Optimisation of supply chains for single vendor–multibuyer consignment stock policy under controllable lead time using genetic algorithm

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Abstract: In this article, we develop a controllable-lead-time inventory model where the lead time is assumed to be dependent because at the time of contract with a manufacturer, the retailer may intend to reduce the lead time, for which he will pay an additional cost to accomplish an increased production rate. The lead time of Consignment Stock (CS) strategy has been controlled to minimise Joint Total Expected Cost (JTEC), and other decision variables such as quantity transported, lead time, number of transport operations and delay deliveries under stochastic environment have been simultaneously optimised so as to gain competitive advantage in the business strategy. Numerical examples and sensitivity analysis are presented to illustrate the solution procedure.

Keywords: Consignment Stock; CS; single vendor–multibuyer model; Genetic Algorithm; GA; Supply Chain; SC; stochastic demand; control lead time; crashing cost.

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1 Introduction

In today's globalised economy, businesses are looking for ways to optimise the Supply Chain (SC) network by means of the integration and cooperation of network echelons (Drucker, 1998; Douglas and Cooper, 2000). Inventory is one of the most widely discussed issues for improving SC efficiency. Wal-Mart and Procter and Gamble popularised it in the late 1980s. Since the holding of inventories in a SC can cost anywhere between 20% to 40% of product value, the effective management of inventory is critical in SC operations (Ballou, 1992). In this environment, Supply Chain Management (SCM) has become an effective business tool to reduce SC network inventory cost. Houlihan (1985) is credited with coining the term SC and having insightful concepts with a strong case for viewing the SC as a strategy for global business decisions. The SCM is generally viewed as a strategy for integrating network businesses that work together to source products and ultimately distribute products and services to the customer at the right quantities, right place and right time (Simchi-Levi *et al.*, 2000). Each echelon of the SC performs an independent business with integrated information sharing among all the echelons and holds some inventories, which may be unavoidable owing to existing uncertainty in the business.

In the area of inventory, an effective industrial approach that is quickly gaining ground is the Consignment Stock (CS), in which the vendor stocks his finished products in buyer's warehouse. The vendor will guarantee the quantity stored in the buyer warehouse, which will be kept between a minimum (s) and maximum (S) level with supporting shortages in stochastic demand and lead time. For single vendor–single buyer cases, the demand rate can be assumed to be consistent but this may be reversed in the case of multiple buyers, wherein variation in scope of demand and lead time is quite evident.

The most radical application of the CS approach leads to the minimisation of vendor inventory, as this party will use the buyer's warehouse to stock his finished products. The CS with single vendor–multibuyer model is viewed as a classification of divergent SC with end2multi-end case. CS is a combination of push and pull systems. The vendor adopts the push system whereas the buyer adopts the pull system. The change of ownership commences during the pull system. It is found in the literature that little research has been done on CS. The fundamentals of CS are explained in detail in Braglia and Zavanella (2003), Valentini and Zavanella (2003), Simone and Grubbstrom (2004), and Srinivas and Rao (2004). The CS policy is conveniently adopted for small-sized and less-cost items. Generally, it is best suited for automobile components (Braglia and Zavanella, 2003), fashion products, pharmaceuticals, electronics, Fast Moving Consumer Goods (FMCG), and retail items of super- and hypermarkets (Srinivas and Rao, 2004).

2 Literature review

Fisher (1997) and Chopra and Meindl (2001) argue that for 'functional' make-to-stock products, management should focus on reducing operating costs.

Corbett (2001) is credited with being the first person to give the fundamentals of CS policy, whereas Valentini and Zavanella (2003) presented an industrial case and performance analysis of CS policy for a single vendor–single buyer case. Braglia and Zavanella (2003) presented an analytical modelling approach which concerns the deterministic single vendor–single buyer case, allowing the analyst to identify the optimal inventory level and shipment policy for minimising total costs.

Piplani and Viswanathan (2003) discussed Supplier Owned Inventory (SOI), which possess the concepts of CS. They evaluated the performance of the policy and concluded that SOI arrangement is always beneficial for the SC as a whole. They showed that SOI would be beneficial to the buyer assuming that they continue to pay the same price to suppliers, but did not discuss its impact on suppliers and the Joint Total Economic Cost (JTEC) as a whole. Simone and Grubbstrom (2004) extended the work of Braglia and Zavanella (2003) by giving the explicit analytical expression of ordering quantity, number of shipments and delay deliveries in two cases: $h_v > h_b$ and $h_v < h_b$, which mean no delay and maximum delay respectively. In fact, in the practical application of the CS model, there will always be $h_v < h_b$, because of downstream movement of the product.

It is found from the literature that there will be considerable savings in JTEC when vendor and buyer cooperate with each other. In order to encourage the buyer to cooperate with the vendor, Goyal (1976) pointed out that a judicious method is essential for allocating costs.

Pan and Yang (2002) are credited with minimising the JTEC of a vendor's and buyer's inventory model with controllable lead time, which is a decision variable; however, shortages are not allowed in their paper. Venkateswaran and Son (2004) proposed strategies to verify the effectiveness of reduced lead time between vendor and buyer. Pan and Yang (2002) and Ryu and Lee (2003) analysed the effect of investment strategies to control lead times. Liao and Shyu (1991) decomposed lead time into 'n' components each having a different crashing cost for reduced lead time. The lead time is the only decision variable in their model. They assumed that the order quantity is predetermined. Ben-Daya and Raouf (1994) considered both lead time and order quantity as decision variables. Their model uses different representations of the relationship between lead time crashing cost and lead time. Ben-Daya and Raouf (1994) considered both lead time and order quantity as decision variables.

Ouyang *et al.* (2004) discussed an integrated vendor-buyer model with stochastic demand to integrate a production inventory model. Shortages are permitted and it is assumed that the lead time is controllable with added cost so as to optimise ordering quantity. Pan and Yang (2002), Ben-Daya and Raouf (1994) and Ouyang *et al.* (2004) considered only three lead-time components. In practical problems there may be many lead-time components within the control of the parties involved. It is in the interests of both parties involved to reduce the lead time as much as economically possible, by techniques such as work study (Goyal, 2003). Most of the published papers have assumed a deterministic environment. When demand during the cycle time is not deterministic but stochastic, the system lead times become important issues and their control leads to some quantitative benefits. The system lead time (Tersine, 1994) consists of order presentation, order transit, supplier lead time, delivery lead time and set-up time. Lead-time crashing facilitates lower lead time, and enables quick response and production line structuring. It also reduces inventories in the SC and improves the coordination between different stages of the network. For general problems, whenever the lead time reduces for either larger or smaller demands for immediate delivery, companies may face stock-out problems, but in the method proposed the stock-out is eliminated or minimised.

Persona *et al.* (2005) proposed an analytical model able to take into account the effects of obsolescence in a SC-based CS model. They used a deterministic single vendor–single buyer CS model as a basis to develop the model. The results showed that the presence of obsolescence reduces the optimal inventory level, specifically for short-life components.

Recently, Srinivas and Rao (2004) extended and analysed the models proposed by Braglia and Zavanella (2003) and Ouyang *et al.* (2004) for single vendor–single buyer inventory models, with emphasis on crashing lead time. Their model suggests that CS with stochastic lead-time reduction yields less JTEC. The literature review papers of Aytug *et al.* (2003) and Chaudhury and Luo (2005) reveal that no attempt has been made to develop a heuristic method such as Genetic Algorithm (GA) to determine inventory levels in SC echelons. The recent paper of Daniel and Rajendran (2005) studied GA, enumeration and Random Search Procedure (RSP) methods for single-product serial SC operating with a base-stock periodic review system to optimise the base-stock inventory levels in the SC so as to minimise the total SC cost, comprising holding and shortage costs at all the installations in the SC. They found that the solution generated by the proposed GA was not significantly different from the optimal solution yielded by complete enumeration, but it is significantly good for deterministic replenishment lead times and the other solution for random replenishment lead times. They did not check for multibuyer stochastic demand and lead-time models.

This paper addresses the problem of CS in SCs to minimise JTEC for a single product, single vendor–multibuyer model. It is an extension of Srinivas and Rao (2004). To simplify the analysis, we have assumed that there is only one entity per tier. In Srinivas and Rao (2004), the authors used both enumeration techniques as well as GA. The former takes up more CPU time (more than a couple of hours) for more than three buyers with five process variables and the latter method takes less than 20 sec. for all models. Hence we have restricted ourselves to the GA method and applied this mode up to ten buyers. Goyal (1974) proposed an enumerative procedure, which requires substantial computational effort to produce an optimal solution. The running time of this procedure grows exponentially with the number of items. However, a heuristic procedure that requires less computation can be adopted successfully. The enumeration technique generally will have to search for the optimal solution in open space.

The most attractive feature of GA (Gen and Cheng, 2000) is its flexibility in handling objective functions with minimal requirements for fine mathematical properties and its ability to deal with real-life problems.

2.1 Notations and assumptions

The necessary notations used in this paper are summarised as follows:

A_v batch set-up cost (\$) (vendor)

A_b order emission cost (\$) (buyer)

h_v vendor stock holding cost ($) per unit per unit time

h_b buyer stock holding cost ($) per unit per unit time

p vendor production rate (continuous)

d_i demand rate in units per unit time seen by the buyer (continuous)

σ standard deviation of demand/unit time

π unit back-order cost ($) for the buyer

L length of the lead time for the buyer

C_L lead-time crashing cost ($) per cycle

k^1 delay deliveries ($\leq$ n)

ϕ normal probability density function

Φ cumulative distribution function

n number of transport operations/production batch

m_i delayed deliveries shifted to another buyer ($\leq$ k)

j_{ij} delivery shifted from *i*-th buyer to *j*-th buyer, $\Sigma J_{ij} = m_i$.

To develop the proposed models the following assumptions are used:

- single-product flow (one set-up for each vendor) with continuous review of inventory replenishment system over an infinite horizon for single vendor–multiple buyers
- buyer and vendor carrying cost is independent of quantity transported but proportional to the holding time
- the demand rate and the delivery lead time for each buyer are continuous variables with known, stationary probability distributions
- shortages during the lead time are permitted on the basis of fixed cost
- demand is normally distributed and there is no order splitting
- $\sum_{i=1}^{y} d_i < \textit{production rate} \ngtr \textit{production capacity}$ (*i.e.*, infinite capacity)
- if demand exceeds on-hand inventory, the situation is considered as shortage
- $\mathrm{h_{bi}} > \mathrm{h_v}, \frac{p}{n_i} \geq \sum \frac{d_i}{n_i}$ and $\mathrm{n_i} \geq 1 \ \forall \ \mathrm{b_i}$
- Production is organised in such a way that the first shipment for each buyer is done in sequence. Following this sequence, the first delivery starts with the first buyer followed by the second, the third and so on. The duration from one delivery to the next is fixed for each buyer.

3 Genetic algorithm: an introduction

We propose a Genetic Algorithm (GA) approach to optimise the CS-based inventory models' JTEC in an SC. This study attempts to perform both performance analysis and optimisation, *i.e.*, various inventory policy settings yielded by the GA are evaluated. GA is a class of evolutionary algorithms that utilise the theories of evolution and natural selection. GA begins with a population of randomly generated strings that represent the problems' possible solutions. Thereafter, each of these strings is evaluated to find its fitness. The initial population is subjected to genetic evolution to procreate the next generation of candidate solutions (Goldberg, 1989). The members of the population are processed by the four main GA operators – reproduction, crossover, mutation and inversion – to create the progenies for the next generation of candidate solutions. The progenies are then evaluated and tested for termination until a satisfactory solution (based on the acceptability or search stoppage criterion) is found; then the search is stopped.

3.1 Working mechanism of GA

Schematic working principle of GA is shown in Figure 1 and the GA consists of six steps, those are:

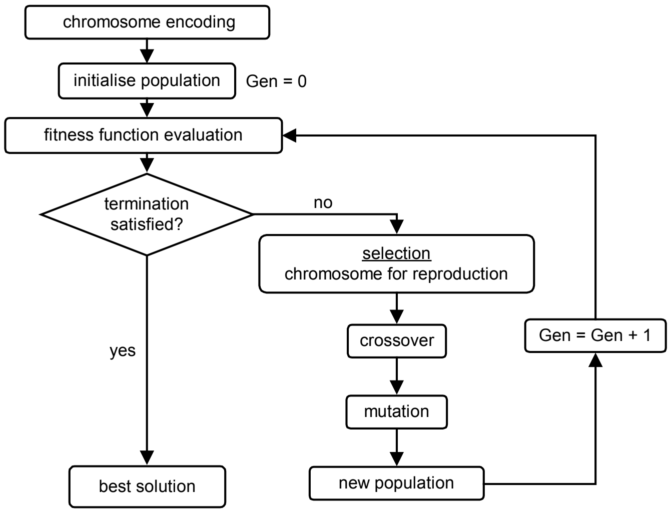
1. Initialise a set of feasible solutions (*i.e.*, initialise a population of chromosomes) randomly.
2. Compute the fitness value $f_t = \dfrac{1}{1 + JTEC_{(n,k^1,c,L)}}$ for every chromosome in the population.
3. Select chromosomes for reproduction by making use of the roulette wheel selection procedure and fitness function value.
4. Apply crossover and mutation on the selected chromosomes to produce new chromosomes.
5. Form next-generation population.
6. If the stopping condition is reached, return to the best solution; if not, go to 2.

GA works on a population or collection of solutions to the given problem. Each individual in the population is called a chromosome. Designing chromosomes is a very important step in GA, which contains decision variables that are to be optimised. The chromosome structures for various models are summarised below:

- Basic CS model $(c, n_i, \Phi_{(z)})$
- CS with delay model $(c, n_i, k_i, \Phi_{(z)})$
- CS with information sharing and delay model $(c, n_i, k_i, m_i, j_{ij}, \Phi_{(z)})$
- CS-LT model $(c, n_i, L_i, \Phi_{(z)})$.

Integer coding is used for *n*, *k*, *m* and *j*, whereas for *c* and $\Phi_{(z)}$ the range is large; therefore binary coding has been considered for these two variables. For converting into binary coding, first multiply with 1000 to remove the decimal point and then convert to binary coding. The population size is fixed at 150–190; crossover rate and mutation rate for the proposed GA are fixed by conducting a pilot study with different combinations of probability of crossover (pc) from 0.7 to 0.8 and probability mutation (pm) 0.05 with respect to four different CS policies. The number of generations is fixed at 500. Crossover is known as 'recombination'; it exchanges information among the strings present in the mating pool and creates new strings. In crossover, two strings are picked from the mating pool and some portions of these strings are exchanged between them.

Figure 1 GA principle schematic flowchart



A crossover operator attempts to produce new strings of superior fitness by effecting large changes in a string in search of the optimum in the solution space. The need for a local search around a current solution also exists and is accomplished by mutation. Mutation is additionally aimed at maintaining diversity in the population. Mutation creates a new solution in the neighbourhood of a current solution by introducing a small change in some aspect of the current solution and helps to ensure that no point in the search space has a zero probability of being examined. The commonly used mutation operator is swap mutation. For binary coding, a normal swap mutation operator is used. All bits in binary number are mutated with pm = 0.05 and a uniform random number between 0 and 1 is generated. If the number is less than the probability of mutation, then that bit is changed from 0 to 1 or vice versa. For integers, all genes in the parent population are mutated with pm = 0.05, by sampling a uniform random number, u. If $u \leq$ mutation rate, then the value of the gene is altered as given below:

$$S_{new} = S_{old}(1-x) + 2xuS_{old}$$

where:

S_{new} = is the new gene after mutation
S_{old} = is the gene before mutation
u = is a uniform random number between 0 and 1
x = denotes the fraction of S_{old}.

It is to be noted that if the computed S_{new} takes a noninteger value, then it is rounded off to the nearest integer. In this study 'x' is set to 0.2. The same repair function as discussed in crossover is used for damaged genes after mutation. After crossover and mutation, the new population is called child population. We now have N chromosomes in the initial population and N chromosomes in the parent population. The best N chromosomes, among the 2N chromosomes in the initial and parent population put together, with respect to JTEC are chosen for entry into the parent population as the surviving chromosomes for the next generation.

An example of a chromosome for the one vendor–four buyers case for the CS with delay model is:

	c	n_1	n_2	n_3	n_4	k_1	k_2	k_3	k_4	$\Phi_{(z)}$
1st parent chromosome:	0.112	5	6	4	2	3	5	1	1	0.980
2nd parent chromosome:	0.122	8	4	5	6	7	2	2	2	0.862

parent strings before crossover:

binary coding	integer coding	binary coding
1 1 | 1 0 0 | 0 0	5 6 4 | 2 3 5 1 | 1	1 | 1 1 1 0 1 0 1 | 1 0 0
1 1 | 1 1 0 | 1 0	8 4 5 | 6 7 2 2 | 2	0 | 1 1 0 1 0 1 0 | 0 1 0

offsprings after two point crossover operator:

1 1 1 1 0 0 0	5 6 4 6 7 2 2 1	0 1 1 0 1 0 1 0 1 0 0
1 1 1 0 0 1 0	8 4 5 2 3 5 1 2	0 1 1 1 0 1 0 1 0 1 0

after decoding:

c	n_1	n_2	n_3	n_4	k_1	k_2	k_3	k_4	$\Phi_{(z)}$
0.120	5	6	4	6	7	2	2	1	0.852
0.114	8	4	5	2	3	5	1	2	0.938

Variables in the offspring after crossover may cross the permissible independent boundary ranges. They are found in the above chromosome $n_i < k_i$, but it should be $n_i \geq k_i$. Therefore a repair function is to be devised to correct these defective chromosomes.

offsprings after repair:

1st child chromosome:	0.120	5	6	4	6	3	2	2	1	0.852
2nd child chromosome:	0.114	8	4	5	2	3	2	1	2	0.938

3.2 Optimum GA parameters

Population size, number of generations, probability of crossover and probability of mutation are the GA parameters. A large population size means a better exploration of the search space, while a large number of generations allows for better exploitation of the promising solutions found. Generally, the larger these parameters are, the better the algorithm will perform, but at the expense of longer run-times, since more fitness evaluations will be involved. Population size is fixed at 150 after experimentation. The termination criterion used is the number of generations that are fixed at 500. The probability of crossover is varied from 0.5 to 1 with a step of 0.1; the optimum value found of 0.7 pm is varied from 0.05 to 0.15 in steps of 0.05; and finally 0.05 is fixed, as it gives optimum cost.

4 Vendor-buyer inventory models

4.1 Consignment stock model

In this model, the vendor uses the buyer's warehouse for keeping the goods produced by the vendor without changing the ownership. To fulfil this concept, the vendor should be close to the buyer production line. This creates a condition of shared benefit; neither the vendor nor the buyer will benefit until the product is sold to an end user. This shared risk-benefit condition will often be enough to convince the buyer to stock the products. The key benefit to the buyer should be obvious – that he/she does not have to tie up his/her capital $h_{b,\ finance}$. This does not mean that there are no inventory carrying costs for the buyer, as he still incurs costs $h_{b,\ stock}$ related to storing and managing the inventory, *i.e.*, both parties incur holding costs, depending on different rates and the length of time for which materials are stocked in SC. Finally, the buyer sees a lower inventory cost per unit, *i.e.*, only $h_{b,\ stock}$ instead of the entire $h_{b,\ stock} + h_{b,\ finance}$. Further, there is no longer any administrative cost per placement of an order. The vendor will have set-up costs and holding costs, whereas the buyer will have order emission costs and holding costs.

The average total cost for this model is:

$$\begin{aligned} T_C^{CS} &= \text{vendor set-up cost} + \text{average vendor holding cost} + \text{buyer ordering cost} \\ &\quad + \text{average buyer holding cost} + \text{safety stock cost} + \text{shortage cost} \end{aligned}$$

$$\begin{aligned} T_C^{CS} &= \frac{s}{c} + h_v \frac{c}{2p}\left(\sum_{i=1}^{y} \frac{D_i^2}{n_i}\right) + \frac{1}{c}\left(\sum_{i=1}^{y} A_i + \sum_{i=1}^{y} n_i A_{ti}\right) \\ &\quad + \frac{h_{b_i}}{2}\left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{i \neq j} \frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\} \\ &\quad + h_{b_i} z \sigma_i \sqrt{L_i} + \frac{1}{c}\sum_{i=1}^{y} \pi_i \sigma_i \sqrt{L_i}\,\Psi(z) \end{aligned} \tag{1}$$

Equation (1) is modified as:

$$\begin{aligned} T_C^{CS} &= \frac{1}{c}\left(s + \sum_{i=1}^{y} A_i + \sum_{i=1}^{y} n_i A_{ti}\right) + h_v \frac{c}{2p}\left(\sum_{i=1}^{y} \frac{D_i^2}{n_i}\right) \\ &\quad + \sum_{i=1}^{y}\left(\frac{h_{b_i}}{2}\left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{i \neq j} \frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right) \\ &\quad + \sum_{i=1}^{y}\left(h_{b_i} z \sigma_i \sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\,\Psi(z)\right) \end{aligned} \tag{2}$$

Equation (2) is written as:

$$T_C^{CS} = \frac{1}{c}G(n) + H(n)c + \sum_{i=1}^{y}\left(h_{b_i} z\sigma_i \sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\Psi(z)\right) \quad (3)$$

where:

$$G(n) = s + \sum_{i=1}^{y}(A_i + n_i A_{ti})$$

$$H(n) = h_v \frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{h_{b_i}}{2}\left\{D_i - (n_i - 1)D_i\left[\frac{D_i}{n_i p} + \sum_{i \neq j}\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right)$$

The minimum cost for optimum values of (c, n and z) will be:

$$T_C^{CS} = \left\{\left(G(n) + \sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\Psi(z)\right)\right)H(n)\right\}^{\frac{1}{2}} + \sum_{i=1}^{y}\left(h_{b_i} z\sigma_i \sqrt{L_i}\right) \quad (4)$$

The maximum level of inventory for buyer i is:

$$b_{i\max} = \left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{j \neq i}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\} + z\sigma_i \sqrt{L_i}. \quad (5)$$

4.2 CS-k^1 model (number of delayed deliveries, $k^1 < n$)

The CS model is not suitable for limited/small periods because the maximum level of the buyer's inventory may be reached even within limited periods. Hence the CS model with delayed delivery period (CS-k^1) is preferred for limited periods. In the CS-k^1 model, the last delivery is delayed until it happens that there is no longer an increase in the level reached. That means we always have to delay the stock whenever the maximum level of inventory stock is reached. The average joint total cost in this model is:

$$T_C^{CS-k^1} = \frac{1}{c}G(n) + H(n)c + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\Psi(z)\right) + \sum_{i=1}^{y}\left(h_{b_i} z\sigma_i \sqrt{L_i}\right) \quad (6)$$

where:

$$G(n) = s + \sum_{i=1}^{y}(A_i + n_i A_{ti})$$

$$H(n) = h_v\left\{\frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{D_i}{n_i}\frac{(p - D_i)}{n_i p}\frac{(k_i^1 + 1)}{2}k_i^1\right)\right\}$$
$$+ \sum_{i=1}^{y}\left\{\frac{h_{b_i}}{2}\left\{(n_i - k_i^1)\frac{D_i}{n_i} - (n_i - k_i^1 - 1)D_i\left[\frac{D_i}{n_i p} + \sum_{j \neq i}\left(\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\}\right\}.$$

The minimum cost for optimum values of (c, n, k^1 and z) will be:

$$T_C^{CS-k^1} = \left\{ \left(G(n) + \left[\sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_i} \Psi(z) \right) \right] \right) H(n) \right\}^{\frac{1}{2}} + \sum_{i=1}^{y} \left(h_{b_i} z \sigma_i \sqrt{L_i} \right). \tag{7}$$

The maximum inventory level for buyer *i* is:

$$b_{i\max} = \left\{ (n_i - k_i^1) \frac{D_i c}{n_i} - (n_i - k_i^1 - 1) D_i \left[\frac{D_i c}{n_i p} + \sum_{j \neq i} \left(\frac{D_j c}{n_j p} \left(\frac{n_j}{n_i} \right) \right) \right] \right\} + z \sigma_i \sqrt{L_i}. \tag{8}$$

Equation (8) ensures that not less than a single delay has been delayed. When $k^1 = 0$, Equation (8) becomes the maximum level of the buyer's stock in the basic CS model (Equation (7)), and when $k^1 = (n - 1)$, Equation (8) matches with the maximum level of the buyer's stock of Hill (1999) model, in which $b_{\max}^{hill} = nq$. The delay-deliveries strategy is much explained in Simone and Grubbstrom (2004). They also provided a quick method for calculating the optimal total number of deliveries and number of deliveries to be delayed and gave more emphasis on inventory holding costs of the vendor and buyer.

4.3 CS with information sharing and delay

Goyal (1976) is credited as the first person to describe integrated models of single vendor–single buyer. Goyal (1977) proposed a Joint Economic Lot Size (JELS) model to minimise total relevant costs, which is compared with total costs incurred if vendor and buyer act independently. Banerjee (1986) generalised Goyal's (1977) model by assuming the vendor with finite rate produces for a buyer on a lot-for-lot basis under deterministic conditions. Goyal (1988) generalised the Banerjee (1986) model by relaxing the assumption of the lot-for-lot policy of the vendor. In an integrated inventory model, one partner's gain exceeds the other partner's loss. Therefore, the net benefit can be shared in some equitable fashion (Goyal and Gupta, 1989). They also summarised the literature on integrated vendor-buyer models.

The model of consignment stock with partial information sharing includes information on demand, shipments and inventory. In SCM one of the most well-known problems is the Bullwhip effect. It can be controlled with partial information sharing. It is known that partial information sharing benefits the vendor more than the buyer owing to a reduction in vendor inventory and also to adjusted shipments between buyers; otherwise the vendor may have to keep the inventory (see Section 5, Table 2). In this view the SC is constructed in such a way that if the buyer does not need a particular scheduled delivery lot, the vendor finds an alternative buyer in the SC network. To fulfil this, the vendor adjusts the exact delivery quantity required by the alternative buyer, *i.e.*, the shifted quantity should be equal to the scheduled quantity of the alternative buyer.

The average total cost in this model is:

$$T_C^{IS} = \frac{1}{c} G(n) + H(n)c + \sum_{i=1}^{y} \left(h_{b_i} z \sigma_i \sqrt{L_i} \right) + \frac{1}{c} \sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_i} \Psi(z) \right) \tag{9}$$

where:

$$G(n) = s + \sum_{i=1}^{y} (A_i + n_i A_{ti}).$$

$$H(n) = h_v \left\{ \frac{1}{2p} \left[\sum_{i=1}^{y} \frac{D_i^2}{n_i} \right] + \sum_{i=1}^{y} \left(\frac{D_i}{n_i} \frac{(p - D_i)}{n_i p} \frac{(k_i^1 - m_i + 1)}{2} (k_i^1 - m_i) \right) \right\}$$
$$+ \sum_{i=1}^{y} \left\{ \frac{h_{bi}}{2} \left\{ \left(n_i - k_i^1 + \sum_{j \neq i} j_{ij} \right) \frac{D_i}{n_i} - \left(n_i - k_i^1 - 1 + \sum_{j \neq i} j_{ij} \right) D_i \left[\frac{D_i}{n_i p} + \sum_{j \neq i} \left(\frac{D_j}{n_j p} \left(\frac{n_j}{n_i} \right) \right) \right] \right\} \right\}.$$

The optimum value of c and $\Phi(z)$ is:

$$c = \sqrt{\frac{G(n) + \sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_i} \Psi(z) \right)}{H(n)}} \tag{10}$$

$$\Phi(z) = 1 - \frac{c \left[\sum_{i=1}^{y} \left(h_{b_i} z \sigma_i \sqrt{L_i} \right) \right]}{\left[\sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_i} \Psi(z) \right) \right]}. \tag{11}$$

From Equation (9) the minimum cost for optimum values (n, k^1, m) is calculated after differentiating as follows:

$$T_C^{IS} = \left\{ \left(G(n) + \left[\sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_i} \Psi(z) \right) \right] \right) H(n) \right\}^{\frac{1}{2}} + \sum_{i=1}^{y} \left(h_{b_i} z \sigma_i \sqrt{L_i} \right). \tag{12}$$

The maximum level of inventory for buyer i is:

$$b_{i\max} = \left\{ \left(n_i - k_i^1 + \sum_{j \neq i} j_{ij} \right) \frac{D_i c}{n_i} - \left(n_i - k_i^1 - 1 + \sum_{j \neq i} j_{ij} \right) D_i \left[\frac{D_i c}{n_i p} + \sum_{j \neq i} \left(\frac{D_j c}{n_j p} \left(\frac{n_j}{n_i} \right) \right) \right] \right\} + z \sigma_i \sqrt{L_i}. \tag{13}$$

4.4 CS-LT model

In this model, the vendor will closely negotiate with a buyer to reduce lead time as much as possible, down to a point where it is acceptable to the buyer with his stable production and delivery schedule. The inventory is reviewed continuously and shortages are allowed with fully backordered. It should be noted that the delivery lead time is null, however the batch is to be produced, so that there exists a ‘system lead time’ other than zero. By adding an additional cost, the lead time can be controlled. Thus the system lead time is drastically reduced one at a time starting from the first independent component because it has minimum unit crashing cost per unit time, and then the second independent component, and so on. It is clear that when lead time is reduced, its corresponding handling cost for that time is reduced. The length of lead time which ensures the order transit arrival even though lead time is crashed and shortages if any are permitted. Since lead time is a decision variable in this model, the extra costs incurred by the vendor will be fully transferred to the buyer if the shortened lead time requested can be viewed as an investment.

The lead time crashing cost per cycle C_L is:

$$T_{CL} = C_i(L_{i-1} - L) + \sum_{j=1}^{i-1} C_j(b_j - a_j) \tag{14}$$

$$L_i = L_0 - \sum_{j=1}^{i-1} C_j(b_j - a_j) \tag{15}$$

where:

$$L_0 = \sum\nolimits_{i=1}^{n} b_j$$

and L_i is the length of the lead time with components 1,2..i which is to be crashed to minimum duration, and $L \in [L_i, L_{i-1}]$ for the *i*-th component has a normal duration 'b_i' and minimum duration 'a_i' and crashing cost per unit time 'c_i', such that $c_1 \leq c_2 \leq \ldots \leq c_n$ (Table 1).

$$T_C^{CS-LT} = \frac{1}{c} G(n) + H(n)c + \sum_{i=1}^{y}\left(h_{b_i} z\sigma_i \sqrt{L_i}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\,\Psi(z)\right) + \frac{1}{c}\sum_{i=1}^{y} C_{Li} \tag{16}$$

where:

$$G(n) = s + \sum_{i=1}^{y} A_i + \sum_{i=1}^{y} n_i A_{ti}$$

$$H(n) = h_v \frac{1}{2p}\left[\sum_{i=1}^{y} \frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{h_{b_i}}{2}\left\{D_i - (n_i - 1)D_i\left[\frac{D_i}{n_i p} + \sum_{i \neq j} \frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right).$$

$$c = \sqrt{\frac{G(n) + \sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\,\Psi(z)\right) + \sum_{i=1}^{y} C_{Li}}{H(n)}} \tag{17}$$

$$\Phi(z) = 1 - \frac{c\left[\sum_{i=1}^{y}\left(h_{b_i} \sigma_i \sqrt{L_i}\right)\right]}{\sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\right)}. \tag{18}$$

The minimum cost for optimum values of (c, n, L, z) is:

$$T_C^{CS-LT} = \left\{\left(G(n) + \sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_i}\,\Psi(z)\right) + \sum_{i=1}^{y} C_{Li}\right) H(n)\right\}^{\frac{1}{2}} + \sum_{i=1}^{y}\left(h_{b_i} z\sigma_i \sqrt{L_i}\right). \tag{19}$$

The maximum level of inventory for buyer *i* is:

$$b_{i\max} = \left\{D_i c - (n_i - 1)D_i\left[\frac{D_i c}{n_i p} + \sum_{j \neq i} \frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\} + z\sigma_i \sqrt{L_i}. \tag{20}$$

Table 1 Lead-time crashing cost

Lead-time component, i	*Leading time (days)*	$(b_i - a_i)$ *days*	*Unit crashing cost* C_i *($/day)*	*Total crashing cost ($)*
1	14	0	0	0
2	10.5	3.5	0.4	1.4
3	7	3.5	1.2	5.6
4	5.25	1.75	5.0	14.35

4.5 Algorithm for lead-time control CS model

In this section, an iterative algorithm (single vendor–two buyers) that includes the crashing expenses is presented to find the minimum JTEC with optimal decision variables:

Step 1 Set $n_{1,2} = 1$.

Step 2 For each $L_{1,2}$ perform Steps (a) to (e):

- a Start with z = 0 (implies $\Psi(z) = 0{:}39894$; which can be obtained by checking the standard normal table $\phi(z) = 0.39894$ and $\Phi(z) = 0.5$).
- b Substitute $\Psi(z)$ into Equation (17) to evaluate c.
- c Using c, determine $\Phi(z)$ from Equation (18), then find z for the next iteration by checking the standard normal table, and hence $\Psi(z)$ for the next iteration.
- d Repeat (b) to (c) until no change occurs in the values of c and z.
- e Find corresponding min. $\text{JTEC}(c, n_2, n_1, L_1, L_2, z) = \text{JTEC}(c^*, n_2, n_1, L_1, L^*_2, z^*)$.

Step 3 For each L_1, L_2, repeat Step (a) to (e) to get $\text{JTEC}(c^*, n_2, n_1, L_1^*, L^*_2, z^*)$.

Step 4 If $\text{JTEC}(c^*, n_2, n_1, L_1, L^*_2, z^*) \leq \text{JTEC}(c_{L1-1}{}^*, n_{2(L1-1)}, n_{1(L1-1)}, L_1-1, L^*_{2(L1-1)}, z_{(L1-1)}{}^*)$, then go to Step 3, otherwise go to Step 5.

Step 5 Set $\text{JTEC}(c^*, n_2, n_1, L^*_1, L^*_2, z^*) = \text{JTEC}(c_{L1-1}{}^*, n_{2(L1-1)}, n_{1(L1-1)}, L_1-1, L^*_{2(L1-1)}, z_{(L1-1)}{}^*)$.

Step 6 Set $n_2 = n_2 + 1$; repeat Steps 2 to 5 to get $\text{JTEC}(c^*, n_2, n_1, L^*_1, L^*_2, z^*)$.

Step 7 If $\text{JTEC}(c^*, n_2, n_1, L^*_1, L^*_2, z^*) \leq \text{JTEC}(c_{n2-1}{}^*, n_{2(n2-1)}, n_{1(n2-1)}, L^*_{1(n2-1)}, L^*_{2(n2-1)}, z_{(n2-1)}{}^*)$, then go to Step 6, otherwise go to Step 8.

Step 8 Set $\text{JTEC}(c^*, n^*_2, n_1, L^*_1, L^*_2, z^*) = \text{JTEC}(c_{n2-1}{}^*, n_{2(n2-1)}, n_{1(n2-1)}, L^*_{1(n2-1)}, L^*_{2(n2-1)}, z_{(n2-1)}{}^*)$.

Step 9 Set $n_1 = n_1 + 1$; repeat Steps 2 to 7 to get $\text{JTEC}(c^*, n^*_2, n_1, L^*_1, L^*_2, z^*)$.

Step 10 If $\text{JTEC}(c^*, n^*_2, n_1, L^*_1, L^*_2, z^*) \leq \text{JTEC}(c_{n1-1}{}^*, n_{2(n1-1)}, n_{1(n1-1)}, L^*_{1(n1-1)}, L^*_{2(n1-1)}, z_{(n1-1)}{}^*)$, then go to Step 9, otherwise go to Step 11.

Step 11 Set $\text{JTEC}(c_{n1-1}{}^*, n^*_{2(n1-1)}, n_{1(n1-1)}, L^*_{1(n1-1)}, L^*_{2(n1-1)}, z_{(n1-1)}{}^*) = \text{JTEC}(c^*, n^*_2, n^*_1, L^*_1, L^*_2, z^*)$; then $(c^*, n^*_2, n^*_1, L^*_1, L^*_2, z^*)$ is the optimal solution.

5 Numerical results

The input values to all the models discussed refer to Ben-Daya and Raouf (1994), Braglia and Zavanella, (2003), Ouyang *et al.* (2004) and Srinivas and Rao (2004): h_v = \$4per unit/year, h_{b_i} = \$5per unit/year, d_i(units/year) = 1000, 1300, p/$\sum d_i$ ratio = 3.2, σ_i = 44.72, 50, A_v = \$400/set up, A_b = \$25/order, π = \$50/unit. The input data is extended to ten buyers with $d_{i=3,4,10}$ = 800, 1000, 1500, 600, 1200, 1500, 1000, 800 and $\sigma_{i=3,4,10}$ = 35.7, 30, 30, 20, 30, 30, 30, 20. A brief summary of results is given in Table 2.

Table 2 Summary of results of up to ten buyers with a single vendor

		Buyer's size								
Variable	*Model*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
JTEC (\$) for (Σb_i)	CS	7106	9683	11 476	13 900	15 274	17 326	19 760	21 780	23 704
	CS-k^1	6855	8908	11 279	13 743	15 150	17 355	19 707	21 767	23 231
	CS-IS-k^1	6545	8544	10 841	13 366	13 651	16 864	19 200	21 325	22 497
	CS-LT	5703	7392	9049	10 995	12 077	13 779	15 616	17 173	18 331
S max. (buyer max. stock)	CS	690	603	564	548	503	489	489	480	459
	CS-k^1	569	485	491	490	456	447	457	459	462
	CS-IS-k^1	593	510	473	511	442	462	455	456	433
	CS-LT	551	480	443	433	389	382	385	372	363
S min. (buyer min. stock)	CS	351	327	316	314	291	289	291	292	283
	CS-k^1	334	309	304	304	282	281	285	285	283
	CS-IS-k^1	317	305	292	295	256	270	277	277	262
	CS-LT	207	194	187	182	172	170	171	170	165
No. of shipments	CS	6	7	8	10	11	13	14	13	13
	CS-k^1	10	12	12	14	17	18	19	19	16
	CS-IS-k^1	12	13	14	17	23	20	22	22	28
	CS-LT	6	7	8	10	10	12	12	12	12
Delay deliveries	CS-k^1	4	6	5	4	4	5	4	3	1
	CS-IS-k^1	6	7	6	7	9	7	8	7	10
J_{ik}	CS-IS-k^1	3	4	3	4	5	4	4	4	5

It is found that the number of shipment deliveries in CS-IS-k^1 is mainly due to partial information sharing, whereas it is almost equal in the cases of CS and CS-LT. Even though the number of shipments is almost equal in the cases of CS and CS-LT, the JTEC cost in CS-LT is much less compared to all other models due to considerable reduction in buyer total cost (Table 2). Savings in cost with CS-k^1 and CS-IS-k^1 policies decrease as uncertainty in demand and lead time increases, whereas for the CS-LT model they increase as uncertainty in demand and lead time increases. Therefore, when uncertainty in demand and lead time is higher, one should prefer the CS-LT policy, as it lowers the lead time (Table 2). Buyers' maximum stock level and safety stock level (minimum stock) in the case of CS-LT are always low. The greatest difference is for CS, then CS-IS-k^1 and CS-k^1. The difference in the case of CS-k^1 and CS-IS-k^1 is controlled owing to delay and information sharing.

The CS-LT policy for single vendor–single buyer terminates the iterative algorithm analysis for a minimum JTEC of $6,335, with two components' lead-time reduction with an aggregate lead time of (6 + 6 + 16) 28 days for a set of given inputs (Srinivas and Rao, 2004). For the same single vendor–single buyer input and the same given input, Ouyang *et al.* (2004) got aggregate lead time as 28 days and JTEC as $6,660.4. For both these models, total lead time is 28 days. In the case of CS-LT single vendor–multibuyer, the lead time is reduced down to minimum, because in the multibuyer case the buyer who takes the lowest lead-time component reduction (10.5 + 7 + 5.25), 22.75 days, is considered in the final output (Tables 2 and 3).

The results show that by having more buyers in the SC, the projected total cost savings $T_C^{CS-LT_p}$ (where $T_C^{CS-LT_p} = \frac{T_C^{CS-LT}}{b_i}$) of one buyer increases by a considerable amount (Table 3) for $b_i = 1$ to 5, and then steadily from $b_i = 6$ to 10. It gives the lowest projected cost ($1,833) when ten buyers exist in the SC network. For the basic CS model in the case of Braglia and Zavanella (2003) it is $2,035; the proposed CS-LT model has 11% cost savings. The savings are due to reduction in shipments and reduction in the buyers' carrying cost. The sensitivity analyses given through Figures 2 to 6 refer to the single vendor–two buyers model. The closer the total demand rate to the production rate, the greater the savings that can be obtained. In other words, by gradually decreasing the ratio of the production rate to demand rate, the percentage of JTEC savings is increased. In contrast, by increasing the value of (p/d), the savings are decreased. However, it does not mean that the savings diminish to zero as (p/d) becomes significantly high, as shown in Figure 2.

Table 3 Comparison of different strategies of Braglia and Zavanella (2003)

	Braglia and Zavanella (2003) (single vendor–single buyer)			*This model* ($T_C^{CS-LT_p}$)*, with buyer size varying from 6 to 10*				
Variable	*CS-*$k^1 = 2$	*CS-*$k^1 = 1$	*CS*	$b_i = 6$	$b_i = 7$	$b_i = 8$	$b_i = 9$	$b_i = 10$
Total cost ($)	1929	2003	2035	2013	1969	1952	1908	1833*
Max. level of buyer stock	164	267	376	389	382	385	372	363
Number of shipments	3	3	4	2	2	2	2	2

The buyer's maximum stock level with minimum JTEC ranges from 1100 to 1400. The minimum total cost in the case of CS-LT is 5703 (two buyers) with buyers' maximum level of 1102 (two buyers). There is a close range for CS-k^1 and CS-IS-k^1 but for basic CS, the minimum total cost occurs at buyers' maximum level, 1430 (two buyers) (Figure 3). From the fundamentals of CS policy the vendor always prefers to have the maximum stock level at the buyer's. Figure 3 gives total system cost in the case of two buyers while increasing the shipment size. The minimum JTEC is for $n = 5$ in the case of the CS-LT model, whereas for CS, CS-k^1 and CS-IS-k^1 it is 6, 10 and 12 respectively (Figure 4).

Figure 2 Effect of (p/d) ratio on the percentage of savings in JTEC

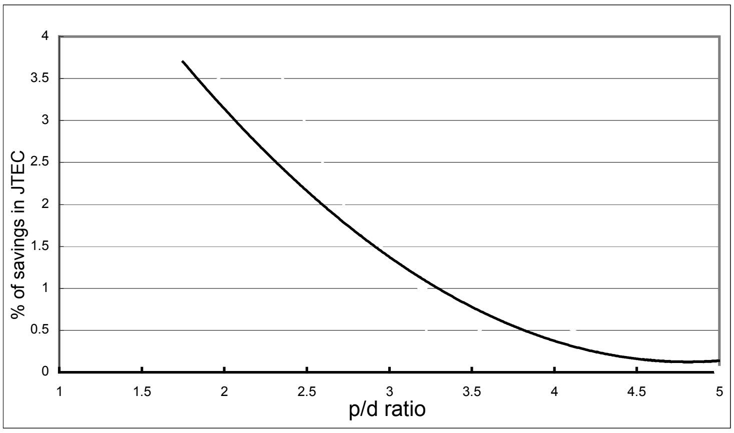


Figure 3 JTEC ($) for different CS policies with buyer's maximum stock

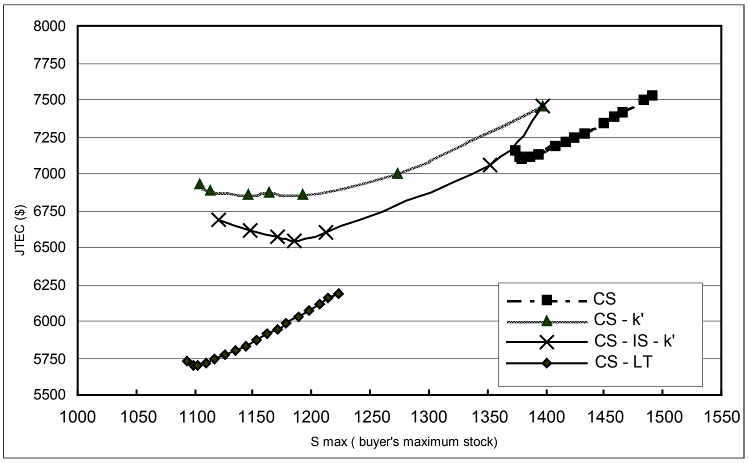


Figure 4 Total system cost for different CS models and number of shipments

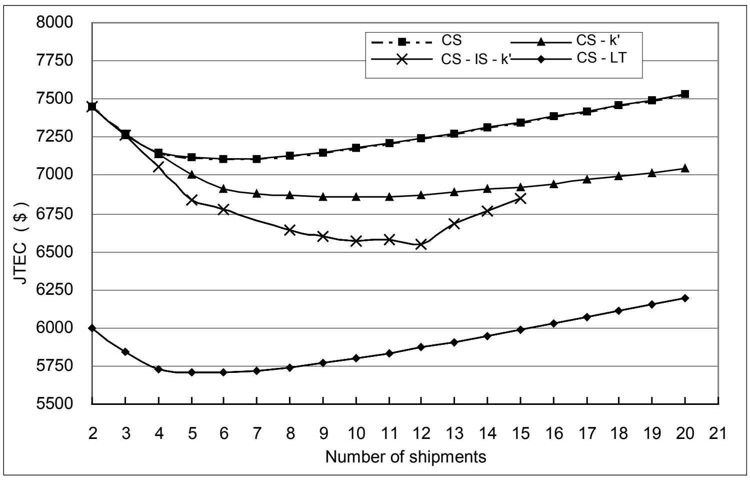


Figure 5 JTEC for CS-k^1 model with different shipments

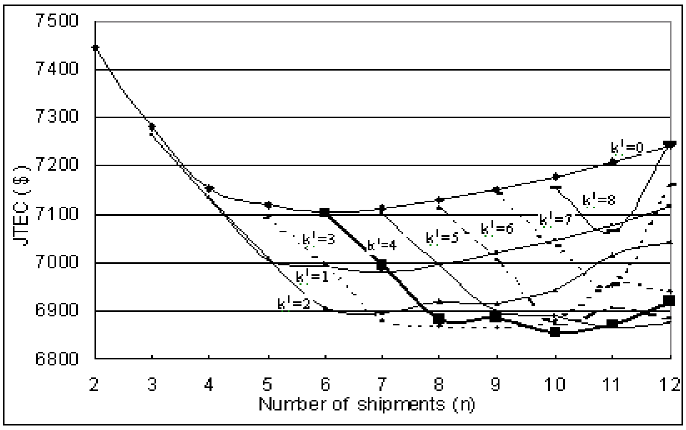
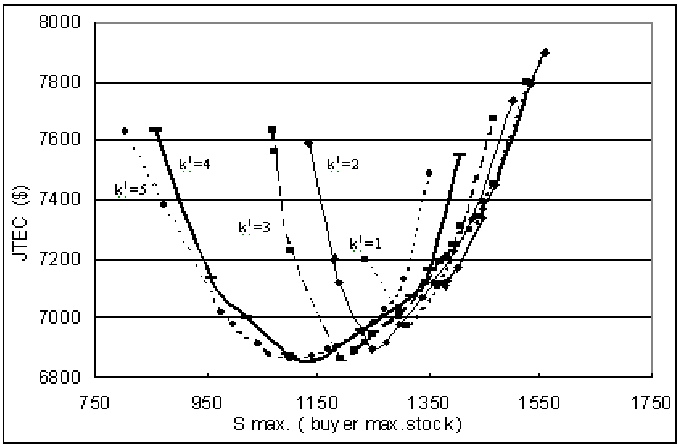


Figure 6 JTEC and maximum level of buyer's stock for a different k^1



CS-$k^1 = 4$ gives the lowest JTEC at $n = 10$. CS-$k^1 \in$ CS $\forall$ $k^1 = 0$ always produces a maximum cost, if it adopts the basic CS model (Figure 5). The minimum JTEC for CS-k^1 model decreases from $k^1 = 8$ to 4 and then increases from $k^1 = 4$ to 1. For $k^1 = 4$ there is a low buyer and vendor inventory cost for all the ranges of maximum buyer inventory levels (Figure 6).

6 Conclusions and future scope

The CS inventory management policy with controllable lead time has proved to be suitable for facing new SCM challenges with stochastic demand for single vendor–multiple buyers. Four types of models have been developed, basic CS, CS with delay, CS with information sharing and delay, and CS-LT. It is found that for multibuyer models with five or more buyers, the total cost savings increases.

Future studies have to be made in the areas of CS with controllable lead times and other policies for single vendor–multiple buyers with multiple products that can be extended to multiple echelons. Radio Frequency Identification (RFID) can also be used as a tool, and is gaining prominence as a pervasive technology with significant potential to deliver business benefits. These include stock availability improvements of >50% and a reduction of ≈ 20% labour cost. The CS policy with RFID in the SC could give extremely good results.

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