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Stokes' first problem for a micropolar fluid through state-space approach

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Abstract

The flow of an incompressible micropolar fluid over a suddenly moved plate is considered under isothermal conditions. State-space technique is used to find the solution of the problem. Inversion of Laplace transform is carried out using a numerical approach. The variation of velocity and microrotation fields is studied with respect to various flow parameters and the results are presented through graphs.

Keywords: Micropolar fluid; State-space technique; Stokes' first problem; Laplace transform; Numerical inversion

1. Introduction

Four decades have passed ever since the theory of micropolar fluid was initiated by Eringen [1] in 1966. It is well known that in many of the real fluids the shear behavior cannot be characterized by Newtonian relationships and hence researchers have proposed diverse non-Newtonian fluid theories to explain the deviation in the behavior of real fluids with that of Newtonian fluids. One such theory is that of micropolar fluids. This theory accounts for the internal characteristics of the substructure particles with the assumption that they are allowed to undergo rotation independent of their linear velocity. Micropolar fluids represent fluids consisting of rigid randomly oriented particles suspended in a viscous medium when the deformation of the particles is ignored. This constitutes a substantial generalization of the Navier–Stokes model. Micropolar fluids belong to the class of fluids with non-symmetric stress tensor which are referred to as polar fluids. This is a class which is more general than the one which we face in classical fluid dynamics. The theory of micropolar fluids may form a suitable non-Newtonian fluid model that can be used to analyze the behavior of lubricants, colloidal suspensions, polymeric fluids, liquid crystals and animal blood. The equations of motion characterizing a micropolar fluid flow are non-linear in nature (as in the case of Newtonian viscous fluids) and are constituted by a coupled system of vector differential equations. In any micropolar fluid flow problem, in addition to the

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Nomenclature

(x, y)	space coordinates
u	velocity of the fluid along the x-direction
U	velocity of the plate
c	microrotation component
ρ	density
t	time
p	pressure
j	gyration parameters
$\bar{q}$	velocity vector
$\bar{\upsilon}$	microrotation vector
$\bar{f}$	body forces per unit mass
$\bar{l}$	body couple per unit mass
λ_1, μ, k	viscosity coefficients
α, β, γ	gyro viscosity coefficients
υ_i	components of the microrotation vector
ω_i	components of the vorticity vector
d_{ij}	components of the rate of strain
δ_{ij}	kronecker symbol
t_{ij}	force stress tensor
m_{ij}	couple stress tensor
ε_{ijk}	Levi-Civita symbol or permutation symbol

usual field variables pressure p and velocity vector $\bar{q}$, we come across another field variable called microrotation vector $\bar{\upsilon}$ which is independent of $\bar{q}$. To understand the departure from the viscous fluid flow model, several problems that were studied in viscous fluid flow theory have also been studied in the realm of micropolar fluids. An account of the earlier developments in polar fluid theory can be seen in [2] of V.K. Stokes and the existing state of art can be seen in the excellent treatise of Lukaszewicz [3].

In this paper, we propose to study the Stokes' first problem for a micropolar fluid using state-space approach, which has been used till recently in modern control systems theory. Consider an infinitely long flat plate above which a fluid exists. Initially both the plate and fluid are assumed to be at rest. Let us suddenly impart a constant velocity to the plate in its own plane. Stokes in 1851 and again Raylegh in 1911 have discussed the fluid motion above the plate independently taking the fluid to be Newtonian [4]. In the literature this problem is referred to as Stokes' first problem. Subsequently in 1962 Tanner [5] considered the above problem with Maxwell fluid in place of the Newtonian fluid. Prezoisi and Joseph [6] and Phan-Thien and Chew [7] studied Stokes' first problem for viscoelastic fluids. In recent years many investigators have studied Stokes' first problem for non-Newtonian fluids with different constitutive equations (see Ref. [8–16]). In this paper we study the problem for an incompressible micropolar fluid whose constitutive equations were proposed by Eringen [1]. We solve the problem through the method of state-space formulation which is more general than the classical Laplace transform and Fourier transform techniques. The state-space theory is applicable to all systems that can be analyzed by integral transforms in time and is successfully employed to study, in particular, problems in modern control theory. As Helmy et al. [17] observe, the state-space approach is useful to study linear systems with time varying parameters in essentially the same manner as the time invariant linear systems.

2. Basic equations for incompressible micropolar fluid flow

The field equations of micropolar fluid dynamics are [1],

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \bar{q}) = 0, \tag{2.1}$$

$$\rho \frac{\mathrm{d}\bar{q}}{\mathrm{d}t} = \rho \bar{f} - \mathrm{grad}(p) + k\mathrm{curl}(\bar{\upsilon}) - (\mu + k)\mathrm{curl}\,\mathrm{curl}(\bar{q}) + (\lambda_1 + 2\mu + k)\mathrm{grad}(\mathrm{div}(\bar{q})), \tag{2.2}$$

$$\rho j \frac{\mathrm{d}\bar{\upsilon}}{\mathrm{d}t} = \rho \bar{l} - 2k\bar{\upsilon} + k\mathrm{curl}(\bar{q}) - \gamma \mathrm{curl}\,\mathrm{curl}(\bar{\upsilon}) + (\alpha + \beta + \gamma)\mathrm{grad}(\mathrm{div}(\bar{\upsilon})). \tag{2.3}$$

In the above, the scalar quantities ρ and j are, respectively, the density and gyration parameters and are assumed constants. The vectors $\bar{q}, \bar{\upsilon}, \bar{f}$ and $\bar{l}$ are the velocity, microrotation, body force per unit mass and body couple per unit mass, respectively. p is the fluid pressure at any point. The material constants λ_1, μ, k are the viscosity coefficients and α, β, γ are the gyroviscosity coefficients. These constants confirm to the inequalities,

$$k \geqslant 0; \quad 2\mu + k \geqslant 0; \quad 3\lambda_1 + 2\mu + k \geqslant 0,$$
$$\gamma \geqslant 0; \quad |\beta| \leqslant \gamma; \quad 3\alpha + \beta + \gamma \geqslant 0.$$

The stress tensor t_{ij} and the couple stress tensor m_{ij} are given by

$$t_{ij} = (-p + \lambda \mathrm{div}(\bar{q}))\delta_{ij} + (2\mu + k)d_{ij} + k\varepsilon_{ijm}[\omega_m - \upsilon_m], \tag{2.4}$$

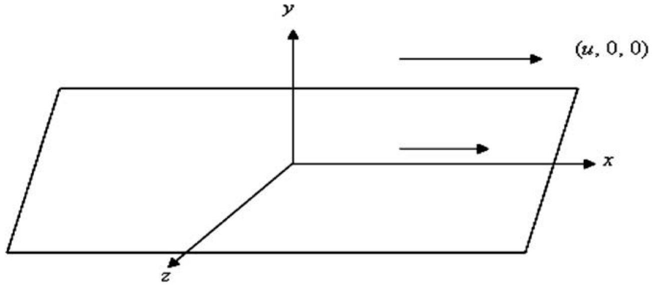
$$m_{ij} = \alpha(\mathrm{div}(\bar{\upsilon}))\delta_{ij} + \beta \upsilon_{i,j} + \gamma \upsilon_{j,i}. \tag{2.5}$$

In Eqs. (2.4) and (2.5), υ_i and ω_i are the components of the micro rotation vector and the vorticity vector respectively, d_{ij} are the components of the rate of strain, δ_{ij} denotes Kronecker symbol, ε_{ijk} is the Levi-Civita symbol and comma denotes covariant differentiation.

The boundary conditions usually employed in the solution of these equations are that the velocity $\bar{q}$ at the boundary equals to that of the velocity $\bar{q_{\mathrm{B}}}$ of the boundary and that the microrotation $\bar{\upsilon}$ at the boundary equals to the rotational velocity $\bar{\upsilon_{\mathrm{B}}}$ of the boundary [18,19]. This choice of the boundary condition can be expressed by the statement that there is no slip at the boundary and that a fluid particle is inflexibly attached to it so that the micro rotational velocity of the particle on the boundary equals to the angular velocity of the boundary. This is based on an analogy with the no slip condition of the classical Fluid Dynamics and it may be referred to as superadherence condition or hyperstick condition.

3. Formulation of the problem

Consider the laminar flow of an incompressible, micropolar fluid above the half-space $y > 0$. Taking the positive y-axis of a Cartesian coordinate system in the upward direction, the fluid flows through the half-space $y > 0$ above and in contact with a flat plate occupying xz-plane. Initially both the fluid and plate are at rest. At time $t = 0+$, the plate suddenly starts to slide slowly in its plane with a constant speed U. Under these conditions, no flow occurs in the y and z directions and the flow velocity at a given point in the half-space depends only on its y-coordinate and time. i.e., the velocity field is of the form $\bar{q} = u(y,t)\boldsymbol{i}$, where $\boldsymbol{i}$ is the unit vector in the x-coordinate direction. The microrotation field will be in the form $\bar{\upsilon} = (0, 0, c(y,t))$. The flow is assumed to be generated by the motion of the flat plate and not by any pressure change. The pressure in the whole space is constant.



This velocity field automatically satisfies the continuity equation (2.1). Now, the equations governing the flow are given by

$$\rho \frac{\partial u}{\partial t} = k \frac{\partial c}{\partial y} + (\mu + k) \frac{\partial^2 u}{\partial y^2}, \tag{3.1}$$

$$\rho j \frac{\partial c}{\partial t} = -2kc - k \frac{\partial u}{\partial y} + \gamma \frac{\partial^2 c}{\partial y^2}, \tag{3.2}$$

with the conditions

$$\begin{aligned} &\text{for } t \leqslant 0, \quad u(y,t) = 0 \quad \text{and} \quad c(y,t) = 0 \quad \text{for all } y, \\ &\text{for } t > 0, \quad u(0,t) = U, \quad c(0,t) = 0, \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} &u(y,t) \to 0 \text{ as } y \to \infty \\ &c(y,t) \to 0 \text{ as } y \to \infty. \end{aligned} \tag{3.4}$$

Using the non-dimensional scheme

$$u = Uu^*, \quad y = \frac{\mu + k}{\rho U} y^*, \quad c = \frac{\rho U^2}{\mu + k} c^* \quad \text{and} \quad t = \frac{\mu + k}{\rho U^2} t^*. \tag{3.5}$$

Eqs. (3.1) and (3.2) reduce to

$$\frac{\partial u^*}{\partial t^*} = m \frac{\partial c^*}{\partial y^*} + \frac{\partial^2 u^*}{\partial y^{*2}}, \tag{3.6}$$

$$\frac{\partial c^*}{\partial t^*} = -2nn_2 c^* - nn_2 \frac{\partial u^*}{\partial y^*} + n_2 \frac{\partial^2 c^*}{\partial y^{*2}}, \tag{3.7}$$

where

$$m = \frac{k}{\mu + k}, \quad n = \frac{k}{\gamma} \left(\frac{\mu + k}{\rho U} \right)^2 \quad \text{and} \quad n_2 = \frac{\gamma}{j(\mu + k)}.$$

Dropping *s, we get,

$$\frac{\partial u}{\partial t} = m \frac{\partial c}{\partial y} + \frac{\partial^2 u}{\partial y^2}, \tag{3.8}$$

$$\frac{\partial c}{\partial t} = -2nn_2 c - nn_2 \frac{\partial u}{\partial y} + n_2 \frac{\partial^2 c}{\partial y^2}, \tag{3.9}$$

with conditions,

$$u(y,t) = 0 \quad \text{and} \quad c(y,t) = 0 \quad \text{for all} \quad y \quad \text{and for } t \leqslant 0, \tag{3.10}$$

and

$$\begin{aligned} &u(0,t) = 1, \quad c(0,t) = 0, \\ &u(y,t) \to 0 \quad \text{and} \quad c(y,t) \to 0 \quad \text{as } y \to \infty \quad \text{for } t > 0. \end{aligned} \tag{3.11}$$

4. Solution of the problem

We shall now recast Eqs. (3.8)–(3.11) in an alternative form using state-space technique. Taking Laplace transform to Eqs. (3.8), (3.9), and (3.11) with respect to t and using initial conditions (3.10), we obtain,

$$\frac{d^2\bar{u}}{dy^2} + m\frac{d\bar{c}}{dy} - s\bar{u} = 0, \tag{4.1}$$

$$\frac{d^2\bar{c}}{dy^2} - n\frac{d\bar{u}}{dy} - \left(2n + \frac{s}{n_2}\right)\bar{c} = 0, \tag{4.2}$$

with the conditions,

$$\bar{u}(0,s) = \frac{1}{s}, \quad \bar{c}(0,s) = 0, \quad \bar{u}(y,s) \to 0 \quad \text{as } y \to \infty \quad \text{and} \quad \bar{c}(y,s) \to 0 \quad \text{as } y \to \infty. \tag{4.3}$$

Introducing the variables,

$$\bar{u'} = \frac{d\bar{u}}{dy}, \tag{4.4}$$

and

$$\bar{c'} = \frac{d\bar{c}}{dy}. \tag{4.5}$$

Eqs. (4.1) and (4.2) reduce to

$$\frac{d\bar{u'}}{dy} = s\bar{u} - m\bar{c'}, \tag{4.6}$$

$$\frac{d\bar{c'}}{dy} = \left(2n + \frac{s}{n_2}\right)\bar{c} + n\bar{u'}. \tag{4.7}$$

Writing Eqs. (4.4)–(4.7) in matrix form, we get,

$$\frac{d}{dy}\begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{u'} \\ \bar{c'} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ s & 0 & 0 & -m \\ 0 & \left(2n + \frac{s}{n_2}\right) & n & 0 \end{pmatrix}\begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{u'} \\ \bar{c'} \end{pmatrix}, \tag{4.8}$$

or

$$\frac{d}{dy}\bar{V}(y,s) = A(s)\bar{V}(y,s) \tag{4.9}$$

where

$$A(s) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ s & 0 & 0 & -m \\ 0 & p & n & 0 \end{pmatrix}, \quad \bar{V}(y,s) = \begin{pmatrix} \bar{u}(y,s) \\ \bar{c}(y,s) \\ \bar{u'}(y,s) \\ \bar{c'}(y,s) \end{pmatrix} \quad \text{and} \quad p = 2n + \frac{s}{n_2}. \tag{4.10}$$

Following the state-space technique as is used in problems dealing with modern control theory [20], we obtain the formal solution of Eq. (4.9) as

$$\bar{V}(y,s) = \exp[A(s)y]\bar{V}(0,s). \tag{4.11}$$

To enable us to determine the matrix $\exp[A(s)y]$, we note that the characteristic equation of the matrix $A(s)$ is

$$k^4 - (s + p - mn)k^2 + sp = 0. \tag{4.12}$$

The characteristic roots $\pm k_1$, $\pm k_2$ of the Eq. (4.12) satisfy the relations

$$k_1^2 + k_2^2 = s + p - mn,$$

$$k_1^2 k_2^2 = sp,$$

where k_1, k_2 are taken as the roots with positive real parts.

The Maclaurin's series expansion of $\exp[A(s)y]$ is given by

$$\exp[A(s)y] = \sum_{r=0}^{\infty} \frac{[A(s)y]^r}{r!}. \tag{4.13}$$

Therefore, the infinite series (4.13) can be written in the form

$$\exp[A(s)y] = L(y,s) = a_0 I + a_1 A + a_2 A^2 + a_3 A^3, \tag{4.14}$$

using the Cayley–Hamilton theorem where I is the unit matrix of order 4 and a_0, a_1, $a_2$anda_3 are some parameters depending on y and s. The characteristic roots $\pm k_1$, $\pm k_2$ satisfy the Eq. (4.14) and hence by replacing the matrix A with its characteristic roots $\pm k_1$, $\pm k_2$ therein, we get the following system of linear equations:

$$\left.\begin{aligned} \exp[k_1 y] &= a_0 + a_1 k_1 + a_2 k_1^2 + a_3 k_1^3, \\ \exp[-k_1 y] &= a_0 - a_1 k_1 + a_2 k_1^2 - a_3 k_1^3, \\ \exp[k_2 y] &= a_0 + a_1 k_2 + a_2 k_2^2 + a_3 k_2^3, \\ \exp[-k_2 y] &= a_0 - a_1 k_2 + a_2 k_2^2 - a_3 k_2^3, \end{aligned}\right\} \tag{4.15}$$

for the determination of a_0, a_1, a_2 and a_3. The above system can easily be solved and we get

$$\left.\begin{aligned} a_0 &= \tfrac{1}{F}\left[k_1^2 \cosh(k_2 y) - k_2^2 \cosh(k_1 y)\right], \\ a_1 &= \tfrac{1}{F}\left[\tfrac{k_1^2}{k_2} \sinh(k_2 y) - \tfrac{k_2^2}{k_1} \sinh(k_1 y)\right], \\ a_2 &= \tfrac{1}{F}[\cosh(k_1 y) - \cosh(k_2 y)], \\ a_3 &= \tfrac{1}{F}\left[\tfrac{1}{k_1} \sinh(k_1 y) - \tfrac{1}{k_2} \sinh(k_2 y)\right], \end{aligned}\right\} \tag{4.16}$$

where

$$F = k_1^2 - k_2^2.$$

Substituting these into Eq. (4.11), computing A^2, A^3 and substituting these in Eq. (4.14), we obtain the matrix elements (L_{ij}; $i, j = 1,2,3,4$) of the matrix $L(y, s)$ as

$$L_{11} = \frac{1}{F}\left\{(k_1^2 - s)\cosh(k_2 y) - (k_2^2 - s)\cosh(k_1 y)\right\},$$

$$L_{12} = \frac{-mp}{F}\left\{\frac{1}{k_1}\sinh(k_1 y) - \frac{1}{k_2}\sinh(k_2 y)\right\},$$

$$L_{13} = \frac{1}{F}\left\{\left(\frac{k_1^2 - (s - mn)}{k_2}\right)\sinh(k_2 y) - \left(\frac{k_2^2 - (s - mn)}{k_1}\right)\sinh(k_1 y)\right\},$$

$$L_{14} = \frac{-m}{F}\{\cosh(k_1 y) - \cosh(k_2 y)\},$$

$$L_{21} = \frac{ns}{F}\left\{\frac{1}{k_1}\sinh(k_1 y) - \frac{1}{k_2}\sinh(k_2 y)\right\},$$

$$L_{22} = \frac{1}{F}\left\{\left(k_1^2 - p\right)\cosh(k_2 y) - \left(k_2^2 - p\right)\cosh(k_1 y)\right\}; \quad L_{23} = -\frac{n}{m}L_{14},$$

$$L_{24} = \frac{1}{F}\left\{\left(\frac{k_1^2 - (p - mn)}{k_2}\right)\sinh(k_2 y) - \left(\frac{k_2^2 - (p - mn)}{k_1}\right)\sinh(k_1 y)\right\},$$

$$L_{31} = sL_{13}; \quad L_{32} = pL_{14}; \quad L_{33} = L_{11} + nL_{14}$$

$$L_{34} = \frac{m}{F}\{k_2\sinh(k_2 y) - k_1\sinh(k_1 y)\}; \quad L_{41} = -\frac{ns}{m}L_{14},$$

$$L_{42} = pL_{24}; \quad L_{43} = -\frac{n}{m}L_{34}; \quad L_{44} = L_{22} + nL_{14},$$

with this, we get the solution of (4.9) in the form

$$\bar{V}(y,s) = L(y,s)\bar{V}(0,s), \tag{4.17}$$

where L_{ij}s are explicitly written as above. We note that the solution on the right hand side above contains terms $e^{k_1y}, e^{k_2y}, e^{-k_1y}$ and e^{-k_2y}only. The terms involving $e^{k_1y}, e^{k_2y}, e^{-k_1y}$ and e^{-k_2y} can all be individually grouped and the solution $\bar{V}(y,s)$ can be written from Eq. (4.17). It can be noticed that the solution involves $\bar{u}(0,s), \bar{c}(0,s), \bar{u}'(0,s)$ and $\bar{c}'(0,s)$ out of which $\bar{u}(0,s), \bar{c}(0,s)$ are known and the other two are unknown. However, the regularity of $\bar{u}(y,s)$, $\bar{c}(y,s)$ far away from the plate at $y=\infty$, enables us to determine $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$ as detailed below.

Using Eq. (4.17) and expressions of L_{ij}, we note that

$$\begin{aligned}\bar{u}(y,s) = &-\frac{e^{k_1y}}{2F}\left\{\frac{k_2^2-s}{s}+\frac{p-k_1^2}{k_1}\bar{u}'(0,s)+m\bar{c}'(0,s)\right\}+\frac{e^{k_2y}}{2F}\left\{\frac{k_1^2-s}{s}+\frac{p-k_2^2}{k_2}\bar{u}'(0,s)+m\bar{c}'(0,s)\right\}\\ &-\frac{e^{-k_1y}}{2F}\left\{\frac{k_2^2-s}{s}-\frac{p-k_1^2}{k_1}\bar{u}'(0,s)+m\bar{c}'(0,s)\right\}\\ &+\frac{e^{-k_2y}}{2F}\left\{\frac{k_1^2-s}{s}-\frac{p-k_2^2}{k_2}\bar{u}'(0,s)+m\bar{c}'(0,s)\right\},\end{aligned} \tag{4.18}$$

and

$$\begin{aligned}\bar{c}(y,s) = &\frac{e^{k_1y}}{2F}\left\{\frac{n}{k_1}+n\bar{u}'(0,s)-\frac{s-k_1^2}{k_1}\bar{c}'(0,s)\right\}-\frac{e^{k_2y}}{2F}\left\{\frac{n}{k_2}+n\bar{u}'(0,s)-\frac{s-k_2^2}{k_2}\bar{c}'(0,s)\right\}\\ &+\frac{e^{-k_1y}}{2F}\left\{-\frac{n}{k_1}+n\bar{u}'(0,s)+\frac{s-k_1^2}{k_1}\bar{c}'(0,s)\right\}+\frac{e^{-k_2y}}{2F}\left\{\frac{n}{k_2}-n\bar{u}'(0,s)-\frac{s-k_2^2}{k_2}\bar{c}'(0,s)\right\}.\end{aligned} \tag{4.19}$$

Similarly the expressions for $\bar{u}'(y,s)$ and $\bar{c}'(y,s)$ can be written.

Since $\bar{u}(y,s)$ and $\bar{c}(y,s)$ must satisfy the regularity condition at $y=\infty$, the coefficients of e^{k_1y} and e^{k_2y} must be zeros in view of the positive nature of the real parts of k_1 and k_2. This leads to the determination of $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$ as

$$\left.\begin{aligned}\bar{u}'(0,s) &= -\frac{(k_1+k_2)}{(k_1k_2+s)},\\ \bar{c}'(0,s) &= \frac{n}{(k_1k_2+s)}.\end{aligned}\right\} \tag{4.20}$$

It can be verified that these expressions will also make the coefficients of e^{k_1y} and e^{k_2y} in $\bar{u}'(y,s)$and$\bar{c}'(y,s)$ equal to zero.

The above expressions (4.20) for $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$ can also be obtained by an alternate procedure.

In view of the finiteness of the solution as $y\to\infty$, we have to drop the positive exponentials that are unbounded at infinity. This is equivalent to the replacement of $\sinh(ky)$ and $\cosh(ky)$, respectively, by $(-1/2)\exp(-ky)$ and $(1/2)\exp(-ky)$. This leads us to the relevant expression in place of $L(y,s) = \exp[A(s)y]$. Let this be denoted by $L^*(y,s)$. Using this in Eq. (4.17), we get

$$\bar{V}(y,s) = L^*(y,s)\bar{V}(0,s), \tag{4.21}$$

which is a solution of Eq. (4.9) satisfying the regularity condition at $y=\infty$.

The two components $\bar{u}(0,s)$ and $\bar{c}(0,s)$ of the vector $\bar{V}(0,s)$ are given in (4.3). To obtain the remaining components $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$ of the vector $\bar{V}(0,s)$, we substitute $y=0$ into Eq. (4.21) to get a system of two equations in the two unknowns $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$. Solving this system, we get the same expressions for $\bar{u}'(0,s)$ and $\bar{c}'(0,s)$ as in (4.20).

The velocity and the microrotation in the Laplace transform domain can now be obtained by using (4.18) and (4.19), taking into account (4.20). These, after considerable algebra are seen to be

$$\bar{u}(y,s) = \frac{1}{s(C_1 - C_2)}[C_1 e^{-k_2 y} - C_2 e^{-k_1 y}], \tag{4.22}$$

$$\bar{c}(y,s) = \frac{k_1 k_2 n}{s(C_1 - C_2)}[e^{-k_2 y} - e^{-k_1 y}], \tag{4.23}$$

where

$$C_1 = k_1(p - k_2^2) \quad \text{and} \quad C_2 = k_2(p - k_1^2).$$

The expressions for $\bar{u}(y,s)$ and $\bar{c}(y,s)$ are in terms of k_1, k_2, p, C_1 and C_2 each one of which depends upon s and hence the analytical inversion of these expressions is not possible. We have to adopt a suitable numerical procedure to get $u(y,t)$ and $c(y,t)$ for different values of y and t. We have chosen the procedure suggested by Honig and Hirdes [21] to determine numerically $u(y,t)$ and $c(y,t)$.

4.1. Numerical inversion of the Laplace transforms

In order to invert $\bar{u}(y,s)$ and $\bar{c}(y,s)$, we adopt the numerical inversion technique proposed by Honig and Hirdes [21]. Using this method, the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by

$$f(t) = \frac{e^{c^* t}}{t_1}\left[\frac{1}{2}\bar{f}(c^*) + \mathrm{Re}\left(\sum_{k=1}^{N}\bar{f}\left(c^* + \frac{\mathrm{i}k\pi}{t_1}\right)\exp\left(\frac{\mathrm{i}k\pi t}{t_1}\right)\right)\right], \quad 0 < t_1 < 2t,$$

where c^* is an arbitrary constant greater than all the real parts of the singularities of $f(t)$ and N is sufficiently large integer chosen such that,

$$e^{c^* t}\mathrm{Re}\left[\bar{f}\left(c^* + \frac{\mathrm{i}N\pi}{t_1}\right)\exp\left(\frac{\mathrm{i}N\pi t}{t_1}\right)\right] < \varepsilon,$$

where ε is a prescribed small positive number that corresponds to the degree of accuracy required.

5. Results and discussion

Using the numerical procedure cited, to invert the expressions of velocity and microrotaion components in Laplace transform domain, the variation of the velocity component u and the microrotation component c is plotted for different values of y and t for various values of microrotation parameters. As the distance y is increasing, the velocity component is decreasing and tending to zero. This is exactly the observation that was made in the case of Newtonian viscous fluid [4]. Fig. 1 shows the variation of the velocity for different times with the micropolarity parameters m, n and n_2 fixed. The velocity for any y is increasing with respect to time.

The case $m = 0$ with the suppression of the other micropolarity parameters as well results in the classical case of Newtonian fluid discussed by Schlichting [4]. Fig. 2 indicates the variation in velocity u for a fixed time t and fixed n, n_2 as m varies. Here as m increases the velocity decreases. This implies that, as microrotation viscosity k increases, it reduces the velocity at any point of the fluid. Thus, an increase in microrotational viscosity has a decreasing effect on the velocity component in comparison with that of the Newtonian fluid case. This decrease is due to the presence of microrotation, since some portion of the energy is dissipated due to the friction between the rotating particles (see page 218 of Grzegorz Lukaszewicz [3]).

Fig. 3 shows the variation in velocity for different values of n at a fixed time and for fixed values of m and n_2. Here also the velocity decreases as n increases and tends to zero as y increases. Fig. 4 shows the variation of velocity with distance for different values of n_2 at fixed t, m and n. In Fig. 4, the velocity profiles for different values of n_2 seem to be almost overlapping.

Figs. 5–8 depict the variation of microrotation with distance. The microrotation in all these figures is initially decreasing; subsequently reaching a maximum and latter as distance increases is dying down. This trend is seen to be true whether we vary time fixing all other parameters, vary the micropolarity parameter m fixing time and other parameters, varying n fixing time and other parameters or varying n_2 and fixing time and other parameters. Far away from the moving plate, the fluid velocity and microrotaional velocity are both vanishing.

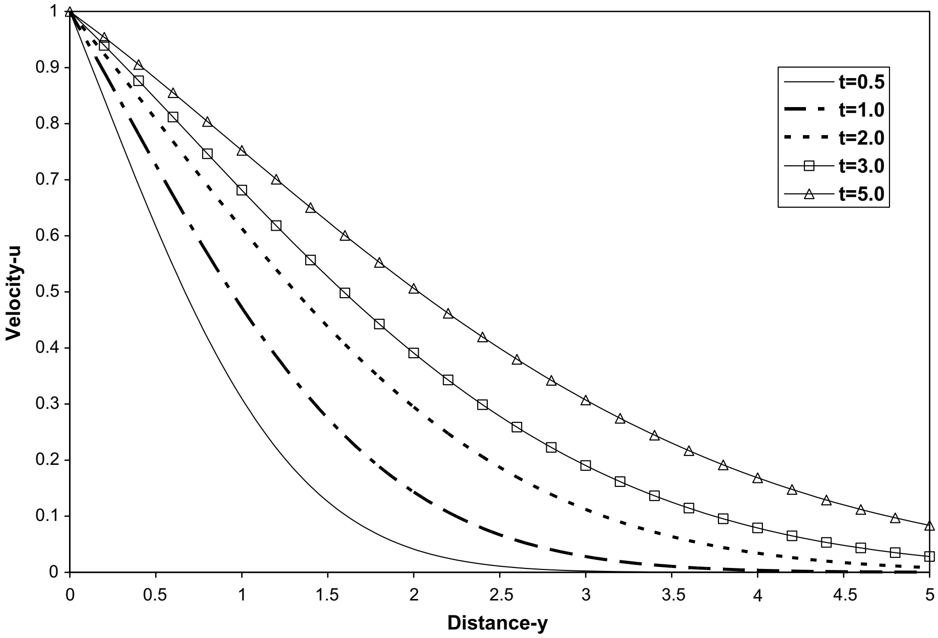


Fig. 1. Variation of velocity with distance for different times at $m = 0.5$; $n = 1.0$; $n_2 = 0.5$.

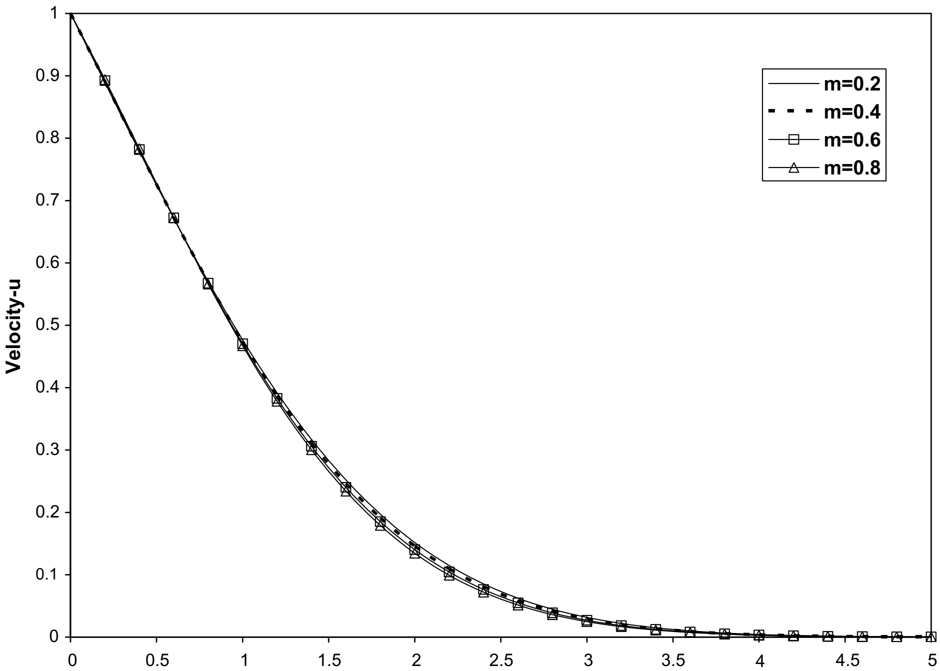


Fig. 2. Variation of velocity with distance for different values of m at $t = 1.0$; $n = 1.0$; $n_2 = 0.5$.

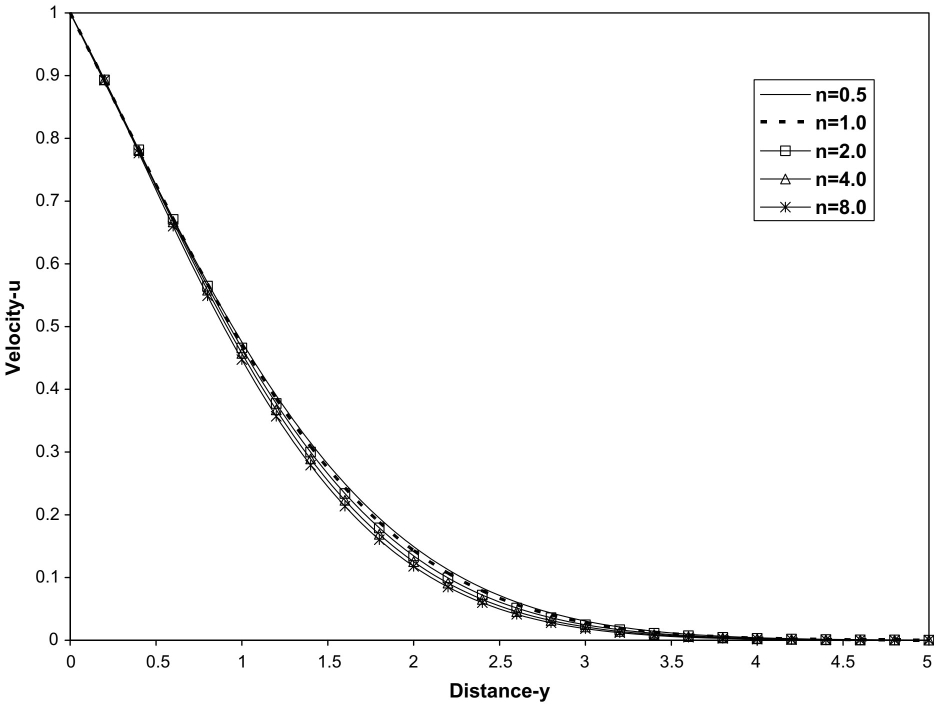


Fig. 3. Variation of velocity with distance for different values of n at $t = 1$; $m = 0.5$; $n_2 = 0.5$.

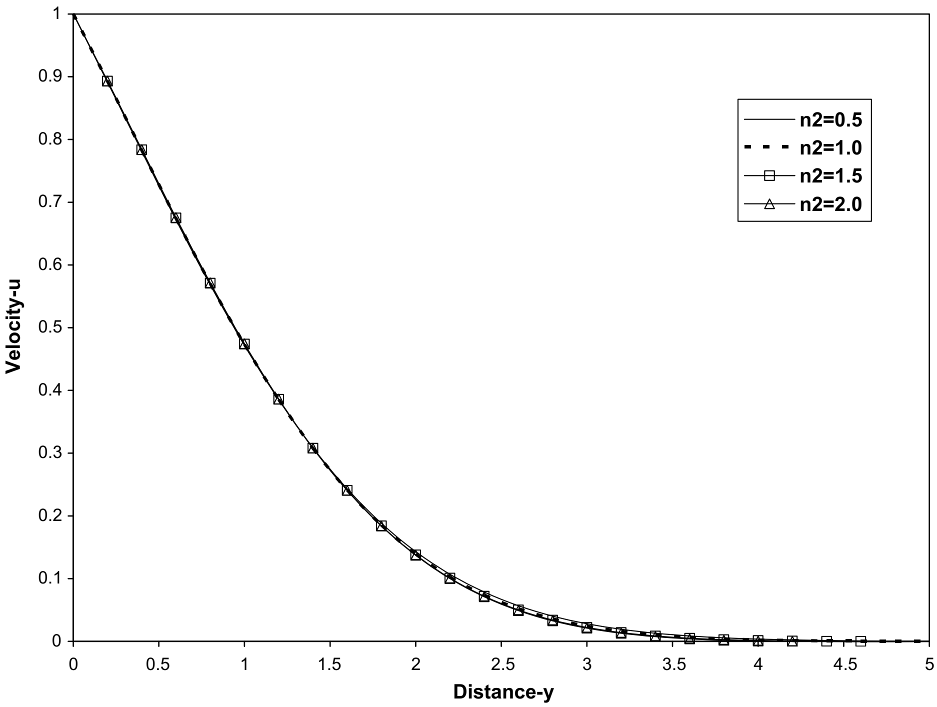


Fig. 4. Variation of velocity with distance for different values of n_2 at $t = 1$; $m = 0.5$; $n = 1$.

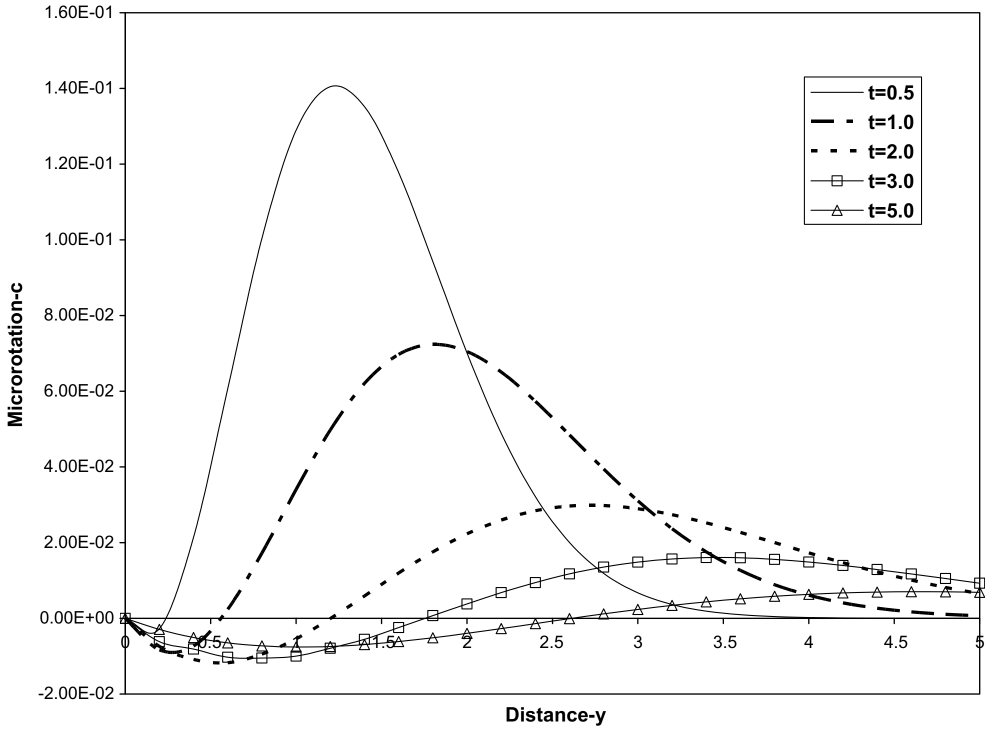


Fig. 5. Variation of microrotation with distance for different times when $m = 0.5$; $n = 1$; $n_2 = 0.5$.

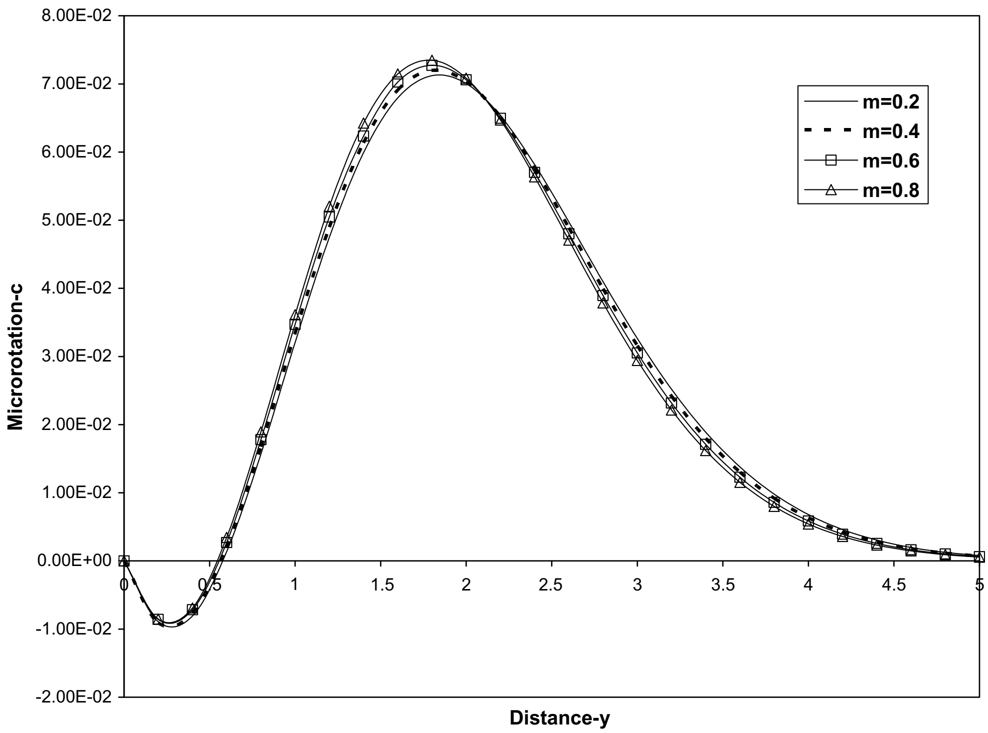


Fig. 6. Variation of microrotation with distance for different values of m when $t = 1$; $n = 1$; $n_2 = 0.5$.

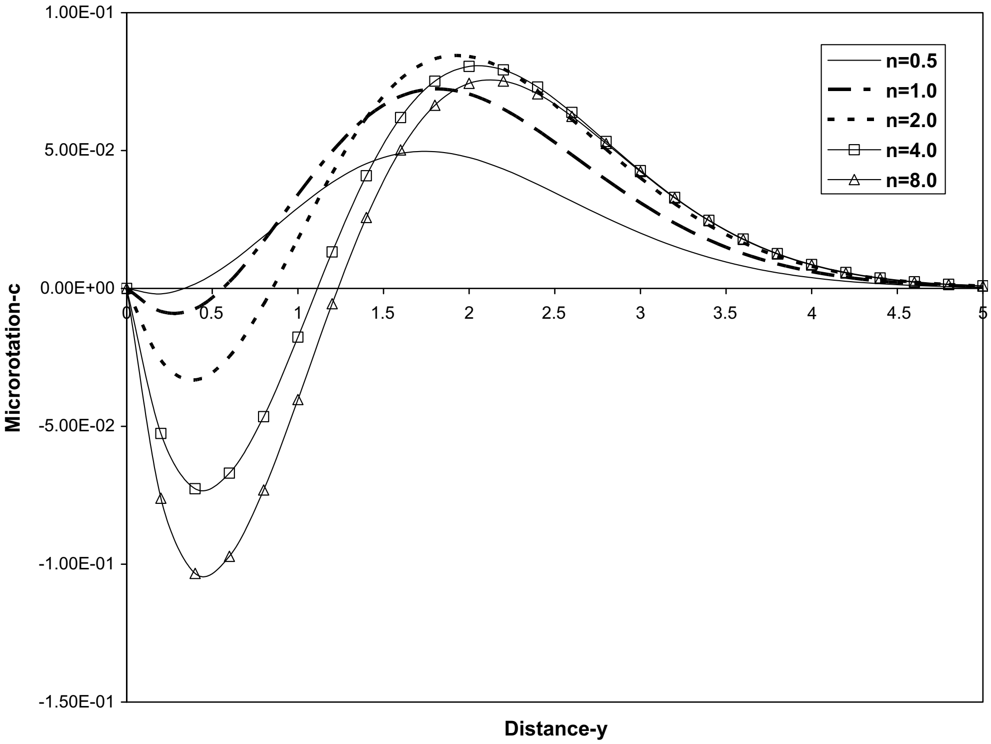


Fig. 7. Variation of microrotation with distance for different values of n when $t = 1$; $m = 0.5$; $n_2 = 0.5$.

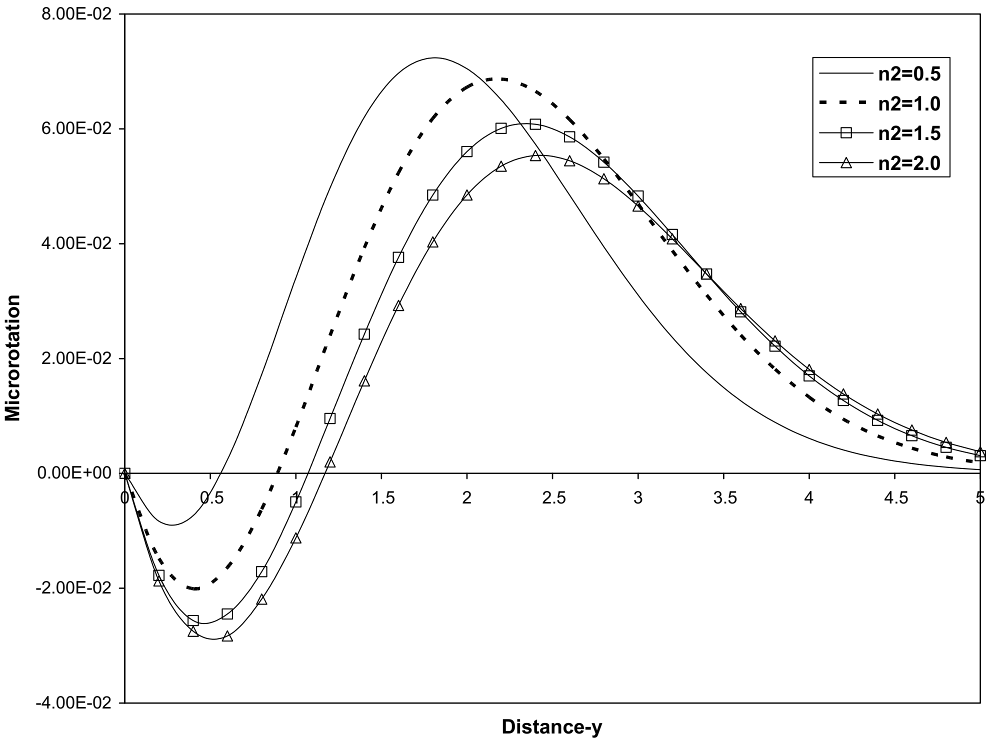


Fig. 8. Variation of microrotation with distance for different values of n_2 when $t = 1$; $m = 0.5$; $n = 1$.

6. Conclusions

The Stokes' first problem for an incompressible micropolar fluid is solved making use of the state-space approach. The micropolar fluid flow equations, in view of their coupled nature, are to be solved, in general, by decoupling. The state-space formulation has enabled us to solve the problem directly. The velocity and microrotation components are obtained in the Laplace transform domain and their expressions are inverted numerically using the numerical inversion technique due to Honig and Hirdes [21]. It is noticed that the velocity in the case of micropolar fluid flow decreases in comparison with the Newtonian fluid flow case as the microrotation of the particles in the medium causes a dissipation of energy leading to the observed decrease in the velocity.

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