reynolds number
W	fluid mass flow rate, kg/s
f	friction factor
Nu	Nusselt number
H	total water flow length, m
U	overall heat transfer coefficient, W/m ² °C

Nomenclature (continued)

P	pressure, Pa
NTU	number of heat transfer Units
k_b	Boltzman constant (1.3807×10^{-23}), J/K
Re_m	Reynolds number $\frac{1}{\nu_f} \sqrt{\frac{18 k_b T}{\Pi \rho_p d_p}}$
<i>Greek</i>	
ρ	density, kg/m ³
ϕ	volume fraction
ψ	sphericity
μ	viscosity, m ² /s
α	conductivity ratio
ε	thermal effectiveness
<i>Subscripts</i>	
nf	nanofluids
p	nanoparticle
f	basefluid
fin	fin
i	inlet condition
o	outlet condition
a	air