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Development of a model for drying of solids in a continuous fluidized bed dryer

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A mathematical model to describe the drying characteristics of solids in a continuous fluidized bed dryer has been developed. The RTD model proposed by Pydi Setty *et al.*[1] is used to obtain the required drying model. Experimental data in a continuous fluidized bed dryer are satisfactorily matched with the model developed. Using the model, one can predict the average moisture content relative to the initial moisture content in a continuous fluidized bed dryer for the given operating conditions.

Drying describes the process of thermally removing volatile substances (moisture) to yield a solid product. Moisture held in loose chemical combination, present as a liquid solution within the solid or even trapped in the microstructure of the solid, which exerts a vapour pressure less than that of a pure liquid, is called bound moisture. When a wet solid is subjected to thermal drying, two processes occur simultaneously: (i) transfer of energy (mostly as heat) from the surrounding environment to evaporate the surface moisture and (ii) transfer of internal moisture to the surface of the solid and its subsequent evaporation due to process(i).

The rate at which drying is accomplished is governed by the rate at which the two processes proceed. Energy transfer as heat from the surroundings to the wet solid can occur as a result of conduction, convection and radiation and in some cases as a result of combination of these effects. Industrial dryers differ in type and design, depending on the principal method of heat transfer employed. In most cases heat is transferred to the surface of the wet solid and then to the interior. However, in dielectric or microwave freeze drying, energy is supplied to generate heat internally within the solid and flows to the exterior surfaces.

In process (i), the removal of water as vapour from the material surface, depends on the external conditions of temperature, air humidity and flow and area of exposed surface. In process (ii), the movement of moisture internally within the solid is a function of the physical nature of the solid, the temperature and its moisture content. In a drying operation anyone of these processes may be the limiting factor governing the rate of drying, although both of them proceed simultaneously throughout the drying cycle.

The use of fluidized bed drying for granular materials is well-established. Fluidized bed drying has the advantages of high intensity of drying and high thermal efficiency with uniform and close control of temperature in the bed. It requires less drying time due to high rates of heat and mass transfer and provides a wide choice in the drying time of the material. In addition, fluidized bed drying also offers other advantages like:

(i) The even flow of fluidized particles permits continuous, automatically controlled large-scale operation with ease in handling of feed and product.
(ii) There are no mechanically moving parts, thus requiring less maintenance.
(iii) Heat transfer rates between fluidized bed and immersed objects are high.
(iv) Rapid mixing of solids to nearly isothermal conditions throughout the fluidized bed and reliable control of the drying process can be achieved easily.

In spite of several advantages mentioned above, fluidized bed drying suffers from disadvantages like high pressure drop and attrition of solids, erosion of containing surfaces and non-uniform moisture content in the product as a result of the distribution of residence times of the individual particles in a continuous fluidized bed.

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Drying kinetics in a continuous fluidized bed is different from that of a batch dryer as variation of residence times are involved in a continuous fluidized bed. To overcome this drawback internal baffles have been provided in the industrial fluidized bed dryers of circular cross-section[2,3] and of rectangular cross-section[4,5]. Vanceck *et al.*[6], approximated solids mixing in a continuous bed to ideal mixing during their studies on drying kinetics. Beran and Lutcha[4] approximated solids mixing to a finite axial Peclet number in a rectangular fluidized bed. The authors[4] studied literaturė on drying of crystalline ammonium sulphate, assuming constant rate period preceding the falling rate period for the drying kinetics. Suzuki *et al.*[7] from their experimental data on vibratory fluidized bed indicated that the single falling rate period following the constant rate period, would suffice to describe the drying kinetics.

Chandran *et al.*[8] developed a kinetic model for the drying of solids in fluidized beds assuming a falling rate period following a constant rate period. Experimental data using batch and continuous single and spiral fluidized beds are satisfactorily matched with the assumed drying kinetics. The authors[8] developed drying kinetic models for both flat fluidized bed and a spiral fluidized bed. For a flat fluidized bed, drying model was developed by the authors[8] using ideal mixing model of Vanecek *et al.*[9] and for a spiral fluidized bed dryer, a model was developed using the axial dispersion model of Setty[10].

Vanecek *et al.*[9] has compared the performance of batch and continuous fluidized beds and made an observation that for a given drying time, the batch operation gives lower average moisture content in the product than the continuous operation. Similar conclusion was made by Beran and Lutcha[4] for a rectangular fluidized bed dryer. Romankov[11], however reported improved performance under mixed flow conditions as compared to batch drying.

Babu and Setty[12] reported experimental investigation in a continuous fluidized bed dryer and the development of a drying kinetic model using RTD kinetic models like tanks-in-series model and fractional tank extension model. The authors[12] developed generalized expressions, which can be used to predict the average moisture content of solids in a continuous fluidized bed dryer.

The present work involves development of a mathematical model to describe the drying kinetics in a continuous fluidized bed dryer using the RTD model proposed by Setty *et al.*[1], known as CSTR-PFR-in-series model. Generalized expressions are developed, which can be used to predict the average moisture content of solids in a continuous fluidized bed dryer. Derivation of these equations are given in Appendix.

Experimental Procedure

The experimental set-up consists of a fluidization column of 89 mm i.d. and 1280 mm in height made up of iron. It consists of fluidization and air chamber separated by a perforated plate with 3 mm perforations arranged on 6 mm triangular pitch. The plate is provided with a stainless steel wire mesh of 100 mesh size with a vertical baffle fixed to it. Air passes from the compressor through the rotameter into the fluidization column through the air chamber.

Heating arrangement was provided by a heating coil rounded along the outside surface of the column and connected to a variac, which in turn is connected to an electric source. Copper-Constantan thermocouples were used for temperature measurement and the instrument was calibrated for temperature correction.

The wet solids were admitted into the column through a down flow pipe located at the column periphery from the hopper at a known flow rate. The solids fed at one end of the column, fluidize, get dried and move along the distributor plate and exit through the distributor weir at the other end which protrudes through the air chamber and the samples were collected at the outlet. The samples were analyzed for moisture content. Table 1 shows the data of a typical experimental run[13]. Schematic diagram of the experimental set-up is given elsewhere[12,13].

Results and Discussion

Assumptions

(i) The drying rate curve has either constant rate period and/or falling rate period.

(ii) The falling rate period is linear and is represented by a single line from the critical moisture content to the equilibrium moisture content.

(iii) The feed conditions of the gas remain unaltered during the drying process. The experiments were performed under constant drying conditions.

Model development

The average moisture content in the product relative to the initial moisture content for continuous drying of solids with distribution of residence times for the solids may be written as[8] :

$$(\bar{c}/c_0)=\int_0^{\infty}(c/c_0)_b\,E(\theta)d\theta \qquad \ldots (1)$$

where $(c/c_0)_b$ denotes the moisture content in the product relative to the initial moisture content under batch operation. $E(\theta)$ is the exit age distribution, which accounts for the residence time distribution of solids resulting due to mixing characteristics within the bed. In the present work, the RTD model, namely, CSTR-PFR-in-series-model[1] is used to obtain a generalized expression to predict the average moisture content for the product in a continuous fluidized bed dryer. CSTR-PFR-in-series-model[1] describes the RTD of solids in a continuous fluidized bed dryer provided with some sort of internals like baffles or a spiral internal, etc. In this model CSTR accounts for the fluidization (mixing) phenomena, while PFR accounts for the diffusional movement of solids induced by the baffle plates. The present drying kinetic model can also be used for a similar type of fluidized bed provided with internals. In a continuous operation, different individual particles take individual travel paths during their travel from inlet to the exit. Age distribution $E(\theta)$ describes the distribution of residence times of particles in a continuous operation. Hence, $E(\theta)$ can be used to predict the drying data in a continuous dryer using the data in a batch dryer applying Eq. (1).

$E(\theta)$ according to CSTR-PFR-in-series-model[1] is given by:

$$E(\theta)=0 \qquad \text{for } \theta<(1-p)$$

$$E(\theta)=\frac{(n/p)^n(\theta-1+p)^{n-1}\left(e^{-(n/p)(\theta-1+p)}\right)}{(n-1)!} \qquad \text{for } \theta\geq(1-p) \qquad \ldots (2)$$

Batch drying

Expressions for batch drying are given as follows:

Constant rate period

$t=(c_0-c)/R$ or $c=c_0-Rt$ and $c\geq c_c$ and $t\geq t_c$ ... (3)

Falling rate period

$$t=\left[\frac{(c_c-c^+)}{R}\right]\ln\left[\frac{(c_c-c^+)}{(c-c^+)}\right] \text{ or}$$

$$c=c^++(c_c-c^+)\exp[-\beta(t-t_c)] \qquad \ldots (4)$$

Continuous drying of solids

Average moisture content relative to initial moisture content is given by:

$$(\bar{c}/c_0)=(\bar{c}/c_0)_{\text{constant}}+(\bar{c}/c_0)_{\text{falling}}$$

(A) Assuming integer values for n

Constant rate period

Using Eq. (2), $(\bar{c}/c_0)$ is obtained as:

$$(\bar{c}/c_0)_{\text{constant}}=\left(1-\frac{R\bar{t}}{c_0}\right)\left[1-e^{-na/p}F(n,na/p)\right]+\frac{R\bar{t}p(na/p)^n}{c_0 n!}e^{-na/p} \qquad \ldots (5)$$

Falling rate period

$$(\bar{c}/c_0)_{\text{falling}}=\left[\frac{c^+}{c_0}F(n,an/p)+\frac{(c_c-c^+)F(n,\beta\bar{t}a+na/p)}{c_0(1+\beta\bar{t}p/n)^n}\right]e^{-na/p} \qquad \ldots (6)$$

Combining Eqs (5) and (6), a generalized expression for drying kinetics in a continuous fluidized bed dryer is obtained for the entire period, which is given by :

$$\frac{\bar{c}}{c_0}=\left(1-\frac{R\bar{t}}{c_0}\right)\left[1-e^{-an/p}F(n,an/p)\right]+\frac{R\bar{t}p}{c_0}\frac{(an/p)^n}{n!}e^{-an/p}+\left[\frac{c^+}{c_0}F(n,an/p)+\frac{(c_c-c^+)F(n,(\beta\bar{t}a+na/p))}{c_0(1+\beta\bar{t}p/n)^n}\right] \qquad \ldots (7)$$

Eq. (7) can be modified as follows to obtain expressions for a system exhibiting only constant rate period or only falling rate period.

(i) For a system exhibiting only constant rate period, as a $\rightarrow \infty$, $(\bar{c}/c_0)_{\text{falling}} \rightarrow 0$ and

$$(\bar{c}/c_0) = (1 - R\bar{t}/c_0) \quad \text{... (8)}$$

(ii) As a $\rightarrow$ o corresponds to a system exhibiting only falling rate period, $(\bar{c}/c_o)_{\text{constant}} \rightarrow 0$ and

$$\left(\frac{\bar{c}}{c_0}\right) = \left(\frac{c^+}{c_0}\right) + \frac{\left[(c_c - c^+)/c_0\right]}{(1+\beta\bar{t}/n)^n} \quad \text{... (9)}$$

(B) Assuming non-integer values for n

Let (n−1) = m = integral part of (n-1) and k = (n-1-m) = fractional part of (n-1):

$(\bar{c}/c_0) = (\bar{c}/c_0)_{\text{constant}} + (\bar{c}/c_0)_{\text{falling}} = T_3 + T_4$, where,

$$T_3 = \frac{(n/p)^k}{\Gamma(k)}\left[\left(1 - \frac{R\bar{t}}{c_0}\right)\int_0^\infty e^{-nv/p} v^{k-1} dv - \left\{\left(1 - \frac{R\bar{t}(1-p)}{c_0}\right) F(m+1, ap) - \frac{\bar{t}Rp}{c_0}[F(m+2, ap)]\right\}\right] e^{-na/p} a^k / k \quad \text{... (10)}$$

and

$$T_4 = \frac{c^+}{c_0}\left[1 - \frac{(n/p)^k}{\Gamma(k)}\left\{e^{-nv/p} v^{k-1} dv - e^{-na/p} a^k F(m+1, ap)/k\right\}\right] + \frac{(c_c - c^+)e^{a\bar{t}\beta}}{c_0 x^{m+1}}\left[\frac{1}{x^k} - \frac{(n/p)^k}{\Gamma(k)} \left\{\int_0^a e^{-xnv/p} v^{k-1} dv - e^{-xan/p} a^k F(m+1.axn/p)/k\right\}\right] \quad \text{... (11)}$$

Here, $v=\theta+p-1$ and a $=\theta_c+p-1$ and $\int_0^a e^{-nv/p} v^{k-1} dv$ is an indefinite Gamma function[14],

$$F(m,s) = 1 + \frac{s}{k+1} + \frac{s^2}{(k+1)(k+2)} + \text{--} + \frac{s^{m-1}}{(k+1)(k+2)\ldots(k+m-1)}$$

and $x=(1+\beta\bar{t}\, p/n)$

For a system showing only constant rate period, a $\rightarrow \infty$, $T_4 \rightarrow 0$ and $(\bar{c}/c_0) = (1 - R\bar{t}/c_0)$

For a system showing only falling rate period, a $\rightarrow 0$, $T_3 \rightarrow 0$ and Eq. (9) is obtained.

The models developed in A and B for a system exhibiting only falling rate periods were tested with a typical experiment in the laboratory on drying kinetics in a continuous fluidized bed dryer provided with an internal vertical baffle[13]. It was found that the predicted value of $(\bar{c}/c_0)$ agreed well with the experimental value as seen from Table 1. Value of $(\bar{c}/c_0)_e$, obtained from the experimental c(t) versus t data is calculated as :

$$\left(\frac{\bar{c}}{c_0}\right)_e = \frac{\int [c(t)/c_0] dt}{\int dt} \quad \text{... (12)}$$

Conclusions

The present work involves development of a drying kinetic model for a continuous fluidized bed dryer using the RTD model of Setty *et al.*[1]. Generalized equations have been developed for integer (A) and

Table 1—Experimental data obtained in a continuous fluidized bed dryer[13]

Ambient temperature, °C: 28; Gas flow rate, kg/h: 30; Solids flow rate, kg/h: 2.56×10^{-3}; Bed temperature, °C: 60; Initial moisture content, kg water/kg dry solid: 0.0203

t, s	c
15	0.001201
30	0.001096
45	0.001081
60	0.001075
75	0.001001
90	0.000983
105	0.000932
120	0.000844
135	0.000784
150	0.000651
165	0.000544
180	0.000507
195	0.000100
210	0.000050

Mean residence time[15], s: 77.7814; Experimental average moisture content (Eq. 12), $(\bar{c}/c_0)_e$: 0.02944; Predicted average moisture content (Eq. 9), $(\bar{c}/c_0)_p$: 0.02904; Number of stages obtained from the model: 1.8

non-integer values (B) of 'n' considering both constant rate period and falling rate period and also for the limiting cases of a system exhibiting only constant rate period or only falling rate period for both cases of A and B. There is perfect agreement of predicted value of $(\bar{c}/c_0)$ for both cases of A and B with the experimental value. Hence the generalized equations can be used to predict the values of average moisture content relative to the initial moisture content, $(\bar{c}/c_0)$.

Nomenclature

c	=	moisture content of the solid at time, t, kg moisture/kg dry solid
$\bar{c}$	=	average moisture content of the product, kg water/kg dry solid
c^+	=	equilibrium moisture content, kg water/kg dry solid
c_c	=	critical moisture content of the solids, kg water/kg dry solid
c_0	=	initial moisture content of the solids, kg water/kg dry solid
R	=	constant drying rate, kg water/kg dry solid-s
t	=	clock time, s
$\bar{t}$	=	mean residence time, s
t_c	=	time corresponding to critical moisture content, s

Greek Letters

β	=	$R/(c_c - c^+)$, s^{-1}
ρ_s	=	density of solids, kg/m^3
θ	=	dimensionless time, $(t/\bar{t})$
θ_c	=	dimensionless time corresponding to critical moisture content, $(t/\bar{t})_c$

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Appendix

Expressions for $(\bar{c}/c_0)$ can be obtained by substituting the expression for $E(\theta)$ (Eq. 2) and respective expressions for $(c/c_0)_b$ for different cases.

Continuous drying of solids

(A) Assuming integer values for n

For constant rate and falling rate periods $(\bar{c}/c_0)$ is obtained by substituting Eq. (2) in Eq. (1) as:

Let $T_1 = (\bar{c}/c_0)_{constant}$

$$T_1 = \frac{(n/p)^n}{(n-1)!}\int_{1-p}^{\theta_c}(1 - R\bar{t}\theta/c_0)(\theta + p - 1)^{n-1} e^{-(n/p)(\theta+p-1)}d\theta$$

$$= \frac{(n/p)^n}{(n-1)!}\int_0^a\left[1 - R\bar{t}(v + 1 - p)/c_0\right]v^{n-1}e^{-n\,v/p}dv$$

where $v = (\theta + p - 1)$ and $a = (\theta_c + p - 1)$.

$$T_1 = \frac{(n/p)^n}{(n-1)!}\left[\int_0^a(1 - R\bar{t}(1-p)/c_0)v^{n-1}e^{-(n/p)v}dv - \int_0^a(R\bar{t}/c_0)v^n e^{-(n/p)v}dv\right] \quad \text{... (A1)}$$

Consider,

$$\int_0^a v^{n-1}e^{-(n/p)v}dv$$

$$= e^{-(n/p)v}\frac{(n-1)!}{(n/p)^n}\left[1 + \frac{vn}{p} + \frac{(vn)^2}{p^2 2!} + \frac{(vn)^3}{p^3 3!} + \ldots + \frac{(vn)^{n-1}}{p^{n-1}(n-1)!}\right]$$

$$= \frac{(n-1)!}{(n/p)^n}\left[1 - e^{-(na/p)}F_a(na/p)\right] \quad \text{... (A2)}$$

where $F(n,s) = F_n(s) = \left[1 + s + \frac{s^2}{2!} + \frac{s^3}{3!} + \ldots + \frac{s^{n-1}}{(n-1)!}\right]$

combination of Eqs (A1) and (A2) yields,

$$T_1 = \left[1 - \frac{R\bar{t}(1-p)}{c_0}\right]\left[1 - e^{-na/p}F(n, na/p)\right]$$
$$- \frac{R\bar{t}n}{c_0(n/p)}\left[1 - e^{-na/p}\left(F(n, na/p) + \frac{(na/p)^n}{n!}\right)\right]$$

On simplification, Eq. (5) can be obtained.

Falling rate period

Let $T_2 = (\bar{c}/c_0)_{\text{falling}}$

$$T_2 = \int_0^{\infty}\left[(c^+/c_0) + \{(c_c - c^+)/c_0\}e^{-\beta\bar{t}(\theta-\theta_c)}\right] \times \frac{(n/p)^n(\theta+p-1)^{n-1}}{(n-1)!}e^{-(n/p)(\theta+p-1)}d\theta$$

$$= \frac{(n/p)^n}{(n-1)!}\int_a^{\infty}\left[(c^+/c_0) + \{(c_c - c^+)/c_0\}e^{-\beta\bar{t}(v+1-p-\theta_c)}\right]v^{n-1}e^{-nv/p}dv$$

$$= \frac{(n/p)^n}{(n-1)!}\frac{c^+}{c_0}\left[\int_0^{\infty} v^{n-1}e^{-nv/p}dv - \int_0^{a} v^{n-1}e^{-nv/p}dv\right]$$

$$+ \frac{(n/p)^n}{(n-1)!}\frac{(c_c - c^+)}{c_0}e^{\beta\bar{t}a}\left(\int_0^{\infty} v^{n-1}e^{-(\beta\bar{t}+n/p)v}dv - \int_0^{a} v^{n-1}e^{-(\beta\bar{t}+n/p)v}dv\right)$$

$$= \frac{(n/p)^n}{(n-1)!}\frac{c^+}{c_0}\left[\frac{(n-1)!}{(n/p)^n} - \frac{(n-1)!}{(n/p)^n}\{1 - e^{-na/p}F(n, an/p)\}\right]$$

$$+ \frac{(n/p)^n}{(n-1)!}\frac{(c_c - c^+)}{c_0}e^{\beta\bar{t}a}$$

$$\times\left[\frac{(n-1)!}{(\beta\bar{t}+n/p)^n} - \frac{(n-1)!}{(\beta\bar{t}+n/p)^n}(1 - e^{-(\beta\bar{t}a+an/p)}F(n, \beta\bar{t}+n/p))\right]$$

$$T_2 = \begin{matrix}(c^+/c_0)e^{-na/p}F(n, na/p) + [(c_c - c^+)/c_0]e^{\beta\bar{t}a} \\ (e^{-((n/p)+\beta\bar{t})a}/(1 + (\beta\bar{t}p/n)^n)F(n, \beta\bar{t}+n/p)\end{matrix}$$

On simplification Eq. (6) can be obtained.

(B) Assuming non-integer values for n

Let $T_3 = (\bar{c}/c_0)_{\text{constant}}$ and $T_4 = (\bar{c}/c_0)_{\text{falling}}$, then $(\bar{c}/c_0) = T_3 + T_4$

$$T_3 = \frac{(n/p)^n}{\Gamma(n)}\int_{1-p}^{\theta_c}(1 - R\bar{t}\theta/c_0)(\theta+p-1)^{n-1}e^{-(n/p)(\theta+p-1)}d\theta$$

$$= \frac{(n/p)^n}{\Gamma(n)}\int_0^{a}[1 - R\bar{t}(v+1-p)/c_0]v^{n-1}e^{-nv/p}dv$$

$$T_3 = \frac{(n/p)^n}{\Gamma(n)}[(1 - R\bar{t}(1-p)/c_0)\frac{(n-1)(n-2)...k}{(n/p)^{m+1}}$$

$$(\int_0^{a} e^{-nv/p}v^{k-1}dv - e^{-na/p}a^kF(m+1, ap)/k)$$

$$-(R\bar{t}/c_0)^n\frac{(n-1)(n-2)...k}{(n/p)^{m+2}}(\int_0^{a} e^{-nv/p}v^{k-1}dv - e^{-na/p}a^kF(m+2, ap)/k)]$$

On simplification, Eq. (10) can be obtained.

Here m and k are integral and fractional parts of $(n-1)$, $v = \theta + p - 1$ and $a = \theta_c + p - 1$

$\int_0^{a} e^{-nv/p}v^{k-1}dv$ is an indefinite Garnma function[14] and $F(m,s)$ is as defined in the text.

$$T_4 = \int_{\theta_c}^{\infty}\left[\frac{c^+}{c_0} + \frac{(c_c - c^+)}{c_0}e^{-\beta\bar{t}(\theta-\theta_c)}\right]\frac{(n/p)^n}{\Gamma(n)}(\theta+p-1)^{n-1}e^{-(n/p)(\theta+p-1)}d\theta$$

$$= \frac{(n/p)^n}{\Gamma(n)}(\int_0^{\infty} - \int_0^{a})\left[\frac{c^+}{c_0} + \frac{(c_c - c^+)}{c_0}e^{-\beta\bar{t}(v-a)}\right]e^{-nv/p}v^{n-1}dv$$

$$= \frac{(n/p)^n}{\Gamma(n)}\left[\frac{c^+\Gamma(n)}{c_0(n/p)^n} + \frac{(c_c - c^+)\Gamma(n)}{c_0(\beta\bar{t}+n/p)^n}e^{a\bar{t}\beta}\right] - \frac{(n/p)^n}{\Gamma(n)}$$

$$\left\{\frac{c^+k(k+1)(k+2)...(n-1)}{c_0(n/p)^{m+1}}\left[\int_0^{a} e^{-nv/p}v^{k-1}dv - e^{-na/p}a^kF(m+1, ap)/k\right]\right\}$$

$$+ \frac{(n/p)^n}{\Gamma(n)}$$

$$\left\{\frac{(c_c - c^+)k(k+1)...(n-1)}{c_0(\bar{t}\beta+n/p)^{m+1}}e^{\bar{t}\beta a}\right.$$

$$\left.\times\left[\int_0^{a} e^{-(\bar{t}\beta+n/p)v}v^{k+1}dv - e^{-(\bar{t}\beta+n/p)a}a^kF(m+1, a\bar{t}\beta+na/p)/k\right]\right\}$$

On simplification, Eq. (11) can be obtained.