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Optimisation of cam-follower motion using B-splines

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This paper proposes design of cam-follower velocity curve by using B-spline polynomials. B-spline polynomials are smooth curves defined by control points. Curve shape can be modified by changing the control points. The traditional design method for improving the motion characteristics of the follower is to find an optimum displacement curve for which follower velocity, acceleration curves to be continuous and their peak values as small as possible (i.e. minimum jerk). B-spline polynomials of degree three and six control points are used in design of follower velocity curve. The B-spline curve is used to approximate the various basic curves which have better motion characteristics. A computer-aided design and computer-aided manufacturing (CAD/CAM) system is developed which generates follower motion curves, i.e. displacement, velocity, acceleration, jerk and cam profiles. An analysis is carried out on B-splines with basic curves for maximum accelerations to select the best cam follower motion. It also provides cam profile coordinates to manufacture a cam on computer numerical control (CNC) machines.

Keywords: disc-cams; cam profile, B-spline; computer numerical control (CNC); computer-aided design and computer-aided manufacturing (CAD/CAM); R-D-R-D (rise-dwell-return-dwell)

1. Introduction

A cam is mechanical member used to transform rotary motion into translating or oscillating motion to its follower by direct contact (Roth Bart 1956, Jensen 1965). Cam may be a plate cam or a disc cam which is cut out of a piece of flat metal, drum cam in which a groove is cut on the surface of cylinder, globoidal cam and circular cam. Applications of these cams are found in packaging machines, wire-forming machines, internal combustion engines, mechanical and electronic computers. Requirements for high performance of such machinery demands efficient methods for the design and manufacture of cams.

In a cam–follower mechanism, the load produced by inertia forces induces deflections and creates vibrations. These affect the operating life of the cam. So the design of motion curves to minimise dynamic loading is of importance for high-speed cam mechanisms. It is well known that the velocity and acceleration curves are required to be continuous and to have the peak values as small as possible (i.e. minimum jerk). The purpose of this paper is to develop a CAD/CAM system for cam-follower mechanisms and to design follower velocity curve using B-splines in view of improving motion characteristics.

To analyse the action of a cam, it is necessary to study its displacement–time diagram and its associated velocity and acceleration curves. Some of the basic follower motion curves selected by cam designers for a typical cam follower mechanism shown in Figure 1 are:

(1) Simple harmonic motion (S.H.M.).
(2) Cycloidal motion.
(3) Parabolic motion.
(4) 3–4–5 polynomial.
(5) 4–5–6–7 polynomial.
(6) Double harmonic motion.

Simple harmonic motion (Masood 1999) which has smoothness in velocity and acceleration during the stroke is an advantage inherent in the curve. However, the instantaneous changes in the acceleration at the beginning and end of the stroke tend to cause vibration, noise, and wear. It is, therefore, suitable only for cams at medium or low speed. Cycloidal motion is obtained by rolling a circle on a straight line. It has the smoothest motion among all of the basic curves. The maximum value of the acceleration of the follower for a given rise is somewhat higher than that of the simple harmonic motion.

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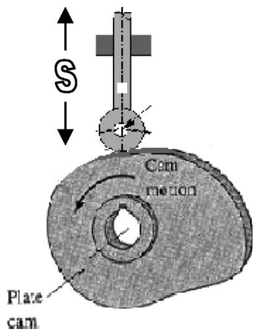

Figure 1. Plate cam.

Cycloidal curve is used often as a basis for designing cams for high-speed machinery because it results in low noise, vibration, and wear. Parabolic motion has constant acceleration and retardation following a parabolic equation. 3–4–5 polynomial is a 5^{th} degree and the 4–5–6–7 polynomial is a 7^{th} degree curve. They have good acceleration characteristics and are used for high-speed cams. Double harmonic motion is a modified simple harmonic curve in which every cycle starts from zero acceleration and can be used for high-speed cams.

1.1. Follower motion equations

Displacement (S) equations (Roth Bart 1956, Jensen 1965) for different follower motions against cam rotation angle θ have been presented as follows,

$$\begin{aligned}
&\text{Simple harmonic:} && S = h/2[1 - \cos(\Pi\theta/\beta)] \\
&\text{Cycloidal:} && S = h/2[\theta/\beta - \cos(2\Pi\theta/\beta)] \\
&\text{Parabolic motion:} && S = 2h[\theta/\beta]^2
\end{aligned}$$

$$\begin{aligned}
&3-4-5 \text{ polynomial:} && S = h[10(\theta/\beta)^3 - 15(\theta/\beta)^4 \\
& && \quad + 6(\theta/\beta)^5] \\
&4-5-6-7 \text{ poly:} && S = h[35(\theta/\beta)^4 - 84(\theta/\beta)^5 \\
& && \quad + 70(\theta/\beta)^6 - 20(\theta/\beta)^7] \\
&\text{Doubleharmonic:} && S = h/2[1 - \cos(\theta/\beta) \\
& && \quad - 0.25(1 - \cos(2\theta/\beta))]
\end{aligned}$$

where h represents lift, β is rise or return angle θ is the cam angle.

2. Literature review

Roth Bart (1956) designed a variable speed mechanism in which the input to the cam was the output of a with-worth quick-return mechanism. Chen (1969, 1972) developed numerical methods to synthesise cam displacement curves. These cam-profile synthesis methods assumed that the cam operated at a constant speed. Dhande *et al.* (1975) provided generalised expressions for equations of the conjugate profile, the pressure angle, and the locations of the cutter. Tesar and Mathew (1976) used analytical functions to synthesise cam profiles. They developed design charts for linear, damped, two degrees of freedom (DOF) models. They also derived motion equations of the follower by considering the case of variable speed cams. Lin *et al.* (1988) have developed a methodology by which cam drawings and NC codes can be automatically created after specifying the cam motion function. The procedure they have developed for cam design is based on a methodology called combined curves fitting.

Sadek and Daadbin (1990) proposed a method of smoothing the profile curve once it had been specified. Polynomial curve fitting was used to replace the profile curve. A two-degree-of freedom model for a cam mechanism is used as a test case. The conclusion is that with curve fitting using polynomials, the cam can cause less vibration than does the original cam and it has less tendency to bounce. However, their work does not deal with the development of diagrams, simulation and the manufacture of cams. Tsay and Wei (1993) developed a CAD system for cylindrical cams with a translating conical follower. Cams can be designed and graphics can be displayed once the follower motion program has been given. Based on the approach previously developed, the contact line of the follower at any angle of rotation can be obtained to find analytical profile expressions. All these procedures were carried out without the assistance of a CAD/CAM system. Tsay and Lin (1996) presented a procedure for the synthesis and analysis of the surface geometry of cylindrical cams with oscillating roller followers.

Chan and Sim (1996) developed a computer-aided design tool for optimum plate cam design. An exploratory search method called the Monte Carlo method was used to optimise the cam design. In this method, random points are generated over the range of all variables of the cam base circle, the width of the cam, the follower roller radius and its offset. The system is an integration of the design calculations and an optimisation algorithm. It provides an optimised solution, a graphical diagram, and a simulation of cam movement but does not provide the data for cam machining.

Tsay and Hwang (1996) developed a tool to synthesise the motion functions of the camoids-follower mechanisms using non-parametric B-splines. The characteristics of these kinds of motion functions are that they possess two independent parameters. To implement the work, this study applies the nonparametric B-spline surface interpolations, whose spline functions are constructed by the closed periodic B-splines and the de Boor's knot sequences in two parametric directions of the motion function, respectively. The rules and the restrictions needed to be noticed in the process of synthesis are established.

Srinivasan and Ge (1998) designed dynamically compensated and robust cam profiles with Bernstein–Bezier harmonics curves. They designed cam profiles to minimise residual vibrations in high cam-follower systems. The traditional Polydyne method is modified and extended to achieve significant improvement in residual vibration characteristics. First, cam displacement curves are represented by Bernstein–Bezier harmonic curves as opposed to polynomial curves. Second, the design procedure is expanded such that the residual vibrations of the resulting cam–follower mechanism is not only extinguished at the design speed but also made insensitive to speed vibrations. These developed harmonic curves are low harmonic content and therefore the resulting cam profiles are less prone to induce resonant vibrations in the follower mechanism.

Masood and Lau (1998) developed a CAD/CAM system for the accurate machining of plate cams within a user specified tolerance. Their work includes the generation of a displacement diagram, the simulation of cam tool path generation and the generation of actual CNC codes. They also proposed a new method for the interpolation of cam profiles called the half angle algorithm, which produces more economical CNC part programs than other interpolation methods.

Neamtu *et al.* (1998) designed Non-Uniform Rational B-Splines (NURBS) cam profiles using trigonometric splines. They show how to design cam profiles using NURBS curves whose support functions are appropriately scaled trigonometric splines. In particular, they discussed the design of cams with various conditions of practical interest, such interpolation conditions, constant diameter, minimal acceleration or jerk and constant dwells. In contrast to general polynomial curves, these NURBS curves have the useful property that offset are of the same type, and hence also have an exact NURBS representation.

Masood (1999) describes a computer-aided design and manufacturing system for the design and production of complex profiles for high-performance drum cams within the specified tolerance. The system graphically generates the cam profile on the cylindrical drum after performing an analysis of the kinematic performance for the prescribed follower motion, using a B-spline representation of follower curves. Eight different types of follower motion for a translating follower are considered. The kinematic performance is based on the criteria of achieving the lowest levels of velocity and acceleration for each curve. The system is also able to simulate the motion of the designed cam graphically. The system also recommends the best cam profile on the basis of comparing the kinematic performance of all the B-spline representations of all the follower motions.

Jao-Hwa Kuang *et al.* (2004) developed dynamic equations of the intermittent-motion of a globoidal cam-driven system. The effects of roller mesh flexibility and cam profile curve on the residual vibration of a globoidal cam system were studied experimentally and numerically. Time varying roller mesh stiffness and damping coefficients were used to account for the periodic variation of the mesh stiffness in the dwell and the active periods respectively. Dynamic responses of a globoidal cam system in the active and the dwell periods were simulated and measured. The effects of cam profile and input shaft speed on the residual vibration were also studied in this work. Results indicated that the proposed model was feasible for the dynamic simulation of a globoidal cam system.

Hua Qiu (2005) proposed a universal optimal approach to cam curve design. The approach consists of four issues that include a cam curve description using uniform B-splines, an objective function in a universal weighting form to integrate the design requirements, an automatic adjusting technique for weighting coefficient values and an improved complex search algorithm to optimise the control points of B-splines. With the approach, it is possible to deal simultaneously with multiple objectives for either kinematical or dynamic optimisation. Application examples on the kinematical and dynamical optimisation of cam curves are presented together with detailed discussions. These examples, especially the dynamic optimisation to control the residual vibration for a cam curve used in an indexing cam mechanism, sufficiently illustrate the effectiveness of the proposed approach. Demeulenaere and De Schutter (2007) introduced inertially compensated cams, of which the motion law is adapted to the camshaft speed fluctuation.

The requirement for high-performance machinery demands efficient methods for the design and manufacture of cams. Conventional methods of design and machining complex cam profiles within a given accuracy are tedious and time consuming. Even programming them on a computer numerical control (CNC) machine is difficult, because of the complexity of the cam profiles. From the literature it is clear that several studies have been carried out on the design, production and performance of cams. Initially, researchers concentrated in conventional design and latter they concentrated on development of CAD/CAM for cam-follower mechanism. But all these works relate to design of basic follower motions. Moreover, they concentrated only on the design of follower basic displacement curve for which velocity and acceleration characteristics are fixed. Users can not change the shape of velocity or acceleration curve and can not select required velocity or acceleration in view of improving the follower motion. None of the researchers were concentrated on development of CAD/CAM system and design of follower velocity curve for cam-follower mechanism using B-splines.

So there is an urgent need to concentrate the work on the development of a CAD/CAM system and design of follower velocity curve using B-splines. Follower motion can be improved by obtaining the continuous velocity or acceleration curves and their peak values are as small as possible. B-spline velocity curve is useful in improving follower motion characteristics. A unique advantage of this curve is that users can change the shape of the follower velocity curve and can obtain required velocity and acceleration by varying the control points that define the curve. The CAD/CAM system developed also provides graphical and numerical representation follower characteristics and cam profiles both for basic as well as B-splines. The system is useful to approximate the basic curve with equivalent B-splines. This is also useful to manufacture the cam on a CNC machine which increases accuracy of the cam profile and reduces the time of manufacture.

3. B-splines

The motion characteristics of the cam follower mechanisms can be improved by reducing the jerk. The jerk causes vibration, more contact stresses and wear and tear in the cam. This is predominant at the transition points, i.e. points where follower motion changes from dwell to rise, rise to dwell, dwell to fall and fall to dwell etc. One method of improving the follower motion characteristics would be to represent the curves by B-Splines (Zeid 1998).

One of the most versatile tools for modelling curves are the B-splines. It has been widely used in modelling of curves and surfaces in CAD/CAM as a standard. It is a smooth spline which offers a common mathematical form for representation and is used for designing standard curves (conic and quadrics, etc), free form curves and surfaces. B-Splines are invariant under translation, rotation, scaling, shear, and parallel and perspective projection. They have the ability to interpolate or approximate a set of given data points. They provide local control of the curve shape as opposed to global control by using a special set of blending functions that provide local influence. Another advantage of B-splines is that they provide the ability to add control points without increasing the degree of the curve. B-splines are free form curves having C^0 (points continuity), C^1 (slope continuity) and C^2 (curvature continuity).

Mathematically, B-splines can be defined by having $(k-1)$ degree and $n + 1$ control points (Zeid 1998) as

$$p(u) = \sum_{i=0}^{n} N_{i,k}(u)\, p_i \qquad 0 <= u <= u_{max} \tag{1}$$

$p(u)$ is the position on the curve at parameter u, p_i is a control point, $N_{i,k}$ (u) is a blending function, which is recursive in nature and polynomial of degree $k-1$. The range of parameter u depends on the number of control points $n + 1$ and the choice for k, so that u varies from 0 to $n-k+2$.

The Blending function has the property of recursion, which is defined as

$$N_{i,k}(u) = [(u - u_i)N_{i,k-1}(u)]/[u_{i+k-1} - u_i] + [(u_{i+k} - u) \times N_{i+1,k-1}(u)]/[u_{i+k} - u_{i+1}] \tag{2}$$

$$\left.\begin{array}{ll} N_{i,1}(u) = 1 & \text{if } u_i < u < u_{i+1}, \\ N_{i,1}(u) = 0 & \text{otherwise} \end{array}\right\} \tag{3}$$

where k controls the degree $(k-1)$ of the resulting polynomial in u and also controls the continuity of the curve. The values u_i are called knot values. They relate the parametric variable u and control points (p_i). The knot values u_j are given by

$$\left.\begin{array}{lll} u_j = 0 & \text{if} & j < k, \\ u_j = j - k + 1 & \text{if} & k <= j <= n, \\ u_j = n - k + 2 & \text{if} & j > n \end{array}\right\} \tag{4}$$

with $0<=j<=n+k$.

Number of knot values (m) $= n + k + 1$,

i.e. $u_i = [u_0, u_1, u_2, u_3, \ldots\ldots\ldots\ldots\ldots, u_{n+k}]$

They are three ways to modify the shape of B-splines. Change the knot vector, move the control points, adding the control points. It is relatively difficult to determine how a curve will respond to changes in the knot vector, this is not the best way to change curve shape. On the other hand, the effect of changing a control point is predictable and intuitive. This is the best way to modify the curve (i.e. velocity curve, which has minimum jerk).

3.1. Creating new velocity curve

In the design of cam follower velocity curve, six control points $(n + 1 = 6)$ of different velocity magnitude and degree three ($k-1=3$) have been selected for the B-spline curve. The six control points are selected in the specified rise angle or return cam angle. The six velocity control points of the follower are taken as V_0, V_1, V_2, V_3, V_4, V_5.

The point on the curve can be defined using equation (1)

$$V(u)=\sum_{i=0}^{5} N_{i,k}(u)V_i \quad 0<=u<=3 \tag{5}$$

Using equations (2) to (5) the parametric equations for velocity curve for three segments can be defined as

$$\begin{aligned}\mathbf{V1(u)} &= (1-u)^3\mathbf{V}_0 + [u(1-u)^2 + 1/2(2-u) \\ &\times\left(-3/2u^2+2u\right)]\mathbf{V}_1 + [u/2\left(-3/2u^2+2u\right) \\ &+ u^2/6(3-u)]\mathbf{V}_2 + \left(u^3/6\right)\mathbf{V}_3 \\ &\text{for } 0<=u<1\end{aligned} \tag{6}$$

$$\begin{aligned}\mathbf{V2(u)} &= \{1/4(2-u)^3\mathbf{V}_1 + [u/4(2-u)^2 + (3-u)/3 \\ &\times\left(-u^2+3u-3/2\right)]\mathbf{V}_2 + [u/3\left(-u^2+3u-3/2\right) \\ &+ 1/4(3-u)(u-1)^2]\mathbf{V}_3 + 1/4(u-1)^3\mathbf{V_4} \\ &\text{for } 1<=u<2\end{aligned} \tag{7}$$

$$\begin{aligned}\mathbf{V3(u)} &= \{1/6(3-u)^3\mathbf{V_2} + [u/6(3-u)^2 + (3-u)/2 \\ &\times(-3/2u^2+7u-15/2)]\mathbf{V_3} + [(u-1)/2 \\ &\times\left(-3/2u^2+7u-15/2\right) + (3-u) \\ &\times(u-2)^2]\mathbf{V_4} + (u-2)^3\mathbf{V_5} \\ &\text{for } 2<=u<=3\end{aligned} \tag{8}$$

V1(u), V2(u), V3(u) are the three segments of follower velocity curve with respect to cam angle in the range of specified cam rise or return angle.

Considering θ = angle of rotation of cam, r_1 – rise angle, r_2 – return angle, the parameter u can be written as $u=\theta/r_1$ where θ varies from 0 to r_1 or $u=\theta/r_2$ where θ varies from 0 to r_2.

Displacement (S) of follower is obtained by integrating above velocity equations (6) to (8) with respect to time (t).

$$S(u)=\int V(u)dt + C = (r_1/w) * \int V(u)du + C \tag{9}$$

where w – angular velocity of follower, $w=2\Pi n/60$, n–RPM of cam, C-constant.

Using the equations (6) to (9), parametric displacement equations of degree four for the three different parts can be obtained.

To find the integrating constants (C) for all the three parts, the following conditions are to be considered.

Initial displacement is zero.

$$S1(u)=0 \quad \text{at } u=0 \tag{10}$$

End point of the first part curve and starting point of the second part curve are same.

$$S1(u)=S2(u) \quad \text{at } u=1 \tag{11}$$

End point of the second part curve and starting point of the third part curve are same.

$$S2(u)=S3(u) \quad \text{at } u=2 \tag{12}$$

The three integrating constants obtained by substituting three conditions from equations 10, 11, 12 into equation (9) are

$$\begin{aligned}C1 &= 0 \\ C2 &= (0.25*V_0 - 0.5*V_1 + 0.375*V_2 - 0.1875 \\ &\quad *V_3 + 0.0625*V_4)*(1/R11) \\ C3 &= (0.25*V_0 + 0.5*V_1 - 2.625*V_2 + 5.8125 \\ &\quad *V_3 - 7.9375*V_4 + 4*V_5)*(1/R11)\end{aligned}$$

where $R11=(3*180*w)/(\Pi*r1)$ and the parameter $u=3*(\theta/r1)$.

The equations for the three parts of the displacement curve are:

$$\begin{aligned}S1(u) &= (r1/w)*\int V1(u)du + C1 \quad \text{for } 0<=u<=1 \\ S2(u) &= (r1/w)*\int V2(u)du + C2 \quad \text{for } 1<=u<=2 \\ S3(u) &= (r1/w)*\int V3(u)du + C3 \quad \text{for } 2<=u<=3\end{aligned} \tag{13}$$

Similarly acceleration (A) of the following is obtained by differentiating velocity equation V(u) with respect to time (t)

$$A(u)=dV(u)/dt=(w/r_1)*dV(u)/du \tag{14}$$

Jerk (J) of the following is obtained by differentiating velocity equation A(u) with respect to time (t)

$$J(u)=dA(u)/dt=(w/r_1)*(w/r1)*dA/du \tag{15}$$

Acceleration, displacement, acceleration and jerk for three segments of curve, i.e. for $0<=u<=1$, $1<=u<=2$, and $2<=u<=3$ are given below

For $0<=u<=1$

$$\begin{aligned}\mathbf{V1(u)} &= (-u\hat{}3 + 3*u\hat{}2 - 3*u + 1)*V_0 \\ &+ ((7/4)*u\hat{}3 - (9/2)*u\hat{}2 + 3*u) \\ &*V_1 + ((-11/12)*u\hat{}3 + (3/2) \\ &*u\hat{}2)*V_2 + (u\hat{}3/6)*V_3\end{aligned} \tag{16}$$

$$\mathbf{S1}(\mathbf{u}) = (1/R11)*(((-u^\wedge 4/4)+u^\wedge 3-(3/2)*u^\wedge 2+u) * V_0 + ((7/16)*u^\wedge 4-(3/2)*u^\wedge 3+(3/2)*u^\wedge 2) * V_1 + ((-11/48)*u^\wedge 4+(1/2)*u^\wedge 3) * V_2 + (u^\wedge 4/24)*V_3) \tag{17}$$

$$\mathbf{A1}(\mathbf{u}) = R11*((-3*u^\wedge 2+6*u-3) * V_0((21/4)*u^\wedge 2-9*u+3) * V_1 + ((-11/4)*u^\wedge 2+3*u) * V_2 + (u^\wedge 2/2)*V_3) \tag{18}$$

$$\mathbf{J1}(\mathbf{u}) = R11*R11 * ((-6*u+6)*V_0 + ((21/2)*u-9)* * V_1 + ((-11/2)u+3)*V_2 + (u*V_3)) \tag{19}$$

For 1<=u<=2

$$\mathbf{V2}(\mathbf{u}) = ((-1/4)*u^\wedge 3+(3/2)*u^\wedge 2-3*u+2) * V_1 + ((7/12)*u^\wedge 3-3*u^\wedge 2+(9/2)*u-(3/2)) * V_2 + ((-7/12)*u^\wedge 3+(9/4)*u^\wedge 2-(9/4) * u+(3/4))*V_3 + ((u^\wedge 3/4)-(3/4) * u^\wedge 2+(3*u/4)-(1/4))*V_4 \tag{20}$$

$$\mathbf{S2}(\mathbf{u}) = C2 + (1/R11)*(((-1/16) * u^\wedge 4 + (1/2)*u^\wedge 3 - (3/2)*u^\wedge 2 + 2*u) * V_1 + ((7/48)*u^\wedge 4 - u^\wedge 3 + (9/4) * u^\wedge 2 - (3/2)*u)*V_2 + ((-7/48) * u^\wedge 4 + (3/4)*u^\wedge 3 - (9/8) * u^\wedge 2 + (3/4)*u) * V_3 + ((u^\wedge 4/16) - (1/4) * u^\wedge 3 + (3*u^\wedge 2/8) - (u/4))*V_4 \tag{21}$$

$$\mathbf{A2}(\mathbf{u}) = R11*(((-3/4)*u^\wedge 2+3*u-3) * V_1 + ((7/4)*u^\wedge 2-6*u+(9/2)) * V_2 + ((-7/4)*u^\wedge 2+(9/2)*u-(9/4)) * V_3 + ((3/4)*u^\wedge 2-(3/2)*u+(3/4))*V \tag{22}$$

$$\mathbf{J2}(\mathbf{u}) = R11*R11 * (((-3/2)*u+3)*V_1 + ((7/2)*u-6) * V_2 + ((-7/2)*u+(9/2)) * V_3 + ((3/2)*u-(3/2))*V_4) \tag{23}$$

For 2<=u<=3

$$\mathbf{V3}(\mathbf{u}) = ((-1/6)*u^\wedge 3+(3/2)*u^\wedge 2-(9/2) * u+(9/2))*V_2 + ((11/12)*u^\wedge 3-(27/4) * u^\wedge 2+(63/4)*u-(45/4))*V_3 + ((-7/4) * u^\wedge 3+(45/4)*u^\wedge 2-(93/4)*u+(63/4)) * V_4 + (u^\wedge 3-6*u^\wedge 2+12*u-8)*V_5 \tag{24}$$

$$\mathbf{S3}(\mathbf{u}) = C3+(1/R11)*(((-u^\wedge 4/24)+(1/2) * u^\wedge 3-(9/4)*u^\wedge 2+(9/2)*u)*V_2 + ((11/48) * u^\wedge 4-(9/4)*u^\wedge 3+(63/8)*u^\wedge 2-(45/4)*u) * V_3 + ((-7/16)*u^\wedge 4+(15/4)*u^\wedge 3-(93/8) * u^\wedge 2+(63/4)*u)*V_4 + ((u^\wedge 4/4)-2*u^\wedge 3+6 * u^\wedge 2-8*u)*V_5 \tag{25}$$

$$\mathbf{A3}(\mathbf{u}) = R11*(((-1/2)*u^\wedge 2+3*u-(9/2)) * \mathbf{V_2} + ((11/4)*u^\wedge 2-(27/2)*u+(63/4)) * \mathbf{V_3} + ((-21/4)*u^\wedge 2+(45/2)*u-(93/4)) * \mathbf{V_4} + (3*u^\wedge 2-12*u+12)*V_5) \tag{26}$$

$$\mathbf{J3}(\mathbf{u}) = R11*R11 * ((-u+3)*V_2 + ((11/2)*u-(27/2)) * V_3 + ((-21/2)*u+(45/2)) * V_4 + (6*u-12)*V_5) \tag{27}$$

3.2. Cam profile calculations

Considering the translating-radial roller follower, the rectangle coordinates (X, Y) for the cam pitch profile are given as

X = (Rp + S) sin θ; Y = (Rp + S) cos θ, where Rp–prime circle radius and S–displacement with respect to cam angle θ.

4. CAD/CAM system

Using above displacement, velocity, acceleration and jerk equations (equations (16) to (27)), a software program is written in Visual Basic language, which gives graphical numerical representation of cam follower characteristics and cam profile for all the basic and B-spline motions.

Program is tested for typical input data (Figure 2):

Rise angle = 120 degrees, Dwell 1 = 60 degrees, Return angle = 120 degrees, Dwell 2 = 60 degrees. Speed (n) = 100 RPM, Lift = 20mm. The six velocity control points selected to approximate the cycloidal motion with equivalent B-spline are,

$V_0 = 0$, $V_1 = 0$, $V_2 = 200$mm/s, $V_3 = 200$m/s, $V_4 = 0$, $V_5 = 0$, Prime circle radius = 30mm.

The Figure 3a shows graphical and numerical representation of follower characteristics for basic cycloidal and its equivalent B-spline for given input. Figure 4 shows the input data for determining cycloidal equivalent B-spline cam profile. Figure 5 shows graphical and numerical representation of cycloidal equivalent cam profile. A comparison is done between regular harmonic curves and cyloidal equivalent B-spline curve for maximum acceleration. This is also explained in Table 1.

5. Analysis of cam-follower characteristics

In the design cam-follower mechanism, cam follower velocity equation is considered as a B-spline polynomial of degree 3 and control points 6. Because acceleration curve obtained is of degree 2, which is smooth curve. The basic cycloidal motion is considered for approximation to obtain equivalent B-spline. Because cycloidal motion is smooth and is used for high speed cams, in approximating the basic cycloidal motion with equivalent B-spline, the first two control points are set to zeros and last two control points set to maximum velocity, so that acceleration curve is continuous and has low maximum acceleration (i.e. minimum jerk).

The two maximum velocity control points are so selected/varied to obtain user required maximum displacement (lift) by assuming constant

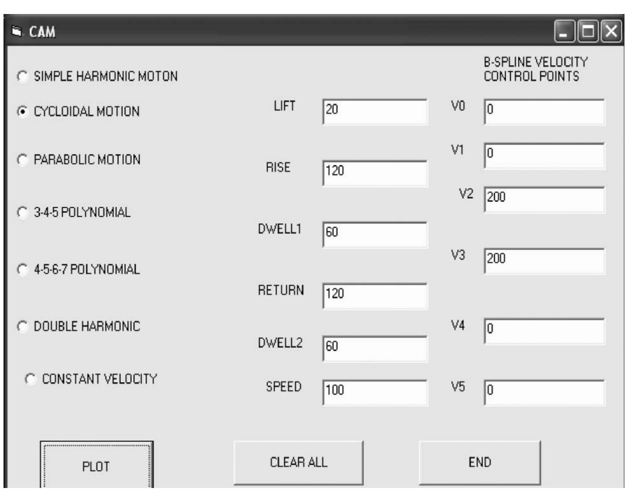

Figure 2. Input data for basic (cycloidal motion) curve and its approximate B-spline curve.

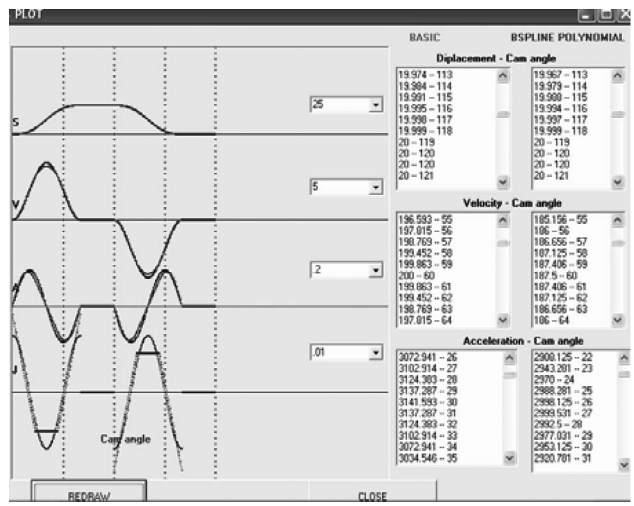

Figure 3. Cam-follower characteristics for basic (cycloidal motion) curve and its approximate B-spline curve.

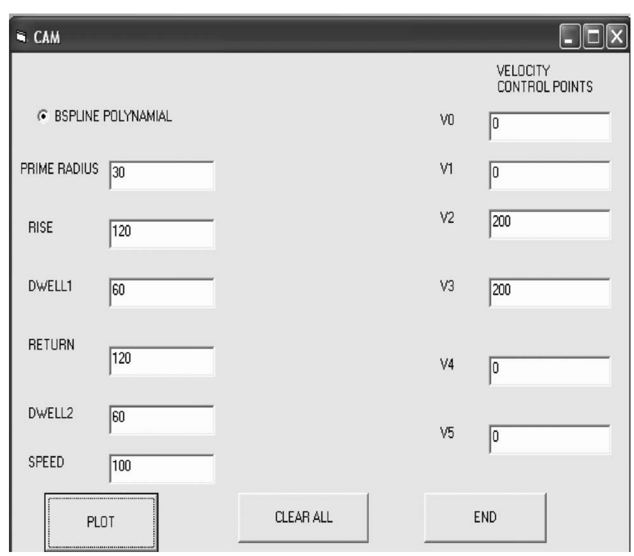

Figure 4. Input data for cycloidal motion equivalent B-spline cam profile.

Rise-Dwell1-Return-Dwell2 angles and cam speed (Figure 2).

i.e. if Speed (n) = 200RPM

The velocity control points $V_0 = 0$, $V_1 = 0$, V_2 400mm/s, $V_3 = 400$mm/s, $V_4 = 0$, $V_5 = 0$, are to be selected to get required maximum displacement (lift) i.e. 20mm.

Table 1 shows the maximum acceleration of the cam follower for different follower motions for cam speeds of 100, 200 and 500 RPM. The cycloidal motion is approximated to the equivalent B-Spline curve. To approximate the cycloidal motion to B-Spline the velocity and acceleration curves are to be continuous. The six velocity control points are so selected as 0, 0, 200, 200, 0, 0 (in mm/s) for cam speed of 100RPM to obtain both velocity and acceleration curves are continuous by keeping the constant lift as shown in Figure 3. From Table 1 it is clear that the maximum

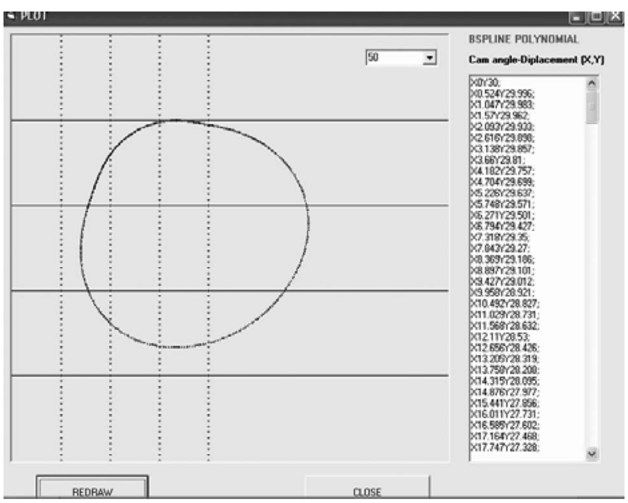

Figure 5. Output data for cycloidal motion equivalent B-spline cam profile (pitch profile and its x, y coordinates, mm).

Table 1. Maximum accelerations of various follower motions for the above input data.

Type of motion	Maximum acceleration (mm/s^2)		
Cam speed (RPM)	100	200	500
SHM	2467.4	9869.6	61685.0
Cycloidal	3141.2	12566.3	78539.8
4–5–6–7 polynomial	3756.4	15025.6	93910.3
Double harmonic motion	4934.8	19739.2	123370.0
B-spline (approximation to Cycloidal)	2999.5	11998.1	74988.2

follower acceleration for cyloidal equivalent B-spline is 2999.50 mm/s^2 and that of basic cycloidal is 3141.28mm/s^2 for cam speed of 100 RPM. That is cycloidal equivalent B-spline has low maximum acceleration curve when compared to basic cyloidal motion. So, cycloidal equivalent B-spline has less jerk and is better for application than basic cyloidal motion. Similarly, other basic follower motions such as simple harmonic motion, 4–5–6–7 polynomial, double harmonic motion are also approximated to equivalent B-spline. Users can adopt the B-spline polynomial for cam-follower mechanisms instead of regular basic curves.

6. Manufacturing

CAD/CAM system provides graphical and numerical representation of cam follower characteristics for typical input data for basic and B-splines as shown in Figure 3. The system also gives a cam profile (Figure 5) and its coordinates about cam centre. These coordinates are useful to manufacture the cam on a Computer Numerical Control (CNC) machine. The coordinates from Figure 5 are selected and can form into a text file. The text file is imported to any cam package (such as ESPRIT, CADEM, and MASTER CAM) and converted into a 3D cam model. Taking the tool path on cam profile surface, the numerical control (NC) codes can be automatically generated. NC code is loaded onto a CNC milling machine to manufacture the same cam. The text file can also be imported to any CAD package (such as Catia, Pro-E, Unigraphics) and converted to IGES format of cam 3D model. The 3D cam model (IGES format) is further converted into STEP format (STandard for Exchange of Product model data). STEP model is directly used to manufacture the cam which eliminates the NC codes. Figure 6 shows a plate cam manufactured on a CNC milling machine. Figure 7 shows STEP format of the 3D plate cam. So the CAD/CAM system is useful to

Figure 6. Plate cam manufactured on CNC machine.

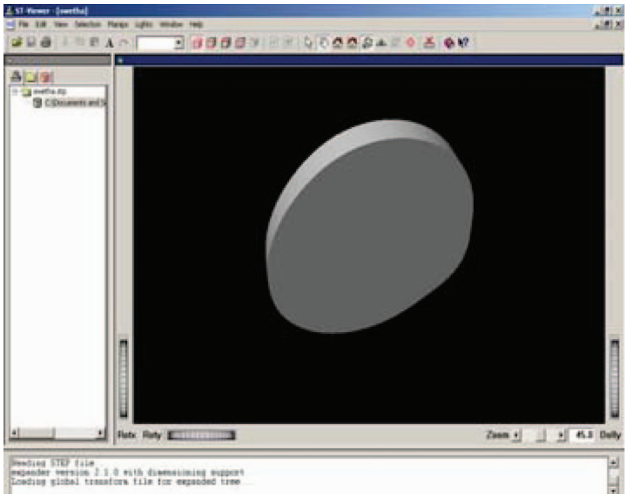

Figure 7. STEP model of plate cam.

manufacture a cam profile with increased accuracy and reduced time of manufacture.

7. Conclusion

A new velocity curve for a cam-follower system is found using B-spline polynomials. An optimum displacement curve is obtained from this new velocity curve. The curve has unique motion characteristics. Jerk of the follower can be also minimised by changing the control points of the B-spline polynomial. User can select or change the velocity of follower as well as the shape of the follower velocity curve for the best follower motion. A CAD/CAM system is developed which provides graphical and numerical representation of cam follower characteristics for all basic and B-splines. It also gives graphical and numerical representation of cam profile. The system is useful in manufacturing the cam on CNC or STEP machines which increase in accuracy the profile and decrease the time of manufacture.

Future direction of the work for cam-follower motion can be improved by designing the follower displacement curves by 5th degree B-splines having eight control points, because acceleration curve obtained is 3rd degree which is free form curve. Work can also extended in the development of a CAD/CAM system in the design of cams using other synthetic curves such as Bezier and NURBS.

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