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Evaluation of alternate objective functions for optimal operation of an irrigation reservoir under a multi-crop environment

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Abstract In the present study, three Stochastic Dynamic Programming (SDP) models with different objective functions were used to develop the operating policy for an irrigation reservoir, namely, Sri Rama Sagar Reservoir on the River Godavari in Andhra Pradesh, India. The reservoir is a major project, meeting the irrigation requirements over a large command area. The three SDP models model the objectives of the reservoir with different levels of mathematical complexity. The performance of the reservoir under these three operating policies is compared through simulation. Three criteria, namely reliability, resilience and average annual deficit of water supply are used to evaluate the performance of the reservoir under the alternative operating policies developed.

Key words irrigation reservoir; multiple crops; operation policy; River Godavari, India; simulation; stochastic dynamic programming; water allocation

INTRODUCTION

Applications of mathematical optimization techniques to water management problems have gained momentum over the years. The use of these mathematical models has greatly aided in providing a good insight into the intricacies of various aspects of problems of water management. The mathematical models have been more widely used in the management of reservoirs, where large volumes of water are stored for use when needed. A comprehensive review of the state of the art of mathematical models used in studying reservoir operation is given in Stedinger *et al.* (1984) and Yeh (1985).

Among the various mathematical models, Stochastic Dynamic Programming (SDP) has become popular in recent years. Studies to examine the relative merits of optimization techniques currently used in reservoir management and design are, however, very few. The proper formulation of the objective function of the optimization model to operate a reservoir is very important. The success or failure of the modelling effort depends largely on the formulation of the objective function, which represents with sufficient accuracy, the goals of the operating policy.

In the present study, three SDP models with different objective functions were used to develop the operating policy for an irrigation reservoir. The performance of the reservoir under these three operating policies is compared through simulation. The formulation of these three models is presented in the following sections.

FORMULATION OF THE MODELS

Three SDP models with different objective functions were produced to develop the operating policy of an irrigation reservoir. The three models vary in their degree of mathematical complexity.

Formulation of Model I

Model I is a relatively simple model which uses a stochastic dynamic programming technique to minimize the expected sum of the squared deficit from a target release. This is the most popularly used objective function for the long term operation of a reservoir (Loucks *et al.*, 1981). The objective function can be written as:

$$\min E[\,(TR_t - R_t)^2],\ \text{if } R_t < TR_t \tag{1}$$

where TR_t is the target release in time period t which is equal to the irrigation demand during that time period, while R_t is the actual release made from the reservoir and $E[]$ is the expectation operator. A backward recursion stochastic dynamic programming model was developed for deriving the steady state operating policy of the reservoir with the above objective function. The optimal releases are related to two state variables: the inflow into the reservoir during the time period and the storage at the beginning of the time period.

Let Q_t represent the inflow into the reservoir during any time period t. This continuous variable Q_t can be discretized into several class intervals. Any value within the range of a class interval can be represented by a single value. Let i be the index to represent the class interval for inflow during the time period t. Thus Q_{it} is the representative flow of the i-th class interval in time period t. Similarly let j be the index for representing the class interval for inflow during time period $t + 1$ and Q_{jt+1} be the representative inflow of the class interval. The other state variable is S_t, which is the reservoir storage at the beginning of time period t. Let the indices k and l represent the class intervals for the storage at the beginning of time period t and $t + 1$, respectively. Thus S_{kt}, and $S_{l,t+1}$ are the representative values of the storage in the respective class intervals.

Given the initial storage volume S_{kt}, the inflows Q_{it} and final storage volume $S_{l,t+1}$ in period t, the release R_{kilt} is determined by the continuity equation:

$$R_{kilt} = S_{kt} + Q_{it} - E_{klt} - S_{l,t+1} \tag{2}$$

where E_{klt} is the possible evaporation loss which depends on the initial and final storage volumes in period t. For a given initial storage state k and inflow state i, some of the final storage states l may not be feasible as they result in negative value of R_{kilt}. The general backward recursive equation for this model can be written as follows

$$f_t^n(k,i) = \min_{\{l\}}\left[(TR_t - R_t)^2 + \sum_j p_{ij}^t \times f_{t+1}^{n-1}(l,j)\right] \tag{3}$$

where $\{l\}$ represents the feasible set of final storage states l and p_{ij}^t is the transition probability of inflow from state i in time period t to state j in time period $t + 1$. Equation (3) is solved recursively until a steady state operating policy is obtained.

Formulation of Model II

Model II is also a stochastic dynamic programming approach but with a different objective function. The objective function is based on the crop production function proposed by Hall & Dracup (1970) and Doorenbos & Kassam (1979) and is given by:

$$1-\frac{y_a}{y_m}=k_y\left(1-\frac{E_a}{E_m}\right) \tag{4}$$

where y_a is the actual yield, y_m is the potential yield, E_a is the actual evapotranspiration needs provided, E_m is the maximum evapotranspiration needs of the crop, and k_y is the yield response factor. The yield response factor reflects the sensitivity of the crop to a water deficit.

Model II considers only an average crop production function over all the crops. The objective function of this model is given by:

$$\min E[ky_{at}(1-TE_{at}/TE_{mt})] \tag{5}$$

where TE_{mt} is the total evapotranspiration needs of all the crops in period t, TE_{at} is the actual evapotranspiration needs provided to all the crops, and ky_{at} is the weighted average of the crop response function of all the crops.

The water available for irrigation X_{kilt} from the total water released from the reservoir R_{kilt} depends upon the irrigation efficiency β and is given by:

$$X_{kilt}=\beta\times R_{kilt} \tag{6}$$

The effective rainfall P_t contribution to the water needs of the crops is taken into consideration while computing the actual evapotranspiration needs provided to all the crops. Thus TE_{at} is given by:

$$TE_{at}=X_{kilt}+P_t\times CAI_t \tag{7}$$

where CAI_t is the area under irrigation in period t. The general backward recursive equation of Model II is given by:

$$f_t^n(k,i)=\min_{\{l\}}\left[ky_{at}\left(1-\frac{TE_{at}}{TE_{mt}}\right)+\sum_j p_{ij}^t\times f_{t+1}^{n-1}(l,j)\right] \text{ for all } k,i \tag{8}$$

Formulation of Model III

Model III is a stochastic dynamic programming model developed for optimal operation of an irrigation reservoir under multiple crop scenarios (Vedula & Mujumdar, 1992; Umamahesh & Sreenivasulu, 1997). The model is formulated conceptually to operate in two phases. In the first phase the model uses a deterministic dynamic programming technique to allocate the given release from the reservoir among all crops to optimize the impact of allocation within a period. The second phase uses a stochastic dynamic programming technique to determine a steady state operating policy of the reservoir so as to optimize the overall impact of the allocation over a full year. The result of this two phase analysis is a set of decisions indicating the reservoir release to be made in each time period and its distribution among various crops.

In the first phase a deterministic dynamic programming (DDP) model is used to allocate the available water among the crops whenever competition for water exists. The objective function of the DDP model is based on the crop production given in equation (4) and is given by:

$$\min \sum_{c=1}^{nc} ky_{ct} \left[1 - \frac{E_{act}}{E_{mct}} \right] = B \tag{9}$$

where c is the index for crop and nc is the number of crops in period t. The value of B is zero, if the water available X is greater than or equal to the water requirement of all the crops. The value of B is greater than zero if competition among crops exists and is to be evaluated.

Each crop constitutes a stage in this DDP model and the state variable is the volume of water q_r available at a given stage r, for allocation among all stages up to and including that stage. If $g_r(q_r)$ is the minimum value of the objective function for a given q_r and $q(r)$ is the water allocated to the crop corresponding to the r-th stage then the general recursive equation for the rth stage is given by:

$$g_r(q_r) = \min \left[ky_{nc-r+1,t} \left(1 - \frac{E_a}{E_m} \right)_{nc-r+1,t} + g_{r-1}(q_r - q(r)) \right], \tag{10}$$

where $0 \leq q(r) \leq q_r \leq X$.

The actual evapotranspiration needs provided to the crop are computed by considering the effective rainfall during the period and the initial moisture present in the root zone of the crop. The details of this computation are given by Umamahesh & Sreenivasulu (1997). The recursive equation given by (10) is solved to obtain the optimal allocation of the available quantity of water X given the initial soil moisture level. This equation is solved for all possible releases from the reservoir, for all possible initial moisture levels for each time period t. The results of this phase are used in the second phase.

The second phase of Model III uses a SDP model to determine the optimal operation policy of an irrigation reservoir under a multi-crop environment. The optimal releases from the reservoir are related to three state variables, namely, the inflow into the reservoir in time period t, the storage state of the reservoir at the beginning of time period t and the average initial soil moisture in the command area at the beginning of the time period. Like the other two state variables, the initial soil moisture θ_t is discretized into class intervals and m and n represent the class intervals for soil moisture at the beginning of time periods t and $t + 1$, respectively, while θ_{mt} and θ_{nt+1} are the representative values of soil moisture in the respective class intervals.

The objective function of the SDP model is to minimize the expected value of B, a measure of the system performance determined from the DDP model of the first phase. The system performance B is a function of reservoir release, R_{kilt} and soil moisture θ_t during period t. The objective function of the SDP model can be written as:

$$\min E[B(k, i, l, m, t)] \text{ for all } k, i, m \tag{11}$$

With the usual notation the general backward recursive equation for the SDP model is given by:

$$f_t^n(k,i,m) = \min_{\{l\}}\left[B(k,i,l,m,t) + \sum_j p_{ij}^t \times f_{t+1}^{n-1}(l,j,n)\right] \text{ for all } k,\ i,\ m \tag{12}$$

The soil moisture at the beginning of time period $t + 1$, n, can be determined for any given k, i, l and m using soil moisture continuity.

Equation (12) is solved recursively until a steady state solution is reached defining the optimal operating policy of the reservoir.

EVALUATION OF THE MODELS

The performance of the three models can be evaluated and compared using simulations. Three performance indicators, namely reliability, resilience, and the expected annual deficit, can be used to evaluate the performance of these models. Reliability is defined as the probability that system performance is satisfactory. Resilience is defined as the probability of system recovery from a failure when it occurs. The system output is satisfactory when the release from the reservoir is at least equal to the irrigation demands. The magnitude of failure is measured in terms of expected annual deficit.

SYSTEM DESCRIPTION

The three stochastic dynamic programming models discussed in the previous section were applied to the Sri Rama Sagar Reservoir on the River Godavari in the state of Andhra Pradesh, India. Although it is a multi purpose reservoir, it is predominantly an irrigation reservoir, with hydropower and municipal water supply consuming only a small part of the releases from the reservoir. Table 1 shows the details of the cropping pattern adapted in the command area of the project.

Table 1 Details of the cropping pattern.

Crop name	Area under the crop in hectares
Sugarcane	14 880
Rice (K)	40 548
Maize(K)	57 320
Groundnut (K)	147 032
Chilly	104 032
Cotton	18 850
Sorghum (K)	2 738
Maize (R)	62 726
Groundnut (R)	101 977
Sorghum (R)	9 672
Pulses	28 588
Green gram	37 733
Red gram	6 123

K: Kharif season (June–October), R: Rabi season (October–March).

The average annual flow into the reservoir is 11 707 million cubic metres (Mm^3) with a standard deviation of 9073 Mm^3. The active storage capacity of the reservoir is 2320 Mm^3, which is discretized into 20 states in each time period (month). The soils in the command area mostly consist of red sandy loams with a field capacity of 15% and a permanent wilting point of 5%. The available soil moisture of 10% is divided into five states.

The water requirements of the crops are estimated using the modified Penman method. The growth stages of the crops and their yield response factors for different growth stages are adopted from Doorenbos & Kassam (1979). Monthly inflows into the reservoir measured over a period of 15 years are available at the site. These flows are modelled using a Periodic Auto Regressive Moving Average model (PARMA1,1)] and 100 sequences, each of 15 years, are generated for use in the simulations.

RESULTS AND DISCUSSION

The three stochastic dynamic programming models were applied to the Sri Rama Sagar Reservoir and optimal operating policies were obtained. The operation of the reservoir under each of the three operating policies was simulated using the synthetic data generated. The three performance indicators discussed above were estimated for all the three operating policies and are presented in Table 2.

The results indicate that there is fairly large difference between the performance of the reservoir under the operating policy developed using Model III and the operating policies developed using the other two models. It is evident that Model III gives a better operating policy. The operating policy developed using Model I is inferior to the other two operating policies.

All the three models use a stochastic dynamic programming technique, but they differ in terms of the objective function and mathematical complexity. Model I is a relatively simple model while Model III is relatively more complex. The purpose of this study was to examine the effect of increasing detail in the model on the system performance.

Table 2 Performance of the reservoir under the alternative operating policies.

Performance index	Model I	Model II	Model III
Reliability	0.536	0.628	0.694
Resilience	0.464	0.533	0.565
Average annual deficit (Mm^3)	725.6	656.2	526.9

CONCLUSIONS

Three stochastic dynamic programming models with varying degrees of mathematical complexity were developed. The performance of the reservoir under the operating policies derived from each of the models was compared. The results indicate that Model III performs better than the other two models and thus can be used to develop operating policies of irrigation reservoirs.

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