

Peristaltic flow and heat transfer in a vertical porous annulus, with long wave approximation

K. Vajravelu^{a,*}, G. Radhakrishnamacharya^b, V. Radhakrishnamurty^c

^aDepartment of Mathematics, University of Central Florida, Orlando, FL 32816, USA

^bDepartment of Mathematics & Humanities, National Institute of Technology, Warangal 506004, India

^cDepartment of Mathematics, Sir C.R. Reddy College of Engineering, Eluru 534007, India

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Abstract

In this paper, we study the interaction of peristalsis with heat transfer for the flow of a viscous fluid in a vertical porous annular region between two concentric tubes. Long wavelength approximation (that is, the wavelength of the peristaltic wave is large in comparison with the radius of the tube) is used to linearise the governing equations. Using the perturbation method, the solutions are obtained for the velocity and the temperature fields. Also, the closed form expressions are derived for the pressure–flow relationship and the heat transfer at the wall. The effect of pressure drop on flux is observed to be almost negligible for peristaltic waves of large amplitude; however, the mean flux is found to increase by 10–12% as the free convection parameter increases from 1 to 2. Also, the heat transfer at the wall is affected significantly by the amplitude of the peristaltic wave. This warrants further study on the effects of peristalsis on the flow and heat transfer characteristics.

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1. Introduction

Peristalsis is a mechanism to pump fluids by means of moving contractions on the tube wall. It appears to be the major mechanism for urine transport in the ureter, the motion of spermatozoa in the cervical canal, bile in the bile duct, etc. Engineering devices like finger pumps and roller pumps work on this principle. Peristaltic transport of toxic liquid is used in the nuclear industry so as not to contaminate outside environment. In view of its importance, several authors [1–6] have studied peristalsis in both mechanical and physiological situations. In particular, Shapiro et al. [2] analysed peristaltic pumping at low Reynolds number and observed the interesting phenomena of reflux and trapping. Shukla et al. [3] have studied the effect of peristalsis on the movement of micro-organisms and its application to spermatozoa transport. The interaction of peristalsis and heat transfer has also received some attention [7] as it might be relevant in processes like hemodialysis and oxygenation.

Though the actual mechanism for the transport of water from the ground to upper branches of tall trees is not well understood, it is speculated that peristalsis and free convection contribute to this motion. The diameters of the trunks of the trees are found to vary with time. In view of this, some investigators [8–11] have studied peristalsis with reference to water transport in trees. The translocation of water involves its motion through the porous matrix of the tree. Recently, Radhakrishnamurty et al. [12] have investigated flow through vertical porous tube with peristalsis and heat transfer. It is also found that in trees there is a core region through which water does not flow and water flows only through the outer region.

Keeping this in view, peristaltic flow through vertical porous annuli is considered. Perturbation solutions are sought in terms of free convection and the porosity parameters. The momentum and energy equations are solved and closed form solutions are obtained. The effects of free convection parameter (G_m), amplitude ratio (ε), porosity parameter (σ^2) and Eckert number (E_m) on mean flux (\bar{Q}) for a prescribed pressure drop (Δp) are studied. We observe that for large values of ε , the effect of Δp on \bar{Q} is negligible. However, for given values of other

* Corresponding author. Tel.: +1 407 823 5089.

E-mail address: vajravelu@pegasus.cc.ucf.edu (K. Vajravelu).

parameters, \bar{Q} increases by about 10–12% as G_m increases from 1 to 2.

2. Formulation of the problem

The motion of a viscous, Newtonian, incompressible fluid through a porous vertical annular region between two concentric tubes is considered (see Fig. 1). It is assumed that sinusoidal waves of very large wavelength travel along the outer boundary of the region. The axisymmetric cylindrical polar coordinate system (X, R) is considered such that the X -coordinate is along the axis of the tube and R is the radial coordinate. The inner and outer tubes are maintained at constant temperatures T_1 and T_0 , respectively.

The deformation of the outer wall due to the propagation of an infinite train of peristaltic waves is represented by

$$R = H(X, t) = a_0 + b \sin(2\pi/\lambda)(X - ct), \quad (1)$$

where a_0 is the mean radius of the outer tube, a_1 is the radius of the inner tube, b is the amplitude, λ is the wavelength, c is the speed of the wave and t is the time.

The governing equations for the present problem are:
the momentum equation

$$\rho \frac{d\bar{v}}{dt} = -\nabla p + \mu \nabla^2 \bar{v} - \frac{\mu}{k_0} \bar{v} + \rho \beta \bar{g}(T - T_0); \quad (2)$$

the continuity equation

$$\nabla \cdot \bar{v} = 0; \quad (3)$$

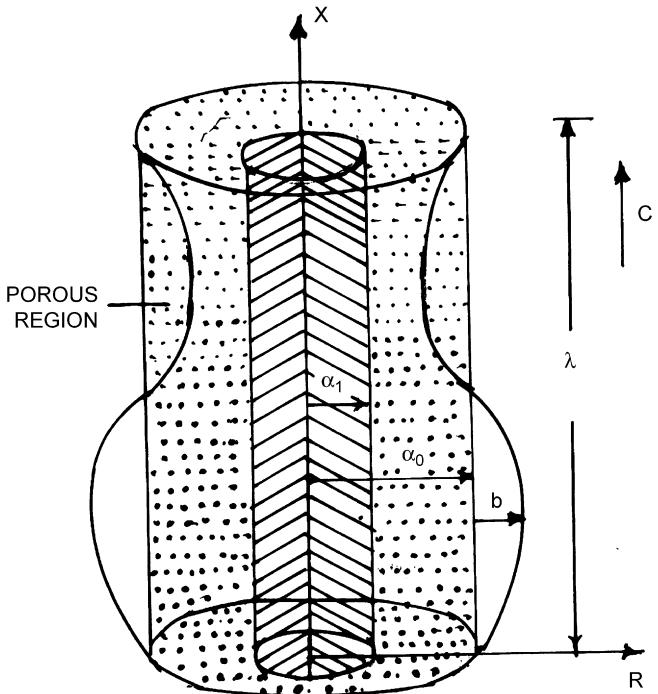


Fig. 1. Flow geometry.

and the energy equation

$$C \rho \frac{dT}{dt} = k \nabla^2 T + \mu \varphi + \frac{\mu}{k_0} v^2, \quad (4)$$

where ρ is the density, \bar{v} is the velocity of the fluid, p is the pressure, μ is the coefficient of viscosity, k_0 is the permeability of the medium, β is the coefficient of expansion, \bar{g} is the acceleration due to gravity, T is the temperature, C is specific heat, k is the thermal conductivity of the fluid, and φ is viscous dissipation.

By using the long wavelength approximation, Eqs. (2)–(4) can be reduced to (for details see [12])

$$0 = -\frac{\partial p}{\partial X} + \frac{\mu}{R} \frac{\partial}{\partial R} \left(R \frac{\partial W}{\partial R} \right) - \frac{\mu}{k_0} W + \rho g \beta (T - T_0), \quad (5)$$

$$0 = \frac{\partial W}{\partial X} + \frac{U}{R} + \frac{\partial U}{\partial R}, \quad (6)$$

$$0 = \frac{k}{R} \frac{\partial}{\partial R} \left(R \frac{\partial T}{\partial R} \right) + \mu \left(\frac{\partial W}{\partial R} \right)^2 + \frac{\mu}{K_0} W^2, \quad (7)$$

where W and U are velocity components of the fluid in X and R directions, respectively.

The boundary conditions are

$$\left. \begin{array}{l} W = 0 \quad \text{at } R = a_1 \quad (\text{radius of the inner tube}) \\ W = 0 \quad \text{at } R = H(x, t), \end{array} \right\} \quad (8)$$

$$\left. \begin{array}{l} T = T_1 \quad \text{at } R = a_1, \\ T = T_0 \quad \text{at } R = H. \end{array} \right\} \quad (9)$$

We introduce the following transformation:

$$\left. \begin{array}{l} x = X - ct, \quad r = R, \\ w = W - c, \quad u = U, \end{array} \right\} \quad (10)$$

and the following non-dimensional quantities:

$$\left. \begin{array}{l} x' = \frac{x}{\lambda}, \quad r' = \frac{r}{a_0}, \quad w' = \frac{w}{c}, \quad u' = \frac{\lambda u}{a_0 c}, \\ \theta = \frac{T - T_0}{T_1 - T_0}, \quad p' = \frac{p}{\mu c \lambda / a_0^2}, \quad a'_1 = \frac{a_1}{a_0}, \\ \eta(x) = \frac{h(x)}{a_0}, \end{array} \right\} \quad (11)$$

where $h(x) = a_0 + b \sin(2\pi x/\lambda)$.

Eqs. (1), (5)–(7), on using (10) and (11), can be written as (after dropping the primes),

$$r = \eta(x) = 1 + \varepsilon \sin 2\pi x, \quad (12)$$

$$0 = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) - \sigma^2 (w + 1) + G_m \theta, \quad (13)$$

$$O = \frac{\partial w}{\partial x} + \frac{u}{r} + \frac{\partial u}{\partial r}, \quad (14)$$

$$O = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta}{\partial r} \right) + E_m \left(\frac{\partial w}{\partial r} \right)^2 + \sigma^2 E_m (w + 1)^2. \quad (15)$$

The corresponding non-dimensional boundary conditions are

$$\left. \begin{array}{l} w = -1 \quad \text{at } r = a_1, \\ w = -1 \quad \text{at } r = \eta(x), \end{array} \right\} \quad (16)$$

$$\left. \begin{array}{l} \theta = 1 \quad \text{at } r = a_1, \\ \theta = 0 \quad \text{at } r = \eta(x), \end{array} \right\} \quad (17)$$

where $\sigma^2 = a_0^2/K_0$ (porosity parameter), $G_m = g\beta T_0 a_0^3/v^2$ (Grashof number), $E_m = c^2/k(T_1 - T_0)$ (Eckert number) and $\varepsilon = b/a_0$ (amplitude ratio).

3. Method of solution

It is not possible to get closed form solutions for Eqs. (13) and (15) for arbitrary values of all the parameters. We seek perturbation solutions in terms of the free convection parameter (G_m) and porosity parameter (σ^2) as follows:

$$f = (f_{00} + G_m f_{01} + \dots) + \sigma^2 (f_{10} + G_m f_{11} + \dots) + \dots, \quad (18)$$

where f is any flow variable.

Substituting (18) in Eqs. (13)–(17), and collecting the coefficients of various powers of G_m and σ^2 , we get the following sets of equations:

Zeroth order:

$$\left. \begin{array}{l} 0 = -\frac{\partial p_{00}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w_{00}}{\partial r} \right), \\ 0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta_{00}}{\partial r} \right) + E_m \left(\frac{\partial w_{00}}{\partial r} \right)^2, \\ w_{00} = -1 \quad \text{at } r = a_1, \\ w_{00} = -1 \quad \text{at } r = \eta(x), \\ \theta_{00} = 1 \quad \text{at } r = a_1, \\ \theta_{00} = 0 \quad \text{at } r = \eta(x). \end{array} \right\} \quad (19)$$

First order:

$$\left. \begin{array}{l} 0 = -\frac{\partial p_{01}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w_{01}}{\partial r} \right) + \theta_{00}, \\ 0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta_{01}}{\partial r} \right) + 2E_m \frac{\partial w_{00}}{\partial r} \frac{\partial w_{01}}{\partial r}, \\ w_{01} = 0 \quad \text{at } r = a_1, \\ w_{01} = 0 \quad \text{at } r = \eta(x), \\ \theta_{01} = 0 \quad \text{at } r = a_1, \\ \theta_{01} = 0 \quad \text{at } r = \eta(x), \end{array} \right\} \quad (20)$$

$$\left. \begin{array}{l} 0 = -\frac{\partial p_{10}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w_{10}}{\partial r} \right) - (w_{00} + 1), \\ 0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta_{10}}{\partial r} \right) + 2E_m \frac{\partial w_{00}}{\partial r} \frac{\partial w_{10}}{\partial r} + E_m (w_{00} + 1)^2, \\ w_{10} = 0 \quad \text{at } r = a_1, \\ w_{10} = 0 \quad \text{at } r = \eta(x), \\ \theta_{10} = 0 \quad \text{at } r = a_1, \\ \theta_{10} = 0 \quad \text{at } r = \eta(x). \end{array} \right\} \quad (21)$$

Solving the above sets of equations, we get

$$w_{00} = \frac{a_{11}(a_{14} + a_{15}a_{16})}{4} - 1, \quad (22)$$

$$v_{01} = a_{12}(a_{14} + a_{15}a_{16}) + a_{14}(b_{11} - b_{12} - \log \eta) + b_{12}a_{17},$$

$$v_{02} = \frac{E_m a_{11}^2}{16} \left[\frac{r^6 - \eta^6}{144} + \frac{a_{15}^2}{4} \left\{ \frac{(r \log r)^2 - (\eta \log \eta)^2}{2} \right. \right. \\ \left. \left. - a_{17} + \frac{3a_{14}}{4} \right\} \right],$$

$$w_{01} = \frac{v_{01}}{4} + v_{02} + \frac{E_m a_{11}^2 a_{15} (\eta^4 - a_1^4)}{16} + b_{13}a_{16}, \quad (23)$$

$$w_{10} = (a_{11}/4) \left[(a_{14} + a_{15}a_{16}) + \frac{r^4 + 3\eta^4 - 4r^2\eta^2}{16} \right. \\ \left. + \frac{r^2 - a_1^2}{4 \log(a_1/\eta)} (r^2 a_{16} - a_{14}) \right] + b_{13}a_{16}, \quad (24)$$

$$t_{00} = -\frac{E_m a_{11}^2}{16} \left[\frac{r^4 - \eta^4}{64} + a_{15} \left\{ a_{15} \frac{(\log r)^2 - (\log \eta)^2}{2} \right. \right. \\ \left. \left. + (r^2 - \eta^2) \right\} \right],$$

$$\theta_{00} = t_{00} + b_{12}a_{16}, \quad (25)$$

$$t_{01} = \frac{a_{11}r^4(a_{12} - b_{11} + b_{12}(1 - a_{16}))}{64} \\ + \frac{E_m a_{11}}{16} \left[\frac{r^8}{768} + \frac{a_{15}r^4}{16} \left\{ \frac{(\log r)^2}{2} - \log r + \frac{11}{16} \right\} \right. \\ \left. + \frac{a_{15}r^6}{72} \right],$$

$$t_{02} = \frac{r^2}{8} \left(a_{12} - b_{11} - b_{12} \left(a_{16} - \frac{3}{2} \right) \right) \\ + \frac{E_m a_{11}^2}{16} \left[\frac{r^6}{864} + \frac{a_{15}^2 r^2}{16} ((\log r)^2 - 3 \log r + 3) \right. \\ \left. + \frac{a_{15}r^4}{64} \right],$$

$$\theta_{01} = -2E_m \left[t_{01} + \frac{a_{11}b_{14}r^2}{8} + b_{18} \left(t_{02} + \frac{b_{14}(\log r)^2}{2} \right) \right] \\ + b_{15} \log r + b_{16}, \quad (26)$$

$$t_{10} = \frac{a_{11}a_{13}r^4}{32} + \frac{a_{11}^2}{4} \left[\frac{r^6}{144} - \frac{r^4\eta^2}{32} + \frac{a_{15}r^4}{32}(a_{16} - 1) \right] \\ + \frac{a_{11}b_{17}r^2}{4} + \frac{a_{11}^2 r^6}{576},$$

$$t_{11} = b_{18} \left[\frac{a_{13}r^2}{4} + \frac{a_{11}}{4} \left\{ \frac{r^4}{32} - \frac{r^2\eta^2}{4} \right. \right. \\ \left. \left. + \frac{a_{15}r^2}{4} \left(a_{16} - \frac{3}{2} \right) \right\} + b_{17}(\log r)^2 \right],$$

$$\begin{aligned}
t_{12} &= \frac{b_{18}^2 r^2}{2} \left[\frac{(\log r)^2}{2} - \log r + \frac{3}{4} \right] \\
&\quad + \frac{r^2(b_{19} + 1)}{4} + \frac{b_{18} a_{11} r^4}{32} \left(\log r - \frac{1}{2} \right), \\
t_{13} &= (b_{18}(b_{19} + 1)r^2(\log r - 1)/2) + (a_{11}(b_{19} + 1)r^4/32), \\
\theta_{10} &= -E_m[t_{10} + t_{11} + t_{12} + t_{13}] + b_{20} \log r + b_{21}, \\
a_{11} &= \frac{\partial p_{00}}{\partial x}, \quad a_{12} = \frac{\partial p_{01}}{\partial x}, \quad a_{13} = \frac{\partial p_{10}}{\partial x}, \\
a_{14} &= r^2 - \eta^2, \quad a_{15} = \frac{\eta^2 - a_1^2}{\log(a_1/\eta)}, \\
a_{16} &= \log(r/\eta), \quad a_{17} = r^2 \log r - \eta^2 \log \eta.
\end{aligned} \tag{27}$$

The expressions for b_i ($i=11-21$) are not presented for brevity.

The non-dimensional flux Q is defined by

$$Q = \frac{Q'}{\pi c a_0^2} = \int_0^\eta 2r w \, dr, \tag{28}$$

and the mean flux is given by

$$\bar{Q} = \int_0^1 Q \, dt. \tag{29}$$

Substituting the expression for w in Eqs. (28) and (29), and after simplification, we get

$$\bar{Q} = (\bar{Q}_{00} + G_m \bar{Q}_{01} + \dots) + \sigma^2 (\bar{Q}_{10} + \dots) + \dots, \tag{30}$$

where

$$\bar{Q}_{00} = \frac{\Delta p_{00} + 8B}{8A} + \frac{\varepsilon^2}{2} - a_1^2 + 1, \tag{31}$$

$$\begin{aligned}
\bar{Q}_{10} &= \frac{\Delta p_{01}}{8A} + \frac{(\eta^2 - a_1^2)}{8} \left[\frac{(3\eta^2 - 5a_1^2)b_{12}}{4} - b_{11}(\eta^2 - a_1^2) \right] \\
&\quad + E_m \alpha \left[\frac{4\eta^6 a_1^2 - 3\eta^8 - a_1^8}{576} + SC \right. \\
&\quad \left. + S \left(\frac{3a_1^2 \eta^4 - 2\eta^6 - a_1^6}{48} \right) \right] \\
&\quad + b_{13} \left[\frac{a_1^2 - \eta^2}{2} a_1 \log(a_1/\eta) \right], \tag{32}
\end{aligned}$$

$$\bar{Q}_{10} = \frac{\Delta p_{10}}{8A} + \alpha D + F_0 \left[a_1^2 \log(\eta/a_1) + \frac{a_1^2 - \eta^2}{2} \right], \tag{33}$$

$$A = \int_0^1 \frac{dx}{a_1^4 - \eta^4 - S^2},$$

$$B = \int_0^1 \frac{dx}{a_1^2 + \eta^2 - S},$$

$$S = \frac{\eta^2 - a_1^2}{\log(a_1/\eta)},$$

$$\begin{aligned}
C_1 &= 2\eta^4 \log \eta + 20a_1^4 \log a_1 - 8\eta^4 (\log \eta)^2 - 8a_1^4 (\log a_1)^4 \\
&\quad + 16a_1^2 \eta^2 (\log \eta)^2 - 32\eta^2 \log \eta,
\end{aligned}$$

$$C_2 = (7\eta^2 - 17a_1^2)(\eta^2 - a_1^2),$$

$$C = \frac{C_1 - C_2}{128},$$

$$D = \frac{5a_1^6}{48} + \frac{\eta^6}{12} - \frac{3a_1 \eta^4}{16} + \left(\frac{5a_1^4 + 3\eta^4 - 8a_1^2 \eta^2}{32} \right) S,$$

$$\alpha = \frac{(\frac{1}{2}\pi)\bar{Q}_{00} + \eta - a_1}{a_1 \eta^2 - (a_1^3/3) - (2\eta^2/3) + S[a_1 \log(a_1/\eta) + a_1 - \eta]}.$$

Here, $\Delta p = (\Delta p_{00} + G_m \Delta p_{01} + \dots) + \sigma^2 (\Delta p_{10} + \dots) + \dots$ is the non-dimensional pressure drop over one wavelength and is given by

$$\Delta p = \frac{\Delta p_\lambda}{\mu c \lambda / a_0^2},$$

where $\Delta p_\lambda = \int_0^\lambda (\partial p / \partial x) dx$.

The heat transfer coefficient Z defined at the outer wall, in non-dimensional form, defined by

$$Z = \left[\frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial r} \frac{\partial \eta}{\partial x} \right]_{r=\eta}, \tag{34}$$

is finally obtained, after simplification, as

$$Z = (Z_{00} + G_m Z_{01} + \dots) + \sigma^2 (Z_{10} + \dots) + \dots \tag{35}$$

where

$$Z_{00} = -E_m \alpha^2 \left[\eta^3 + S^2 \frac{\log \eta}{\eta} + 2S\eta + \frac{b_{12}}{\eta} \right] \frac{d\eta}{dx}, \tag{36}$$

$$\begin{aligned}
Z_{01} &= -2E_m \frac{d\eta}{dx} \left[\frac{\eta^3}{16} F_3 \alpha - b_{12} \alpha \left(\frac{4 \log \eta - 3\eta^2}{64} \right) \right] \\
&\quad + E_m \alpha \left[-\frac{\eta^7}{96} + \frac{S\eta^3}{64} ((8 \log \eta^2) - 12 \log \eta - 7) \right] \\
&\quad + F_1 \alpha \frac{\eta}{4} + b_{18} \left[F_3 \frac{\eta^4}{4} - b_{12} \frac{\eta(\log \eta - 1)}{4} \right. \\
&\quad \left. + E_m \alpha^2 \left[\frac{\eta^5}{144} - \frac{S^2}{16} (\eta \log \eta)^2 - 4(\log \eta + 3) + S \frac{\eta^4}{16} \right] \right. \\
&\quad \left. + \frac{F_1 \log \eta}{\eta} \right] + \frac{b_{15}}{\eta} \frac{d\eta}{dx}, \tag{37}
\end{aligned}$$

$$\begin{aligned}
Z_{11} &= \frac{\alpha F_3 \eta^3}{8} - \alpha \left\{ \frac{7\eta^5}{48} + \frac{3S\eta^3}{16} \right\} + \frac{\alpha \eta}{2} \\
&\quad + b_{18} \left[\frac{\eta}{2} + \alpha \left\{ -\frac{3\eta^3}{8} + S \left(\frac{7\eta \log \eta}{16} - \frac{\eta}{2} \right) \right\} \right],
\end{aligned}$$

$$Z_{12} = (b_{19} + 1)^2 \frac{\eta}{2} + \frac{b_{18}\alpha}{32} (4\eta^3 \log \eta - \eta^3) + b_{18}(b_{18} + b_{19}) \left(\frac{-\eta}{2} + \eta \log \eta - \alpha(b_{19} + 1) \frac{\eta^3}{8} \right),$$

$$Z_{10} = -E_m \frac{d\eta}{dx} [Z_{11} + b_{18}Z_{12}] + \frac{F_2}{\eta} \frac{d\eta}{dx}. \quad (38)$$

The expressions for F_0 , F_1 , F_2 and F_3 appearing in the above equations are not presented here for brevity.

4. Results and discussion

The effects of pressure drop Δp , G_m , ε and σ^2 on mean flux have been presented in Figs. 2–5. Notice that for fixed values of all other parameters, the mean flux increases with amplitude as well as with pressure drop. However, variation of flux with Δp is found to be insignificant for large values of ε . That is, the effect of pressure drop on flux is negligible for peristaltic waves of large amplitude. Further, it is seen that flux increases with free convection and Eckert number. However, it may be noted that the increase of \bar{Q} with E_m is not significant. Flux decreases as channel porosity parameter (σ^2) increases.

Variations of the heat transfer coefficient at the wall have been presented in Tables 1–3. For given values of all other parameters, we can see that the heat transfer coefficient first

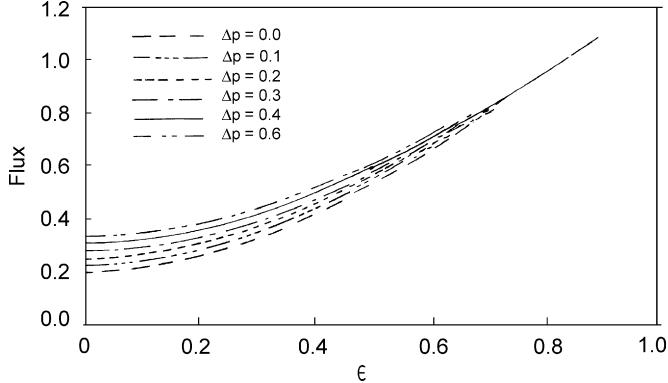


Fig. 2. Variation of mean flux with ε ($G_m = 1$, $\sigma^2 = 1$, $E_m = 2$).

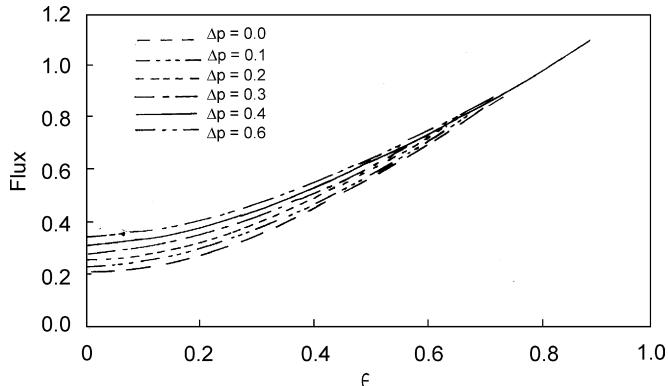


Fig. 3. Variation of mean flux with ε ($G_m = 1$, $\sigma^2 = 1$, $E_m = 4$).

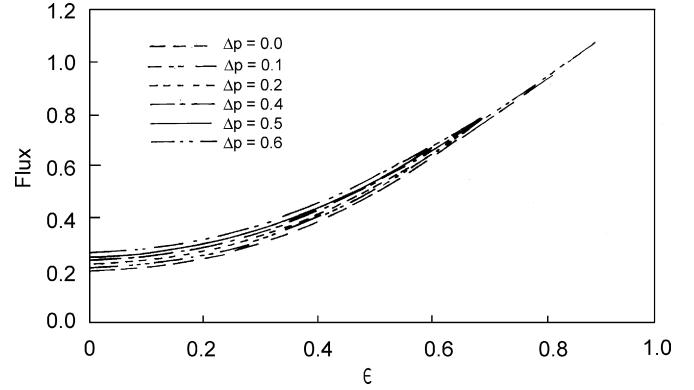


Fig. 4. Variation of mean flux with ε ($G_m = 1$, $\sigma^2 = 2$, $E_m = 2$).

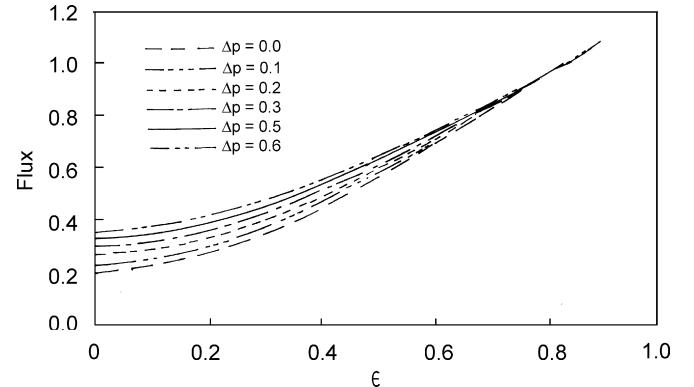


Fig. 5. Variation of mean flux with ε ($G_m = 2$, $\sigma^2 = 1$, $E_m = 2$).

Table 1
Heat transfer variation with σ^2 ($E_m = 3$, $G_m = 3$, $\varepsilon = 0.1$)

X	$\sigma^2 = 1$	$\sigma^2 = 3$
0.0	0.67875	0.679012
0.4	1.48593	1.486193
0.8	0.26149	0.26254

Table 2
Heat transfer variation with G_m ($E_m = 3$, $\sigma^2 = 2$, $\varepsilon = 0.1$)

X	$G_m = 1$	$G_m = 5$
0.0	0.68451	0.693262
0.4	1.49195	1.493177
0.8	0.26259	0.26544

Table 3
Heat transfer variation with ε ($E_m = 3$, $G_m = 3$, $\sigma^2 = 2$)

X	$\varepsilon = 0.1$	$\varepsilon = 0.2$
0.0	0.67888	1.357762
0.4	1.48606	1.867313
0.8	0.26152	0.67841

increases with X and then decreases, which may be due to peristalsis. Notice further that the heat transfer at the wall increases with an increase in the free convection parameter (G_m), porosity parameter (σ^2) and amplitude ratio parameter (ε): increase of Z with G_m and σ^2 is almost negligible. However, the mean flux (\bar{Q}) is found to increase by 10–12% as the free convection parameter (G_m) increases from 1 to 2, for given values of all other parameters.

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