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Abstract. Transient free convective boundary layer flow of micropolar fluids past a semi-infinite cylinder is analysed in the present study. The transformed dimensionless governing equations for the flow, microrotation and heat transfer characteristics are solved by using the implicit scheme. The obtained results concerning velocity, microrotation and temperature across the boundary layer are illustrated graphically for different values of the parameters and the dependence of the flow and temperature fields from these parameters is discussed.

Keywords: Unsteady, natural convection, micropolar, vertical cylinder
PACS: 44.05.+e; 44.20.+b; 44.25.+f

INTRODUCTION

The transient natural convection flows over vertical bodies have a wide range of applications in engineering and technology. In manufacturing processes such as hot extrusion, metal forming and crystal growing, heat transfer effects plays an important role. Free convection flow of air bathing a vertical cylinder with a prescribed surface temperature was first presented by Sparrow and Gregg [1] by applying the similarity method and power series expansion. Velusamy and Garg [2] presented the numerical solution for transient natural convection over heat generating vertical cylinders of various thermal capacities and radii. Ganesan and Rani [3] has investigated the unsteady natural convection flow over a vertical cylinder with variable heat and mass transfer using the finite difference method. All the above mentioned investigations deal with the flows Newtonian fluid model. With the above discussion in mind, the purpose of the present paper is to examine analytically the natural convection flow of a non-Newtonian fluid over a semi-infinite vertical cylinder.

A micropolar fluid obeys the constitutive equations of the considered non-Newtonian fluid model. In the micropolar fluid model, apart from the classical velocity field, a microrotation vector and a gyration parameter are introduced in order to investigate the kinematics of microrotation. Such fluid model may be applied to explain the flow of colloidal solutions, liquid crystals, fluids with additives, suspension solutions, animal blood, etc. Unlike the other fluids, micropolar fluids are fluids with microstructure belonging to a class of fluids with non-symmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a porous medium. The governing equations here are highly non-linear and coupled. They are solved by the Crank-Nicolson implicit method. Expressions for velocity components and temperature are developed. The effects of various sundry parameters are systematically examined through graphs.

MATHEMATICAL FORMULATION

An unsteady two-dimensional laminar natural convection boundary layer flow of a viscous incompressible micropolar fluid past an isothermal semi-infinite vertical cylinder of radius r_0 is considered. The x and r-axis are measured vertically upward along the axis of the cylinder and perpendicular to the axis of the cylinder, respectively. The origin of x is taken to be at the leading edge of the cylinder, where the boundary layer thickness is zero. The surrounding stationary fluid temperature is assumed to be of the ambient temperature (T'_∞). Initially, i.e., at time $t'=0$ it is assumed that the cylinder and the fluid are of the same temperature T'_∞. When $t'>0$, the temperature of the cylinder is maintained to be T'_w ($>T'_\infty$) which gives rise to a buoyancy force. It is assumed that the effect of viscous dissipation is negligible in the energy equation.

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With the above assumptions and under the usual boundary layer approximation together with the Boussinesq approximation, the governing equations for the steady, laminar, incompressible, micropolar fluid flow along the vertical cylinder are

$$\frac{\partial (ru)}{\partial x}+\frac{\partial (rv)}{\partial r}=0 \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=\rho g\beta(T'-T'_{\infty})+(\mu+k_1)\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})+k_1\frac{\partial N'}{\partial r} \tag{2}$$

$$\rho j\left(\frac{\partial N'}{\partial t'}+u\frac{\partial N'}{\partial x}+v\frac{\partial N'}{\partial r}\right)=\gamma\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial N'}{\partial r})-k_1\left(2N'+\frac{\partial u}{\partial r}-\frac{\partial v}{\partial x}\right) \tag{3}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T'}{\partial r}) \tag{4}$$

where u and v are the velocity components parallel to x and r coordinates, respectively, N' is the microrotation / angular velocity, whose direction of rotation is in the xy-plane, g is the acceleration due to the gravity, β is the volumetric coefficient of thermal expansion, ρ is the density, α is the thermal diffusivity, μ is the dynamic viscosity and j, γ and k_1 the respective microinertial per unit mass, spin gradient viscosity and vortex viscosity. The above partial differential equations subject to the following conditions:

$$\begin{aligned}
&t'\le 0: u=0,\ v=0,\ N'=0, T'=T'_w \ \text{ for all x and r}\\
&t'>0: u=0,\ v=0,\ N'=-\frac{1}{2}\frac{\partial u}{\partial r}, T'=T'_w \ \text{ at } r=r_0\\
&u=0,\ v=0,\ N'=0, T'=T'_w \ \text{ at } x=0\\
&u\to 0,\ v\to 0,\ N'\to 0, T'\to 0 \ \text{ as } r\to\infty
\end{aligned} \tag{5}$$

The non-dimensional quantities are defined as

$$X=\frac{x}{r_0},\ R=\frac{r}{r_0},\ U=\frac{u\,r_0}{\nu Gr^{1/2}},\ V=\frac{v\,r_0}{\nu Gr^{1/2}},\ t=\frac{\nu t' Gr^{1/2}}{r_0^2},\ K=\frac{k_1}{\rho\upsilon};\ N=\frac{N' r_0^2}{\upsilon Gr^{12}};$$

$$\gamma=(\mu+k_1/2)\,j=\mu(1+K/2)j;\ j=r_0^2,\ T=\frac{T'-T'_{\infty}}{T'_w-T'_{\infty}},\ Gr=\frac{g\beta r_0^3(T'_w-T'_{\infty})}{\upsilon^2},\ Pr=\frac{\upsilon}{\alpha}, \tag{6}$$

where $K = k/\mu (> 0)$ the vortex viscosity or the material parameter, υ is the kinematic viscosity, Gr and Pr denote the Grashof number and the Prandtl number, respectively.

In non-dimensional form, the above equations (1)—(5) reduce to

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial R}+\frac{V}{R}=0 \tag{7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial R}=T+\frac{(1+K)}{Gr^{1/2}}\left(\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}\right)+\frac{K}{Gr^{1/2}}\frac{\partial N}{\partial R} \tag{8}$$

$$\frac{\partial N}{\partial t}+U\frac{\partial N}{\partial X}+V\frac{\partial N}{\partial R}=\frac{(1+K/2)}{Gr^{1/2}}\left(\frac{\partial^2 N}{\partial R^2}+\frac{1}{R}\frac{\partial N}{\partial R}\right)-\frac{K}{Gr^{1/2}}\left(2N+\frac{\partial U}{\partial R}-\frac{\partial V}{\partial X}\right) \tag{9}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial R}=\frac{1}{Pr\,Gr^{1/2}}\left(\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}\right) \tag{10}$$

with the initial and boundary conditions

$$\begin{aligned}
&t\le 0: U=0,\ V=0,\ N=0, T=0 \ \text{ for all X and R}\\
&t>0: U=0,\ V=0,\ N=-\frac{1}{2}\frac{\partial U}{\partial R}, T=1 \text{ at } R=1\\
&U=0,\ V=0, N=0, T=0 \ \text{ at } X=0\\
&U\to 0,\ V\to 0,\ N\to 0, T\to 0 \ \text{ as } R\to\infty
\end{aligned} \tag{11}$$

NUMERICAL SOLUTION OF THE PROBLEM

In order to solve the unsteady coupled non-linear governing equations (7)-(10) an implicit finite difference scheme of Crank-Nicolson type has been employed. The finite difference equations corresponding to equations (7)-(10) are as follows:

$$\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta X}+\frac{V_{i,j}^{k+1}-V_{i,j-1}^{k+1}+V_{i,j}^{k}-V_{i,j-1}^{k}}{2\,\Delta R}+\frac{V_{i,j}^{k+1}}{1+(j-1)\Delta R}=0 \tag{12}$$

$$\frac{U_{i,j}^{k+1}-U_{i,j}^{k}}{\Delta t}+\frac{U_{i,j}^{k}}{2\,\Delta X}(U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k})+\frac{V_{i,j}^{k}}{4\,\Delta R}(U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k})$$

$$=\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}+\frac{1+K}{Gr^{1/2}}\frac{U_{i,j-1}^{k+1}-2U_{i,j}^{k+1}+U_{i,j+1}^{k+1}+U_{i,j-1}^{k}-2U_{i,j}^{k}+U_{i,j+1}^{k}}{2(\Delta R)^2}$$

$$+\frac{1+K}{Gr^{1/2}}\frac{U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}}{4[1+(j-1)\Delta R]\Delta R}+\frac{K}{Gr^{1/2}}\frac{N_{i,j+1}^{k+1}-N_{i,j-1}^{k+1}+N_{i,j+1}^{k}-N_{i,j-1}^{k}}{4\Delta R} \tag{13}$$

$$\frac{N_{i,j}^{k+1}-N_{i,j}^{k}}{\Delta t}+\frac{U_{i,j}^{k}}{2\,\Delta X}(N_{i,j}^{k+1}-N_{i-1,j}^{k+1}+N_{i,j}^{k}-N_{i-1,j}^{k})+\frac{V_{i,j}^{k}}{4\,\Delta R}(N_{i,j+1}^{k+1}-N_{i,j-1}^{k+1}+N_{i,j+1}^{k}-N_{i,j-1}^{k})$$

$$=\frac{(1+K/2)}{Gr^{1/2}}\frac{N_{i,j-1}^{k+1}-2N_{i,j}^{k+1}+N_{i,j+1}^{k+1}+N_{i,j-1}^{k}-2NS_{i,j}^{k}+N_{i,j+1}^{k}}{2(\Delta R)^2}+\frac{(1+K/2)}{Gr^{1/2}}\frac{N_{i,j+1}^{k+1}-N_{i,j-1}^{k+1}+N_{i,j+1}^{k}-N_{i,j-1}^{k}}{4[1+(j-1)\Delta R]\Delta R}$$

$$-2\frac{K}{Gr^{1/2}}N_{i,j}^{k}-\frac{K}{Gr^{1/2}}\frac{U_{i,j+1}^{k+1}-U_{i,j-1}^{k+1}+U_{i,j+1}^{k}-U_{i,j-1}^{k}}{4\Delta R}+\frac{K}{Gr^{1/2}}\frac{V_{i+1,j}^{k+1}-V_{i,j}^{k+1}+V_{i+1,j}^{k}-V_{i,j}^{k}}{2\Delta X} \tag{14}$$

$$\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}+\frac{U_{i,j}^{k}}{2\Delta X}(T_{i,j}^{k+1}-T_{i-1,j}^{k+1}+T_{i,j}^{k}-T_{i-1,j}^{k})+\frac{V_{i,j}^{k}}{4\Delta R}(T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k})$$

$$=\frac{(T_{i,j-1}^{k+1}-2T_{i,j}^{k+1}+T_{i,j+1}^{k+1}+T_{i,j-1}^{k}-2T_{i,j}^{k}+T_{i,j+1}^{k})}{2\Pr Gr^{1/2}(\Delta R)^2}+\frac{(T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k})}{4\Pr Gr^{1/2}[1+(j-1)\Delta R]\Delta R} \tag{15}$$

The region of integration is considered as a rectangle composed of the lines indicating $X_{\min} = 0$, $X_{\max} = 1$, $R_{\min} = 1$ and $R_{\max} = 5$ where $R_{\max}$ practically corresponds to $R = \infty$ which lies very far from the momentum and energy boundary layers. In the above Eqs. (12)-(15) the subscripts i and j designate the grid points along the X and R coordinates, respectively, where $X = i\,\Delta X$ and $R = 1 + (j - 1)\,\Delta R$ and the superscript k designates a value of the time t ($= k\,\Delta t$), with ΔX, ΔR and Δt the mesh size in the X, R and t axes, respectively. In order to obtain an economical and reliable grid system for the computations, a grid independence test has been performed. The steady-state velocity and temperature values obtained with the grid system of 50 x 300 differ in the second decimal place from those with the grid system of 25 x 150, and differ in the fifth decimal place from those with the grid system of 100 x 600. Hence the grid system of 40 x 300 has been selected for all subsequent analyses, with $\Delta X = 0.02$, $\Delta R = 0.02$. Also the time step size dependency has been carried out, which yields $\Delta t = 0.001$ for reliable result. The method of solving the above finite difference equations using the Crank–Nicolson method has been discussed by Ganesan and Rani [3].

RESULTS AND DISCUSSION

To validate the current numerical procedure, the heat transfer results are compared with the results of Heckel *et al.* [4] for the steady-state, isothermal and Newtonian fluid. The comparison results are shown in Table1 and the results are found to be in good agreement.

TABLE 1. Comparison of local heat transfer

Pr	0.1	0.7	7.0	100
Heckel *et al.*[4]	0.5448	0.7820	1.1609	1.8736
Present	0.5446	0.7858	1.1608	1.8731

Figures 1, 2 and 3 show the variation of velocity, temperature and angular velocity profiles with respect to time t, Gr and K. From these figures it is observed that flow characteristics of micropolar fluids ($K > 0$) differs significantly from the Newtonian fluids ($K = 0$). From Fig. 1, it can observed that velocity and temperature increase with the distance but N decreases with the position. It is observed from Figs. 2(a) and 2(b) that the velocity and temperature decrease as Gr increases. It can also be inferred from Fig. 2(c) that for small values of Gr, N reaches the positive value from the negative value with respect to R. From Fig. 3 it can be observed that N increases with increasing values of K.

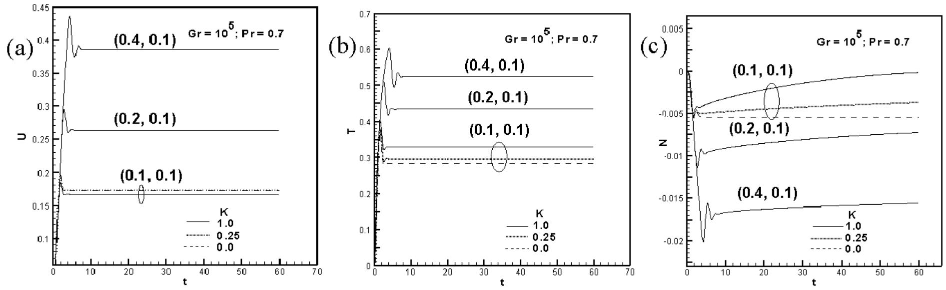

FIGURE 1. (a) Velocity, (b) temperature and (c) angular velocity profiles with respect to time

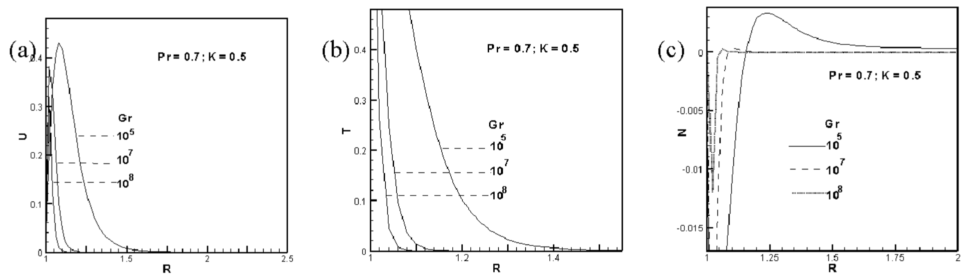

FIGURE 2. Steady state (a) Velocity, (b) temperature and (c) angular velocity profiles with respect to R for different Gr

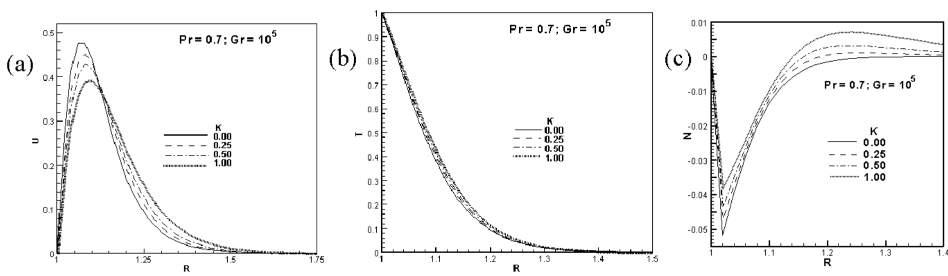

FIGURE 3. Steady state (a) Velocity, (b) temperature and (c) angular velocity profiles with respect to R for different K.

CONCLUSIONS

Unsteady natural convection micropolar flow over a semi-infinite vertical cylinder has been analysed numerically in the present study. The corresponding non-dimensional governing equations are derived. The grid generation and numerical methods for solving the non-dimensional governing equations are detailed. It is observed that the flow characteristics of micropolar fluids differ considerably from that of the Newtonian fluids.

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