Consignment stock policy with controllable lead time for effective inventory management in supply chains

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Abstract: In this paper, we develop an inventory model where the replenishment lead time is assumed to be dependent because at the time of contract with a manufacturer retailer may intend to reduce the lead time for which he pay an additional cost to accomplish the increased production rate. We provide a solution procedure to obtain the efficient ordering strategy in Hill's inventory model in particularly Consignment Stock (CS) policy of Supply Chain Management (SCM) for a single vendor – single buyer under the stochastic nature. The lead time of CS strategy has been controlled to minimise Joint Total Expected Cost (JTEC) and simultaneously optimised other decision variables such as quantity transported, lead time, the number of transport operations and delay deliveries under stochastic environment so as to gain a competitive advantage in the business strategy. Numerical examples are presented to illustrate the solution procedure.

Keywords: Consignment Stock (CS); Hill's inventory model; Supply Chain (SC); stochastic demand; control lead time; crashing cost.

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1 Introduction

In today's global market place, firms are no longer competing as independent entities with unique brand names, but rather as an integral part of entire network links. As such, the ultimate success of a firm will depend on its managerial ability to integrate and coordinate the intricate network of business relationships among Supply Chain (SC) members (Drucker, 1998, Douglas and Cooper, 2000). In this environment, 'Supply Chain Management' (SCM) has become a means of further adding value to products and to gain a competitive advantage in the business strategy. Houlihan (1985) is credited to be the first person for coining the term SC with insight concepts with a strong case for viewing the SC as a strategy is described the holistic approach of integrating the SCM global strategic business decisions.

The SCM is generally viewed as a network of facilities that work together to source, produce and ultimately distribute products and services to the customer. Each echelon of SC perform independent business with information sharing among all the echelons and it holds some inventories which may be unavoidable due to existing uncertainty in the business. In the world of inventory, an effective, approach that is quickly gaining ground is the Consignment Stock (CS). It is a novel approach to the SC networks as it consists of network of tasks/locations. CS of Vendor Managed Inventory (VMI) in which vendor stocks his finished products in buyer's warehouse. The vendor will guarantee for the quantity stored in the buyer's warehouse will be kept between a minimum and maximum level (S) with supporting stock-out costs in stochastic customer demand. The most radical application of CS approach leads to suppression of the vendor inventory, as this party (vendor) will use the buyer's warehouse to stock its finished products. Generally, this warehouse is close to the buyer's production line, so that the material may be picked up when needed. This concept permits the buyer to face demand and/or lead time fluctuations. The CS arrangement has been widely adopted in Italy (Valentini and Zavanella, 2003) and is achieving a consensus in Industrial environments between both small and large companies.

Most of the literature is limited in terms of applicability to the real world SC problems, due to the size and complexity in the problems that they have to handle or because of their underlying assumptions. However, the researchers and practitioners are investigating the new mathematical methodologies. The SC problems are formulated either as deterministic analytical models, if the decision variables are known with certainty, or as stochastic analytical models, when at least one of the decision variables is unknown and is assumed to follow a particular probability distributions.

This paper is structured in six sections: Section 2 describes the literature and the work done in the area of Joint Total Expected Costs (JTECs) of single vendor and single buyer with controllable lead times, VMI, CS and also the Hills inventory model. Section 3 will briefly present the vendor–buyer inventory modelling of Hill's model, CS model, CS with delay deliveries, stochastic case model and CS with controllable lead time model

(CS-LT). In Section 4, an algorithm, which is used for crashing the controllable lead time in CS model is described. Section 5 gives the results and Section 6 gives the conclusions and future studies.

2 Literature review

The VMI is an interesting approach to stock monitoring and control and it has progressively considered and introduced in several companies. It is one of the most widely discussed partnering initiatives for improving multifirm SC efficiency. It was popularised in the late-1980s by Wal-Mart and Procter & Gamble. In most of the literature, the VMI is considered to evaluate the effect of information sharing (partial/full) in an integrated inventory model and improved the decision rules. Goyal (1976) is credited to be the first person to describe integrated models of single vendor–single buyer. Goyal (1977) proposed a Joint Economic Lot Size (JELS) model with the objective of minimising the sum of all relevant costs incurred by vendor and buyer, under the assumption that the pricing structure would then reflect the joint benefit derived when the total cost is compared with the costs incurred if vendor and buyer act independently to optimise their respective costs. Banerjee (1986) generalised Goyal's (1977) model by assuming that the vendor is manufacturing at a finite rate and considered a JELS model where a vendor produces to order for a buyer on a lot-for-lot basis. Liao and Shyu (1991) considered lead time as a decision variable, Ben-Daya and Raouf (1994) considered both lead time and ordering quantity as decision variables. Ouyang et al. (1996) revised the above models and proposed a new model in which shortages are allowed. Moon and Choi (1998), Hariga (1999) and Ben-Daya and Hariga (2004) considered reorder point (R_n) as one of the decision variables.

Many researchers looked at the problem of lead time optimisation following the papers by Liao and Shyu (1991) and Ben-Daya and Raouf (1994). When the demand during the cycle period is not deterministic but stochastic, the system lead times become an important issue and its control leads to some quantitative benefits. The system lead time (Tersine, 1994) consists of

1 order preparation
2 order transit
3 supplier lead time
4 delivery lead time and
5 set-up time.

Pan and Yang (2002) credited for minimising JTEC for lower lead time which is a decision variable, however, shortages are not allowed in his paper. Kelle and Milne (1999) examined the effect of (s, S) ordering policy on the order variability in a SC and provided quantitative tools for estimation of variability increase; however, they have ignored the joint ordering policy. Kim and Benton (1995) considered the effect of lot size on lead time, safety stock and established a linear relationship between lead time and lot size based on observations of Karmarkar (1987). Venkateswaran and Son (2004) proposed strategies to verify the effectiveness of reduced lead time between players (vendor and buyer) on minimising the SC dynamics. The improvement in lead time is typically achieved by reducing production delays through streamlining operations, using customised machines and reducing processing times and transportation delays.

The CS of VMI in buying and payment strategies include various strategies (Frazelle, 2001) such as:

1 central buying-local delivery

2 buying partnerships

3 CS inventory and

4 Electronic Funds Transfer (EFT).

In central buying, all purchasing requirements should be estimated in advance and consolidated across all divisions and departments. This practice permits the organisation to yield as much negotiating leverage as possible with each supplier, yielding the lowest possible unit purchase cost. The buying partnership between two large, non-competing organisations that jointly negotiate for transportation services in all countries they operate in. The EFT or e-cash facilitate consignment inventory programmes with electronic payment on consumption initiated at the point of scale. Finally, CS inventory which is one of the main objectives in buying and payment is to negotiate the most favourable payment terms. Some of the terms are incorporated into CS inventory programmes in which payment for supplier inventory is not released until goods have been sold at the customer location. Late payment terms can yield a significant positive float for the organisation. Moreover, the vendor is used to stock his goods in the buyer's warehouse. This method may lead to a successful strategy for both the buyer and supplier.

Corbett (2001) is credited to be the first person to give about the fundamentals of CS whereas Valentini and Zavanella (2003) presented an industrial case and performance analysis of CS policy for a single vendor and single buyer. Braglia and Zavanella (2003) presented the analytical modeling approach and also some performance evaluation of CS policy, which is an effective alternate to Hill's (1997, 1999) models without considering the crashing expenses due to the controllable lead time components. Ouyang et al. (2004) considered information sharing to integrate production inventory model with shortages is permitted and assumed that the lead time is controllable with added cost so as to optimise ordering quantity and other variables in both normal and free distribution strategies and relaxed the Banerjee's (1986) lot-for-lot basis with a finite production rate of '*nq*' at one set-up but ships in quantity '*q*' to the buyer over '*n*' times. Ryu and Lee (2003) and Pan and Yang (2002) analysed the effect of investment strategies to control lead times. All the previously published research has not considered the lead time control by adding crashing expense in CS strategy of inventory models.

Lead time crashing facilitates lower lead times and also reduces the stock-out probability. Where as in general problems, whenever the lead time reduces for either larger or smaller demand for immediate delivery, companies may face stock-out problems hence it is necessary to increase the reorder point, which is regarded as safety stock. Increasing the safety stock leads to the downstream movement of inventories (Abdel-Malek et al., 2002).

The authors in this paper have formulated an extended framework of Ouyang et al. (2004) and Braglia and Zavanella (2003) for the application of the CS strategy inventory model for a stochastic nature of controllable system lead times, which have been reduced by adding crashing costs to minimise JTEC in SCM. Assuming long-term strategic partnerships between the buyer and the vendor by allowing shortages and reorder point is considered as one of the variables under continuously review system. Some numerical examples are provided to illustrate the benefits the models.

3 Vendor-buyer inventory models

The necessary notations used in this paper are summarised as follows:

A_1 batch set-up cost (vendor)

A_2 order emission cost (buyer)

h_1 vendor stock holding cost

h_2 buyer stock holding cost

p vendor production rate (continuous)

D demand rate seen by the buyer (continuous)

q quantity transported per delivery

T_C average total costs of the system/time

σ standard deviation of demand/unit time

π unit back order cost for the buyer

R_n reorder point of the buyer

L length of the lead-time for the buyer

C_L lead-time crashing cost per cycle

k^1 delay deliveries

x lead time demand

ϕ normal probability density function

Φ cumulative probability density function

n number of transport operations/production batch

3.1 Hill's model

This model minimises the total expenses incurred per year of the single buyer–single vendor system with determining the production and shipment schedule, which minimises the average total cost per unit time of production set-up, shipment and stockholding. The basic assumption is that the vendor only knows the buyer's demand, order frequency and all the variables are assumed to be continuous and no shortages for the buyer under. The standard Hill model will have equal sized shipment. Thus, it concentrates on the number of deliveries made and the vendor stock is the average total stock less than the average buyer stock.

The average total cost using basic geometric considerations for this model is

$$T_C^H = \left(A_1 + nA_2\right)\frac{D}{nq} + h_1\left[\frac{Dq}{p} + nq\frac{(p-D)}{2p}\right] + \left(h_2 - h_1\right)\frac{q}{2} \tag{1}$$

Differentiating Equation (1) with respect to 'q' to find the optimal ordering quantity q^*, which yields

$$q^* = \sqrt{\frac{(A_1 + nA_2)D/n}{h_1\left[(D/p) + ((p-D)n/2p)\right] + ((h_2 - h_1)/2)}} \tag{2}$$

After substituting q^* in Equation (1), the optimum total cost function can be written as

$$T_C^H\left(q^*\right) = 2\sqrt{\left(A_1 + nA_2\right)\frac{D}{n}\left\{h_1\left(\frac{D}{p} + \frac{(p-D)n}{2p}\right) + \frac{h_2 - h_1}{2}\right\}} \tag{3}$$

In this model, the buyer's maximum stock level is equal to q^*.

3.2 CS model

In this model, the vendor uses buyer's warehouse for keeping the goods produced by the vendor without changing the ownership. To fulfill this concept, the vendor should be close to the buyer's production line so that the material may be picked up whenever needed. This creates a condition of the shared benefit, neither the vendor nor the buyer will benefit until the product is sold to an end user. This shared risk/shared benefit condition will often be enough to convince the buyer to stock the products. The key benefit to the buyer should be obvious, that he doesn't have to tie up his capital $h_{2\text{finance}}$. This doesn't mean that there is no inventory carrying costs for the buyer he does still incur costs $h_{2\text{stock}}$ related to storing and managing the inventory that is, both parties incur holding costs, depending on different rates and the length of time for which materials are stocked in a general model of the SC. Finally, the buyer sees a lower inventory cost per unit that is, only $h_{2\text{stock}}$ instead of the entire $h_{2\text{stock}}$ + $h_{2\text{finance}}$ further there is no longer any administrative cost per placing an order as in fact there is no longer any order. The vendor will have set-up cost and holding cost whereas the buyer will have order emission cost and holding cost.

The average total cost for this model is:

$$T_C^{\text{CS}} = \left(A_1 + nA_2\right)\frac{D}{nq} + h_2\left[\frac{Dq}{p} + nq\frac{(p-D)}{2p}\right] - \left(h_2 - h_1\right)\left[\frac{qD}{2p} + q\frac{(p-D)}{np}\right] \tag{4}$$

Differentiating Equation (4) with respect to 'q' to find the optimal ordering quantity q^*

$$q^* = \sqrt{\frac{\left(A_1 + nA_2\right)D/n}{h_2\left[(D/p) + (n(p-D)/2p)\right] - \left(h_2 - h_1\right)\left[(D/2p) + ((p-D)/np)\right]}} \tag{5}$$

Substituting q^* in Equation (4) which gives the optimum cost function as

$$T_C^{\text{CS}}\left(q^*\right) = 2\sqrt{\left[\left(A_1 + nA_2\right)\frac{D}{n}\right]\left[h_2\left\{\frac{D}{p} + \frac{n(p-D)}{2p}\right\} - \left(h_2 - h_1\right)\left(\frac{D}{2p} + \frac{p-D}{np}\right)\right]} \tag{6}$$

In this model, the buyer's maximum stock level is

$$B^{s}_{\max} = nq - (n-1)\frac{qD}{p} \tag{7}$$

3.3 CS-k^1 Model (number of delayed deliveries, $k^1 < n$)

The CS model may not be suitable for the limited (small) periods as the maximum level of the buyer's inventory may reach even for limited periods. Therefore, the CS model with late delivery periods (CS-k^1) is suitable for limited periods. In the CS-k^1 model, the last delivery is delayed until it reaches that there is no longer an increase in the maximum level already reached. This means, we have to delay the stock always whenever the maximum level inventory stock is reached. The average total cost in this model is

$$T_C^{\text{CS-}k^1} = \left(A_1 + nA_2\right)\frac{D}{nq} + h_2\left[\frac{Dq}{p} + \frac{nq(p-D)}{2p}\right] - \left(h_2 - h_1\right)\left[\frac{qD}{2p} + \frac{q(p-D)}{np}\frac{\left(k^1+1\right)k^1}{2}\right] \tag{8}$$

Differentiating Equation (8) with respect to 'q' to find the optimal ordering quantity q^*

$$q^* = \sqrt{\frac{\left(A_1 + nA_2\right)D/n}{h_2\left[(D/n) + (n(p-D)/2p)\right] - \left(h_2 - h_1\right)\left[(D/2p) + ((p-D)/np)\left(k^1\left(k^1+1\right)/2\right)\right]}} \tag{9}$$

Substitute q^* in Equation (8) to find the optimum cost:

$$T_C^{\text{cs-}k^1}\left(q^*, n, k^1\right) = 2\sqrt{\left[\left(A_1 + nA_2\right)\frac{D}{n}\right]\left[h_2\left\{\frac{D}{p} + \frac{n(p-D)}{2p}\right\} - \left(h_2 - h_1\right)\left\{\frac{D}{2p} + \frac{(p-D)}{np}\frac{(k^1+1)k^1}{2}\right\}\right]} \tag{10}$$

The maximum level of the buyer's stock $n \geq k^1$ is

$$B^{s}_{\max} = \left(n - k^1\right)q - \left(n - k^1 - 1\right)\frac{qD}{p} \tag{11}$$

Equation (11) ensures that not less than a single delay has been delayed. When $k^1 = 0$, Equation (11) becomes the maximum level of buyer's stock in basic CS model Equation (7) and when $k^1 = (n - 1)$, Equation (11) matches with maximum level of buyer's stock of Hill's model in which $B^{s}_{\max} = nq$. The late deliveries strategy is much explained in Simone and Robert (2004). They also provided a quick method for calculating the optimal total number of deliveries, the number of deliveries to be delayed and more emphasis made on the inventory holding cost of the vendor and the buyer.

3.4 Stochastic case

Hill's (1999) approach offers the lowest costs in deterministic environment. But most of the time, the business environment will run with uncertainty, which is unavoidable in the point of business strategy. In this situation, the CS obviously gives better results as it functions for the stochastic environment. The total costs in stochastic case model T_C^{SC} will be equal to those of the deterministic case cost, plus the safety stock holding costs when the demand during lead time is normally distributed.

3.5 CS-LT model

In this model, the inventory is reviewed continuously and shortages are allowed with fully backordered. It should be noted that the delivery lead time is null, however, the batch is to be produced, so that there exists a 'System Lead Time' other than zero. Adding an additional cost the lead time can be controlled. Thus, the lead time is crashed one at a time starting from the first component as it has the, minimum unit crashing cost per unit time, and then the second component and so on. The crashing expenses are made available to increase the lead time component. The length of lead time which ensures the order transit arrival even though the lead time is crashed and if any shortages are permitted and backordered. As the lead time is a decision variable in this model, the extra expenses incurred by the vendor will be fully transferred to the buyer if shortened lead time is requested. The most evident difference between Hill's model and the CS approach lies in the location of the stocks, which are preferably located in the vendor's warehouse in the first case, instead of the buyer's, as CS management implies. In the CS-k^1 model, the last delivery is delayed whenever the maximum level inventory is reached and in the CS-LT model, the lead time is crashed by adding an additional cost to minimise JTEC. The total transportation cost is ignored to minimise the complexity of the work.

The lead time crashing cost per cycle C_L

$$T_{C_L} = C_i\left[\left(L_{i-1} - L\right) + \sum_{j=1}^{i-1} C_j\left(b_j - a_j\right)\right] \tag{12}$$

$$L_i = L_0 - \sum_{j=1}^{i-1} C_j\left(b_j - a_j\right) \tag{13}$$

where L_i is the length of the lead time with components 1,2, …, i which is to be crashed to minimum duration and $L \in [L_i, L_{i-1}]$ for ith component has a normal duration 'b_i' and minimum duration 'a_i 'and the crashing cost per unit time 'c_i' (Table 1).

Table 1 Lead time crashing cost (Ouyang et al., 2004)

Lead time component, i	*Normal distribution b_i (days)*	*Minimum duration a_i (days)*	*Unit crashing cost C_i (\$/day)*
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

3.6 The optimal solution after crashing lead time in CS-LT model

The JTEC (q, R_n, L, n) of the buyer (ordering cost + holding cost + shortage cost when $X > R$ + lead time crashing cost) and the vendor (set-up cost + holding cost), can be written as

> JTEC (q, R_n, L, n) total expected cost of the buyer (q, R_n, L) + total expected cost of the vendor (q, R_n, L)

$$\text{JTEC}\left(q,R_n,L,n\right)=\left[\frac{A_2D}{q}+h_2\left(\frac{q}{2}+R_n-DL\right)+\frac{\pi D}{q}E\left(X-R_n\right)+\frac{D}{q}C_L\right] + \left[A_1\frac{D}{nq}+h_1\left(\frac{qD}{2p}+\frac{q(p-D)}{np}\right)\right] \quad (14)$$

$$\text{JTEC}\left(q,R_n,L,n\right)=\frac{D}{q}\left[A_2+\frac{A_1}{n}+\pi E\left(X-R_n\right)+C_L\right] + \frac{q}{2}\left[h_2+h_1\left\{\frac{D}{p}+\frac{2(p-D)}{np}\right\}\right]+h_2\left(R_n-DL\right) \quad (15)$$

Equation (15) is simplified using $G(n)$, $H(n)$. Where

$$G(n)=A_2+\frac{A_1}{n}$$

and

$$H(n)=\left[h_2+h_1\left\{\frac{D}{p}+\frac{2(p-D)}{np}\right\}\right]$$

then JTEC (q, R_n, L, n) can be written as

$$\text{JTEC }\left(q,R_n,L,n\right)=\frac{D}{q}\left[G(n)+\pi E\left(X-R_n\right)+C_L\right]+\frac{q}{2}H(n)+h_2\left(R_n-DL\right) \quad (16)$$

we assumed that the annual average lead time demand 'X' follows a normal distribution with finite mean D_L, Standard deviation of demand during the system lead time interval is $\sigma = \sigma_D\,(q/p)^{1/2}$ and the reorder point $R_n = D_L L + k\sigma\sqrt{L}$. Consequently, by considering the safety factor 'k' as a decision variable instead of 'R_n' and the expected shortage quantity $E(X-Rn)=\sigma\sqrt{L}\psi(k)$ substituting these in Equation (16) results:

$$\text{JTEC}_N(q,k,L,n)=\frac{D}{q}\left[G(n)+\pi\sigma\sqrt{L}\psi(k)+C(L)\right]+\frac{q}{2}H(n)+h_2k\sigma\sqrt{L} \quad (17)$$

Differentiating Equation (17) with respect to (q, k) and substituting we get

$$q=\sqrt{2D\frac{[G(n)+\pi\sigma\sqrt{L}\psi(k)+C(L)]}{H(n)}} \quad (18)$$

and

$$\phi(k)=1-\frac{qh_2}{\pi D} \quad (19)$$

Therefore, JTEC_N equation can be written as

$$\text{JTEC}_N(q,k,L,n)=\sqrt{2DH(n)[G(n)+\pi\sigma\sqrt{L}\psi(k)+C(L)]}+h_2k\sigma\sqrt{L} \quad (20)$$

The optimal value of ‘q’, ‘k’ for given ‘L’ and ‘n’ can be obtained if

$$\text{JTEC}_N(n^*-1) \geq \text{JTEC}_N(n^*) \leq \text{JTEC}_N\left(n^*+1\right)$$

For fixed integer ‘n’, Equation (17) is partially differentiated with respect to ‘q, k, L’ where $L \in (L_i, L_{i-1})$ while further for fixed (q, k, L, n), JTEC_N is a concave function in $L \in (L_i, L_{i-1})$. Hence, for fixed (q, k, n), the minimum JTEC_N per unit time will occur at the end points of interval (L_i, L_{i-1}). On the other hand, that for fixed ‘n’ and $L \in (L_i, L_{i-1})$ taking the second partial derivation of the JTEC_N (q, k, L, n) with respect to (q, k) is a convex function in (q, k). Thus for fixed ‘n’ and $L \in (L_i, L_{i-1})$, the minimum value of JTEC_N (q, k, L, n) will occur at the point (q, k) which satisfies $\sigma^2\text{JTEC}_N(q, k, L, n)/\partial q^2 > 0$ and $\sigma^2\text{JTEC}_N(q, k, L, n)/\partial^2 k > 0$ simultaneously and also $\phi(k) > 0$, $\psi(k) > 0$. Therefore, for fixed ‘n’ and $L \in (L_i, L_{i-1})$, JTEC_N (q, k, L, n) is a convex function in (q, k).

4 Algorithm to lead time control CS model

In this section, an iterative algorithm including the crashing expenses is presented. The algorithm is used to find the optimal solutions of ‘q’ and ‘k’ for given ‘L’ and ‘n’ by solving the Equations (18) and (19) iteratively until convergence is achieved.

Step 1 A set number of transport operations per production batch, let, $n = 1$.

Step 2 For each L_i, $i = 1, 2, 3, \ldots, n$ perform from 1 to 5 to evaluate the JTEC_N.

1. Start with $k_{i1} = 0$, $\psi(k) = \phi(k) - k[1 - \Phi(k)]$ that is, $\psi(k)$ is a *function of*
   [$\phi(k)$, $\Phi(k)$], where $\phi(k)$ is the normal probability density function and $\Phi(k)$ is the cumulative probability density function.
2. Substituting $\psi(k_{i1})$ value into Equation (18)

   $$q_{i1} = \sqrt{2D\frac{[G(n)+\pi\sigma\sqrt{L}\psi(k)+C(L)}{H(n)}}$$

   and determine $q_{i1.}$
3. Utilising q_{i1} and determine $\Phi(k_{i2})$ from equation $\phi(k) = 1 - q_{i1}h_2/\pi D$ then find k_{i2} by checking the standard normal table, and hence $\psi(k_{i2})$.
4. Repeat steps from 2 to 3 until no change occurs in the values of q_i and k_i.
5. Compute the corresponding $\text{JTEC}_N(q_i, k_i, L_i, n)$.

Step 3 Find $\min_{i=0,1,\ldots,n}$ of $\text{JTEC}_N(q_i, k_i, L_i, n)$. If $\text{JTEC}_N(q_n^*, k_n^*, L_n^*, n) = \min_{i=0,1,\cdots,n} \text{JTEC}_N(q_i, k_i, L_i, n)$, Then, (q_n^*, k_n^*, L_n^*, n) is the optimal solution for fixed n.

Step 4 Set $n = n + 1$, repeat steps 2 and 3 to get $\text{JTEC}_N\ (q_n^*,\ k_n^*,\ L_n^*,\ n)$.

Step 5 If $\text{JTEC}_N\ (q_n^*,\ k_n^*,\ L_n^*,\ n) \leq \text{JTEC}_N\ (q_{n-1}^*, k_{n-1}^*, L_{n-1}^*, n-1^*)$, then go to step 4, otherwise go to step 6.

Step 6 Set $\text{JTEC}_N\ (q_n^*,\ k_n^*,\ L_n^*,\ n) = \text{JTEC}_N\ (q_{n-1}^*, k_{n-1}^*, L_{n-1}^*, n-1)$ if so, then $(q_n^*,\ k_n^*,\ L_n^*,\ n^*)$ will be the optimal solution for the vendor and the buyer.

5 Numerical results

The input values to the above-mentioned algorithm are the same as the values reported in Ouyang et al. (1996, 2004) and Ouyang and Wu (1998): A_1 = \$1500/set-up, A_2 = \$200/order, h_1 = \$14/year, h_2 = \$20/year, P = 2000units/year, D = 600units/year, σ = 7units/week, π = \$50/unit and the crashing cost are given in Table 1. CS with a controllable lead time policy for various strategies has been evaluated and the results were given Tables 2–5 and shown in Figures 1–3.

Table 2 Output from algorithm

n	*Lead time L_n (days)*	*Reorder point R_n (days)*	*Service factor k*	*Ordering quantity q_n (units)*	*JTEC \$ (q_n, R_n, L_n, n)*
1	28	57	0.84	294	7226
2	28	61	1.12	184	6463
3	28	63*	1.31	141	6335*
4	28	65	1.16	116	6377

*minimum cost for a set of lead time and reorder point.

Table 3 JTEC_N (\$) of CS-LT for different lead times and transport operations

Lead time (days)	*n = 1*	*n = 2*	*n = 3*	*n = 4*
56	7275	6715	6645	6749
42	7253	6508	6378	6416
28*	7226	6463	6335*	6377
21	7393	6724	6660	6769

*minimum cost for a set of lead time and reorder point.

The JTEC_N of CS-LT model for the number of transport operations per production batch have been evaluated and given in Table 3 and the trend is shown in Figure 1. The JTEC_N is high for $n = 1$ at any lead time as compared to JTEC_N of $n = 2, 3, 4$. When buyer order quantity and lot sizes are equal (i.e. $n = 1$) the JTEC_N of CS-LT model is lower than the models of Banerjee (1986), Pan and Yang (2002) and Ouyang et al. (2004). This is true even at higher values of n (i.e. $n > 1$) (Table 4). In the CS-LT model, JTEC_N is always minimum for $n = 3$ at different lead times. It is found that lowest JTEC_N = \$6335 at lead time = 28 days. The JTEC_N increases when lead time increases from 30 to 50 days and it reaches almost steady state (Figure 1) after 50 days of lead time.

The algorithm procedure (Section 4) used for the CS-LT model yields the results as given in Table 3. From the iteration analysis, we obtain the optimal ordering quantity q^* = 141 units, lead time L^* = 28 days, a number of lots delivered from the vendor to the buyer n^* = 3, reorder point R_n^* = 63 units and the minimum $JTEC_N$ = $6335. Where as for the same input values, Ouyang et al. (2004) got minimum of $JTEC_N$ = $6660.4, q^*=144, R_n^*= 64 and L^*=28 days. To evaluate the performance of the CS-LT model, a detailed comparison of different strategies available in the literature has been given in Table 4.

Table 4 Detailed comparison of strategies performance

Variable	*n = 1*				*n > 1*			
	Banerjee (1986)	*Pan and Yang (2002)*	*Ouyang et al. (2004)*	*This model (CS-LT)*	*Goyal (1988)*	*Pan and Yang (2002)*	*Ouyang et al. (2004)*	*This model (CS-LT)*
Number of deliveries	1	1	1	1	2	3	3	3
Reorder point	–	63	58	57	–	79	64	63
Buyers ordering quantity	290	295	299	294	164	141	144	141
Vendor lot size	290	295	299	294	328	423	432	423
$JTEC_N$	$7948.9	$7708.9	$7466.7	$7226	$7875.1	$6814.3	$6660.4	$6335

Table 5 Comparison of Hill's, CS and CS-LT

Variable	*Hill's model*	*CS model*	*CS-LT model*
Optimal production batch size	415	345	420
Deliveries from vendor stock (q)	277	173	141
Vendor cost ($/year)	3814	2968	3806
Buyers cost ($/year)	2250	3631	2529
JTEC ($/year)	6064	6599	6335

The CS-LT model is further evaluated with Hill's and the CS models for $JTEC_N$ and it is found that Hill's model seek the lowest values. However, it is noted that the CS-LT model will take into account of stochastic demands against Hill's model. A detailed comparison of the CS-LT model obtained with Hill's, CS and CS-k^1 has been given in Table 5 and Figures 2 and 3. The buyer holding cost in the CS model is compared to the Hill's model and the CS-LT model. The reason is that in the CS model the buyer holds inventory for a longer period. The total cost in the CS-LT model occurs when the stock level varies between ≅ 125 and 150 (Figure 2). Hill's model total cost is less, but the number of transport operations per production batch will be one. More over, it is deterministic in its nature.

For zero safety stock, the CS-LT and the Hill's model total expenses are $6113 and $6066 (Figure 3). The CS-LT model and Hill's model is closely vary when safety stock range is 0 and 25. Hill's model and the CS-LT model costs coincide when safety stock is

25 and then after the CS-LT model cost will be less compared with Hill's for increasing safety stock (Figure 3). CS-k^1(k^1 = 3) model finds to give a lower inventory cost for all the ranges of safety stock but it is having a greater limitation is that the last delivery is delayed until it reaches that there is no longer an increase in the maximum level already reached, which is inconvenient in the stochastic environment. It can also be noted that the cost in CS model increases linearly and the starting point is above the CS-LT as well as Hill's model.

Figure 1 Total costs for different '*n*' and lead time values

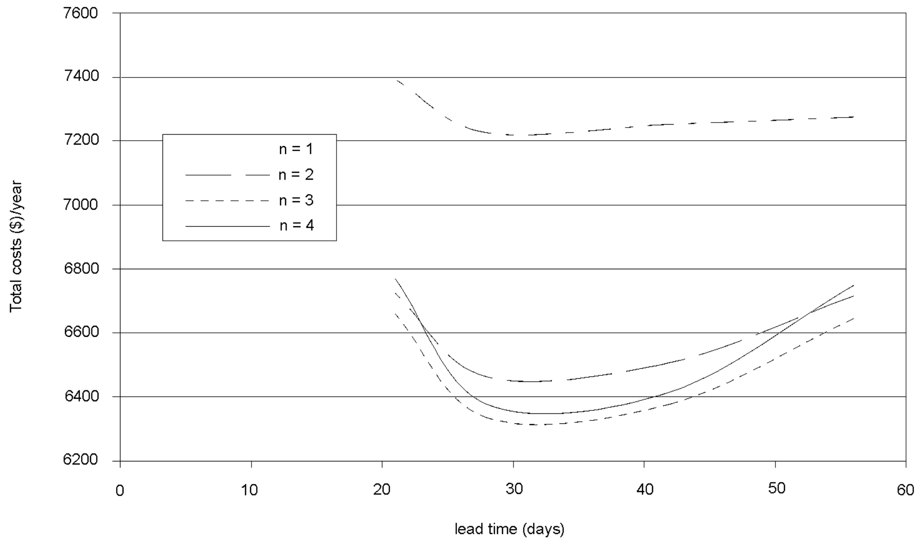


Figure 2 Total costs for different policies and stock levels

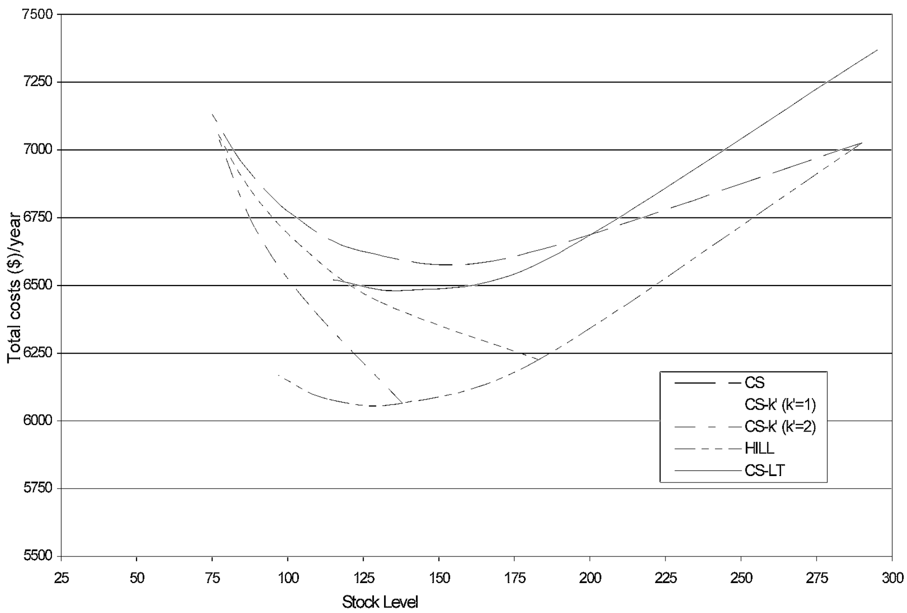
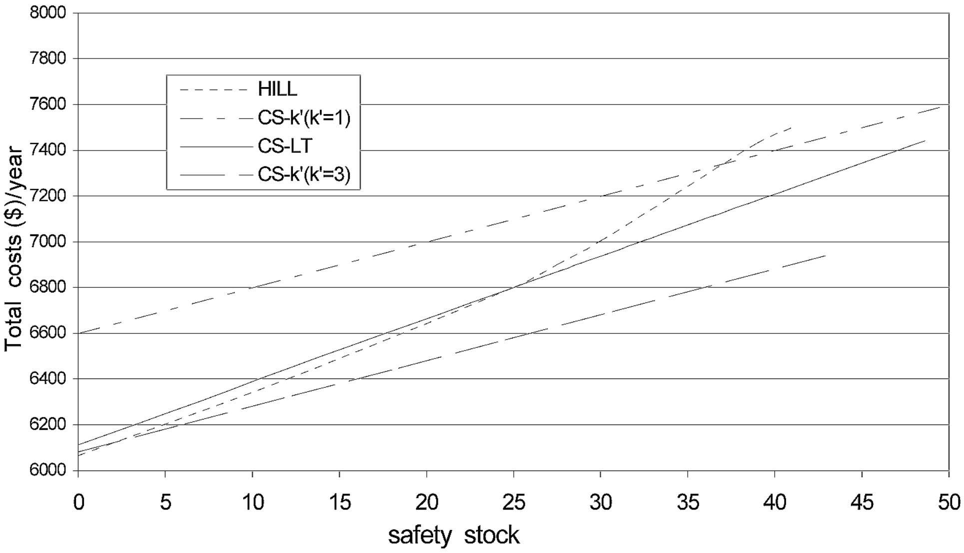


Figure 3 Total expenses with different safety stock quantity for Hills, CS and CS-LT models



6 Conclusions

This paper will provide a simple coherent framework of CS with the controllable lead time to determine JTEC for a single vendor–single buyer policy of SC network. Results obtained helped in understanding of the CS-LT mechanism and comparison was made with till now available models in the literature.

The CS-LT policy could be best suitable for low or reasonable price items and when the demand is stochastic in the nature. It is not suitable when demand is reasonably known and stable. Future studies have to be made in the area of CS with controllable lead times for the multibuyer–multivendor under the stochastic environment. Simulating these models will give effective results, which are used for understanding the various strategies. This study can also be extended to items such as perishable and a short-life electronic goods.

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