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Flow past a porous approximate spherical shell

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Abstract. In this paper, the creeping flow of an incompressible viscous liquid past a porous approximate spherical shell is considered. The flow in the free fluid region outside the shell and in the cavity region of the shell is governed by the Navier–Stokes equation. The flow within the porous annulus region of the shell is governed by Darcy's Law. The boundary conditions used at the interface are continuity of the normal velocity, continuity of the pressure and Beavers and Joseph slip condition. An exact solution for the problem is obtained. An expression for the drag on the porous approximate spherical shell is obtained. The drag experienced by the shell is evaluated numerically for several values of the parameters governing the flow.

Mathematics Subject Classification (2000). 76D07.

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Introduction

Several researchers have studied the flow of fluids past porous particles, as they are of great importance in geophysical, industrial and engineering applications. Some of the applications are flow through porous beds (fixed or fluidized), sedimentation of fine particulate suspensions, modeling of polymer macromolecule coils in solvent, catalytic reactions where porous pellets are used, floc settling process etc. Porous particles are frequently formed by vapor condensation-coagulation process in the atmosphere and in other environmental systems.

The flow problems past porous particles have been modeled by using Stokes version of Navier–Stokes equation for the flow out side the porous particles and Darcy's [1] or Brinkman's [2] equation to describe the flow within the porous particles. Darcy's law states that the filtration velocity is proportional to the pressure gradient in the porous medium. Thus momentum equations in the porous media and the free fluid have different orders. This incompatibility between the internal and external flow equations has resulted in much uncertainty regarding the boundary conditions at the interface between a porous medium and the free fluid. Hence several types of boundary conditions at the interface of the free fluid and porous region to link the different flow regimes were suggested in literature.

Using the condition of continuity of normal velocity and pressure at the surface of the porous sphere and no-slip of tangential velocity component of the free fluid, Joseph and Tao [3] considered the creeping flow past a porous spherical shell immersed in a uniform viscous incompressible fluid. It was noted by Beavers and Joseph [4], in connection with the experimental investigations of viscous flow past planar permeable surfaces, that a slip occurs at the boundary and they proposed a slip boundary condition. Using a statistical approach to extend Darcy's law to non-homogeneous porous media, Saffman [5] gave a theoretical justification of the condition proposed by Beavers and Joseph in the limit of small permeability (i.e. in the limit $k \to 0$, where k is the permeability). Neale *et al.* [7] proved that the condition of Saffman was the most satisfactory one. Therefore for small values of permeability, Saffman's condition is more appropriate than the usual no-slip condition. Using the Saffman condition together with continuity of normal velocity and pressure at the surface of the porous boundary, Rajasekhar *et al* [6] have studied the Stokes flow of a viscous fluid inside a sphere with internal singularities enclosed by a porous spherical shell. A generalization of a boundary condition suggested by Beavers and Joseph for planar boundaries was proposed by Jones [8] for curved surfaces. He used this condition to solve the problem of the slow viscous flow past a porous spherical shell. Sutherland and Tan [9] have used the continuity of the tangential velocity component to study the sedimentation of a porous sphere. Using this boundary condition, the flow past and within permeable spheroid was considered by Vainshtein *et al.* [10]. They have examined the flow past permeable circular disk and elongated rods as limiting cases

The aim of the present paper is to study the flow of incompressible viscous fluid past and within a porous approximate spherical shell. The flow examined is axially symmetric in nature. The flow equations are based on the Stokesian version of Navier–Stokes equation in the general viscous flow regime and the use of Darcy's Model in the porous regions. As boundary conditions, continuity of the velocity, pressure and the slip condition at the interface proposed by Jones are employed.

Formulation of the problem

Consider the spherical polar coordinate system (r, θ, ϕ). Let (e_r, e_θ, e_ϕ) be the unit base vectors of the coordinate system and $h_1 = 1$, $h_2 = r$ and $h_3 = r \sin\theta$ the scale factors.

Consider the creeping flow of an incompressible Newtonian Viscous fluid past and within a porous approximate spherical shell of average external and internal radii a and b ($a > b$). Assume that there is a uniform velocity $\bar{U}$ far away from the shell along the axis of symmetry $\theta = 0$. Let the equation of the outer approximate sphere be $r = a\{1 + \sum_{m=2}^{\infty} \beta_m \vartheta_m(\xi)\} \equiv r_a$ and inner approximate sphere be $r =$

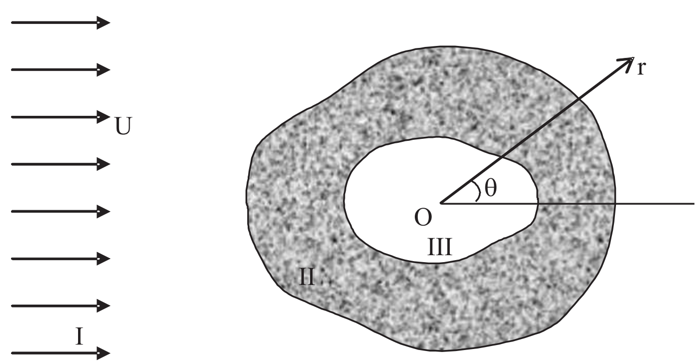

Figure 1. The physical situation and the coordinate system

$b\{1+\sum_{m=2}^{\infty}\gamma_m\vartheta_m(\xi)\}\equiv r_b$ where β_m's and γ_m's are small, $\xi=\cos\theta$ and $\vartheta_n(\xi)$ is the Gegenbauer function of the first kind of order n and degree $-1/2$. If all the β_m's and γ_m's are zero, the approximate spherical shells reduce to spherical shells.

The external region ($r\geq r_a$), the porous region ($r_b\leq r\leq r_a$) and the cavity region ($r\leq r_b$) are denoted by regions I, II and III respectively. Assume that the flow in both the regions I and region III i.e. inside and outside of the approximate spherical shell is governed by Stokes approximation to the Navier–Stokes equation

$$\text{div}\vec{q}^{(i)}=0 \tag{1}$$

$$\text{grad p}^{(i)}+\mu\text{ curl curl }\vec{q}^{(i)}=0, i=1,3 \tag{2}$$

and flow in the region – II is governed by the Darcy's empirical formula

$$\text{div}\vec{q}^{(2)}=0 \tag{3}$$

$$\vec{q}^{(2)}=-\frac{k}{\mu}\text{grad p}^{(2)} \tag{4}$$

where $\vec{q}^{(i)}$ is the fluid velocity, $p^{(i)}$ is the pressure and μ is the coefficient of viscosity and k is the permeability of the medium. The superscripts $i=1,2$ and 3 correspond to the fluid properties in the regions I, II and III respectively.

Since the flow of the fluid is in the meridian plane and the flow is axially symmetric, all the physical quantities are independent of ϕ. Hence we assume that

$$\vec{q}^{(i)}=[u^{(i)}(r,\theta)\vec{e}_r+v^{(i)}(r,\theta)\vec{e}_\theta],\qquad i=1,2,3 \tag{5}$$

To determine the flow velocity and pressure outside and within the porous annular approximate spherical region and within the cavity region, we use the following boundary conditions at the surface of a porous body to link the different flow regimes.

The first condition is that continuity of normal velocity. i.e.,

$$u^{(1)}(r,\theta) = u^{(2)}(r,\theta) \text{ on } r = a[1 + \sum \beta_m \vartheta_m(\xi)]$$

$$u^{(2)}(r,\theta) = u^{(3)}(r,\theta) \text{ on } r = b[1 + \sum \gamma_m \vartheta_m(\xi)] \tag{6}$$

the second condition is that continuity of pressure

$$p^{(1)}(r,\theta) = p^{(2)}(r,\theta) \text{ on } r = a[1 + \sum \beta_m \vartheta_m(\xi)]$$

$$p^{(2)}(r,\theta) = p^{(3)}(r,\theta) \text{ on } r = b[1 + \sum \gamma_m \vartheta_m(\xi)] \tag{7}$$

and the third condition is the slip condition of Beavers and Joseph [4]

$$r\frac{\partial}{\partial r}\left(\frac{v^{(1)}}{r}\right) + \frac{1}{r}\frac{\partial u^{(1)}}{\partial \theta} = \frac{\sigma}{\sqrt{k}}\left(v^{(1)} - v^{(2)}\right) \text{ on } r = a[1 + \sum \beta_m \vartheta_m(\xi)]$$

$$r\frac{\partial}{\partial r}\left(\frac{v^{(3)}}{r}\right) + \frac{1}{r}\frac{\partial u^{(3)}}{\partial \theta} = -\frac{\sigma}{\sqrt{k}}\left(v^{(3)} - v^{(2)}\right) \text{ on } r = b[1 + \sum \gamma_m \vartheta_m(\xi)] \tag{8}$$

where σ is a dimensionless parameter that depends on the properties of the porous medium [8].

In addition to the above boundary conditions, we have the usual regularity conditions at infinity i.e the flow far away from the body is uniform

$$\underset{r\to\infty}{Lt}\, u^{(1)}(r,\theta) = U\cos\theta \qquad \underset{r\to\infty}{Lt}\, v^{(1)}(r,\theta) = -U\sin\theta \tag{9}$$

and the condition that velocity and pressure must have no singularities anywhere in the flow field

Solution of the problem

Introducing the stream functions $\psi^{(i)}$ (r,θ), $i = 1, 2$ through

$$u^{(i)} = \frac{-1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta}; \quad v^{(i)} = \frac{1}{r^{\sin\theta}}\frac{\partial\psi^{(i)}}{\partial r}, \quad i = 1, 2, 3 \tag{10}$$

in the equations (1)–(4) and eliminating the pressure from the resulting equations, we get the following dimensionless equations for $\psi^{(i)}$, $i = 1, 2, 3$

$$E^4\psi^{(1)} = 0 \tag{11}$$

$$E^2\psi^{(2)} = 0 \tag{12}$$

and

$$E^4\psi^{(3)} = 0 \tag{13}$$

where E^2denotes the Stokes stream function operator given by

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta}. \tag{14}$$

For the region I, the solution of (11) which is regular at infinity i.e., far away from the shell and on the axis is given by

$$\psi^{(1)} = \left(r^2 + \frac{A_2}{r} + B_2 r\right)\vartheta_2(\xi) + \sum_{n=3}^{\infty}(A_n r^{-n+1} + B_n r^{-n+3})\vartheta_n(\xi). \tag{15}$$

For the region II, the solution of (12) is given by

$$\psi^{(2)} = \left(C_2 r^2 + \frac{D_2}{r}\right)\vartheta_2(\xi) + \sum_{n=3}^{\infty}(C_n r^n + D_n r^{-n+1})\vartheta_n(\xi). \tag{16}$$

For the region III, the solution of (13) is given by

$$\psi^{(3)} = (E_2 r^2 + F_2 r^4)\vartheta_2(\xi) + \sum_{n=3}^{\infty}(E_n r^n + F_n r^{n+2})\vartheta_n(\xi). \tag{17}$$

Using (15) in (2) and integrating the resulting equations, we get the pressure distribution $p^{(1)}$ out side the body as

$$p^{(1)} = -\frac{B_2}{r^2}P_1(\xi) + \sum_{n=3}^{\infty}\frac{(6-4n)}{n}B_n r^{-n}P_{n-1}(\xi) \tag{18}$$

Similarly, the pressure $p^{(2)}$ within the annulus porous region and $p^{(3)}$ within the cavity region is obtained by using (16) in (4) and (17) in (2) then integrating the resulting equations. Hence $p^{(2)}(r,\theta)$ and $p^{(3)}(r,\theta)$ are given by

$$p^{(2)} = \alpha^2\left[\left(C_2 r - \frac{D_2}{2r^2}\right)P_1(\xi) + \sum_{n=3}^{\infty}\left(C_n\frac{r^{n-1}}{n-1} - \frac{D_n r^{-n}}{n}\right)P_{n-1}(\xi)\right] \tag{19}$$

and

$$p^{(3)} = -10rF_2P_1(\xi) - \sum_{n=3}^{\infty} F_n \frac{(4n+2)r^{n-1}}{(n-1)} P_{n-1}(\xi) \tag{20}$$

where $\alpha^2 = a^2/k$

The boundary conditions (6)-(8) in terms of the stream function in dimensionless form are

$$\psi^{(1)}(r,\theta) = \psi^{(2)}(r,\theta), \quad 2[\psi^{(1)}_{rr} - \frac{1}{r}\psi^{(1)}_r] - E^2\psi^{(1)} = \alpha\sigma(\psi^{(1)}_r - \psi^{(2)}_r)$$
$$p^{(1)}(r,\theta) = p^{(2)}(r,\theta) \text{ on } r = [1 + \sum \beta_m \vartheta_m(\xi)] \tag{21}$$

and

$$\psi^{(2)}(r,\theta) = \psi^{(3)}(r,\theta), \quad 2[\psi^{(3)}_{rr} - \frac{1}{r}\psi^{(3)}_r] - E^2\psi^{(3)} = \alpha\sigma(\psi^{(3)}_r - \psi^{(2)}_r)$$
$$p^{(2)}(r,\theta) = p^{(3)}(r,\theta) \text{ on } r = \eta[1 + \sum \gamma_m \vartheta_m(\xi)] \tag{22}$$

where $\eta = b/a$.

First, we develop the solution corresponding to the boundaries $r = a[1 + \beta_m \vartheta_m(\xi)]$ and $r = b[1 + \gamma_m\vartheta_m(\xi)]$. Assume that the coefficients β_m and γ_m are sufficiently small so that squares and higher powers of β_m and γ_m can be neglected [11]. Comparison of the equations (15), (16) and (17) with those obtained in case of flow of an incompressible viscous fluid past a porous spherical shell, indicates that the terms involving A_n, B_n, C_n, D_n, E_n and F_n for $n > 2$ are the extra terms here which are not present in the case of spherical shell [8]. The body that we are considering is an approximate spherical shell and the flow generated is not expected to be far different from the one generated by flow past a porous spherical shell. Also the coefficients A_n, B_n, C_n, and D_n for $n > 2$ are of order β_m and the coefficients C_n, D_n, E_n, and F_n for $n > 2$ are of order γ_m. Therefore, while implementing the boundary conditions, we ignore the departure from the spherical form and set in (21) $r = 1$ in the terms involving A_n, B_n, C_n, and D_n,for $n > 2$ and in (22) $r = \eta$ in the terms involving C_n, D_n, E_n, and F_n, for $n > 2$.

Expanding the boundary conditions (21) and (22) to the first order in β_m and γ_m and using the observations made above, we get

$$A_2 = \frac{\beta}{(6+\alpha\sigma)}\Big\{2 - \frac{(6+3\alpha\sigma)}{L}[(3+\eta\alpha\sigma)\eta\alpha^2(\alpha^2(1-\eta^3) + 2 - 2\eta^3) + 30\alpha\sigma(\alpha^2+2)]\Big\} \tag{23}$$

$$B_2 = -\alpha^2[\alpha^2(3+\eta\alpha\sigma)\eta(1-\eta^3) + 30\alpha\sigma](6+3\alpha\sigma)/L \tag{24}$$

$$C_2 = [\eta\alpha^2(3+\eta\alpha\sigma) + 30\alpha\sigma](6+3\alpha\sigma)/L \tag{25}$$

$$D_2 = 2\eta^4\alpha^2(3+\eta\alpha\sigma)(6+3\alpha\sigma)/L \tag{26}$$

$$E_2 = [3\eta\alpha^2(3+2\eta\alpha\sigma)+30\alpha\sigma](6+3\alpha\sigma)/L \tag{27}$$

$$F_2 = -3\alpha\sigma\alpha^2(6+3\alpha\sigma)/L \tag{28}$$

where

$$L = 18\eta\{-2\eta^3+\alpha^2(\eta^3-1)-1\}\alpha^2+\alpha^2\sigma^2\{2\eta^2\alpha^4(\eta^3-1)-3(\eta^2+20)\alpha^2-90\}$$
$$+3\alpha\sigma\{2\eta(\eta^4+\eta^3-\eta-1)\alpha^4-(4\eta^5+2\eta^2+3\eta+60)\alpha^2-60) \tag{29}$$

For $n \neq m-2, m, m+2$

$$A_n = B_n = C_n = D_n = E_n = F_n = 0 \tag{30}$$

and the following linear system of equations in A_n, B_n, C_n, D_n, E_n and F_n for $n = m-2, m, m+2$

$$A_n + B_n - C_n - D_n = (-2+A_2-B_2+2C_2-D_2)\beta_m b_n \tag{31}$$

$$[2(n^2-1)+(n-1)\beta]A_n+[2n(n-2)+(n-3)\beta]B_n$$
$$+n\beta C_n-(n-1)\beta D_n = (18A_2+\beta(2+2A_2-2C_2-2D_2))\beta_m b_n \tag{32}$$

$$\frac{4n-6}{n}B_n+\alpha^2\left(\frac{1}{n-1}C_n-\frac{D_n}{n}\right) = (2B_2-\alpha^2(C_2+D_2))\beta_m a_n \tag{33}$$

$$-\eta^n C_n-\eta^{1-\eta}D_n+\eta^n E_n+\eta^{n+2}F_n = (2C_2\eta^2-\frac{D_2}{\eta}-2E_2\eta^2-4\eta^4F_2)\gamma_m b_n \tag{34}$$

$$2[n\{1+\beta\eta(n-2)\}E_n\eta^{n-1}+\{(n^2-1)+\beta\eta(n+2)\}F_n\eta^n$$
$$-\beta\{nC_n\eta^{n-1}-(n-1)D_n\eta^{-n}]$$
$$=(12F_2\eta^2+\beta(2\eta E_2+12F_2\eta^3-2C_2\eta-\frac{2D_2}{\eta^2}))\gamma_m b_n \tag{35}$$

$$-\frac{\alpha^2C_n}{n-1}\eta^{n-1}+\frac{\alpha^2D_n}{n}\eta^{-n}+\frac{(4n+2)}{(n-1)}F_n\eta^{n-1} = (\alpha^2(\eta C_2+D_2/\eta^2)+10\eta F_2)\gamma_m a_n \tag{36}$$

where

$$b_{m-2} = -(m-2)a_{m-2} = -\frac{(m-2)(m-3)}{2(2m-1)(2m-3)} \tag{37}$$

$$b_m = m(m-1)a_m = \frac{m(m-1)}{(2m+1)(2m-3)} \tag{38}$$

$$b_{m+2} = (m+2)a_{m+2} = -\frac{(m+1)(m+2)}{2(2m-1)(2m+1)}. \tag{39}$$

Solving this system of equations, we obtain the values of A_n, B_n, C_n, D_n, E_n, and F_n, for $n = m-2$, m, $m+2$.

Drag on the shell

The drag force acting on the porous approximate spherical shell is given by

$$D = 2\pi a^2 \int_0^\pi \left[\tau_{rr}^{(1)} \cos\theta - \tau_{r\theta}^{(1)} \sin\theta\right]_{r=a[1+\sum \beta_m \vartheta_m(\zeta)]} \sin\theta d\theta. \tag{40}$$

Expanding the integrand in terms of stream function the equation (40) can be written as [11]

$$D = 2a^2\pi \int_0^\pi r^3 \sin^3\theta \frac{\partial}{\partial r}\left(\frac{1}{r^2\sin^2\theta} E^2\psi^{(1)}\right) r d\theta. \tag{41}$$

Using equation (15) and on carrying out the integration it is found to be

$$D = 4\pi\mu U a[\Delta_1 + (1/5)(\Delta_2\beta_2 + \Delta_3\gamma_2) + (2/35)(\Delta_4\beta_4 + \Delta_5\gamma_4)]/L \tag{42}$$

where

$$\Delta_1 = -[3\alpha^2(2+\alpha\sigma)\{3\alpha^2\eta(-1+\eta^3) + \alpha\sigma(-30+\alpha^2\eta^2(-1+\eta^3))\}] \tag{43}$$

$$\begin{aligned}\Delta_2 = &-[90\alpha\sigma(2+\alpha\sigma)\{[9\alpha^2(2+\alpha\sigma)\{3\alpha^2\eta + \alpha\sigma(30+\alpha^2\eta^2)\}]/L\}\\ &+ 2\alpha^4((6+\alpha\sigma)\varepsilon_1 - \varepsilon_2)\eta(-1+\eta^3)(3+\alpha\sigma\eta) + 3\alpha^2\{-20\alpha\sigma((6+\alpha\sigma)\varepsilon_1\\ &- \varepsilon_2) + (3(2+\alpha\sigma)\eta + \alpha\sigma(2+\alpha\sigma)\eta^2 + 12\eta^4\\ &+ 4\alpha\sigma\ \eta^5)\{[9\alpha^2(2+\alpha\sigma)\{3\alpha^2\eta + \alpha\sigma(30+\alpha^2\eta^2)\}]/L\}\}]\end{aligned} \tag{44}$$

$$\begin{aligned}\Delta_3 = &-[3\alpha^2(6+\alpha\sigma)\ \eta\{-20\alpha\sigma\{[54\alpha^2(2+\alpha\sigma)\ \eta^3]/L\}\\ &- 10_\eta\{[18\alpha^3\sigma(2+\alpha\sigma)\eta^2(6+5\alpha\sigma\ \eta)]/L\}\\ &+ (3\eta^2 + \alpha\sigma\ \eta^3)\{-[9\alpha^4(2+\alpha\sigma)\eta^2(3+\alpha\sigma\ \eta)]/L\}\}]\end{aligned} \tag{45}$$

$$\begin{aligned}\Delta_4 = &[-180\alpha\sigma(2+\alpha\sigma)\{[9\alpha^2(2+\alpha\sigma)\{3\alpha^2\eta + \alpha\sigma(30+\alpha^2\eta^2)\}]/L\}\\ &+ \alpha^4((6+\alpha\sigma)\ \varepsilon_1 - \varepsilon_2)\eta(-1+\eta^3)(3+\alpha\sigma\ \eta) - 6\alpha^2\{5\alpha\sigma((6+\alpha\sigma)\ \varepsilon_1 - \varepsilon_2)\end{aligned}$$

$$+ (3(2+\alpha\sigma)\eta + \alpha\sigma(2+\alpha\sigma)\ \eta^2 + 12\eta^4$$
$$+ 4\alpha\sigma\ \eta^5)\{[9\alpha^2(2+\alpha\sigma)\{3\alpha^2\eta + \alpha\sigma(30+\alpha^2\eta^2)\}]/L\}\}] \quad (46)$$

$$\Delta_5 = -[3\alpha^2(6+\alpha\sigma)\ \eta\{\eta(5\{[18\alpha^3\sigma(2+\alpha\sigma)\eta^2(6+5\alpha\sigma\ \eta)]/L\}$$
$$+ 6\eta - [9\alpha^4(2+\alpha\sigma)\eta^2(3+\alpha\sigma\ \eta)]/L\}) + 2\alpha\sigma(5\{[54\alpha^2(2+\alpha\sigma)\ \eta^3]/L\}$$
$$+ \eta^3 - [9\alpha^4(2+\alpha\sigma)\eta 2(3+\alpha\sigma\ \eta)]/L\})\}] \quad (47)$$

$$\varepsilon_1 = [6\alpha^2\{-3\alpha^2\eta + 3(-6+\alpha^2)\eta^4 + \alpha\sigma(-6(5+\eta^5) + \alpha^2(-\eta^2+\eta^5))\}]/L \quad (48)$$

$$\varepsilon_2 = [6\alpha^3\sigma\{5+\alpha\sigma)(30\alpha\sigma + 3\alpha^2\eta + \alpha^3\sigma\eta^2)$$
$$-(-18+\alpha^2(5+\alpha\sigma))(3\eta^4 - \alpha\sigma\eta^5)\}]/L \quad (49)$$

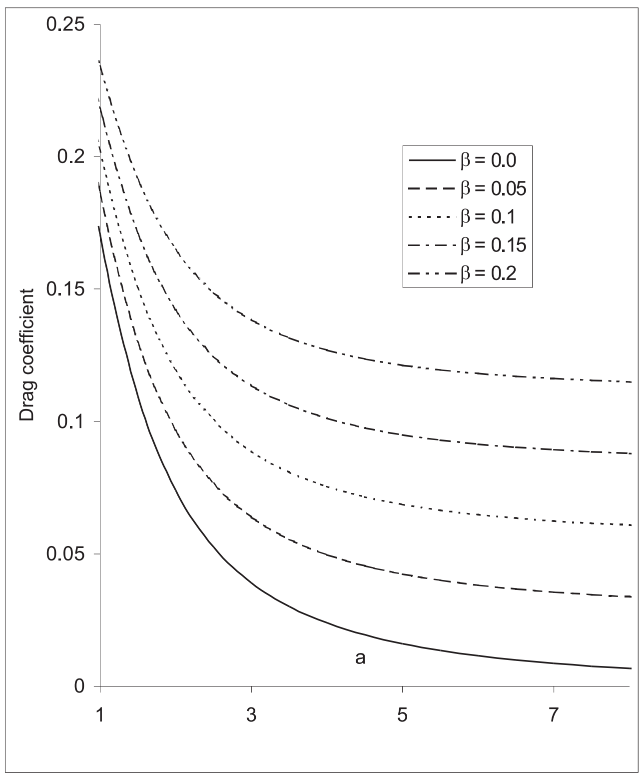

Figure 2. Variation of Drag Coefficient with α for various values of β; $\sigma = 0.75$, $\gamma = 0.0$ and $\eta = 0.6$

It is interesting to note that though the boundary surface is $r = a[1+\sum \beta_m \vartheta_m(\xi)]$

and $r = b\{1 + \sum_{m=2}^{\infty} \gamma_m \vartheta_m(\xi)\ \}$, the coefficients β_2, β_4, γ_2 and γ_4 only, contribute to the drag. This implies that the drag on the porous approximate spherical shell is relatively insensitive to the details of the surface geometry. This is similar to the observation made by Srinivasacharya [12] in case of flow past a porous approximate sphere.

If $\beta_m = \gamma_m = 0$, for $m > 2$, the approximate spherical shell reduces to a spherical shell and the drag on porous spherical shell is

$$-4\pi\mu U a[3\alpha^2(2+\alpha\sigma)\{3\alpha^2\eta(-1+\eta^3)+\alpha\sigma(-30+\alpha^2\eta^2(-1+\eta^3))\}]/L$$

The expression for the drag experienced by a porous spherical shell as obtained by Jones [8] is

$$6\pi\mu a U(2+a\beta)\left[\frac{3k}{2ab}+\frac{3}{20ab}+\frac{b}{20a}-\frac{3b^3}{20a^4\beta}-\frac{b^4}{20a^4}\right]\Big/$$
$$\left[3+a\beta+\frac{3k}{a^2}-\frac{3k\beta}{2a}\right]\left[\frac{3k}{2ab}-\frac{3}{20a\beta}+\frac{b}{20a}\right]-\left[3+a\beta-\frac{6k}{a^2}\right]\left[\frac{3b^3}{20a^4\beta}+\frac{b^4}{20a^4}\right] \tag{50}$$

which can be simplified to

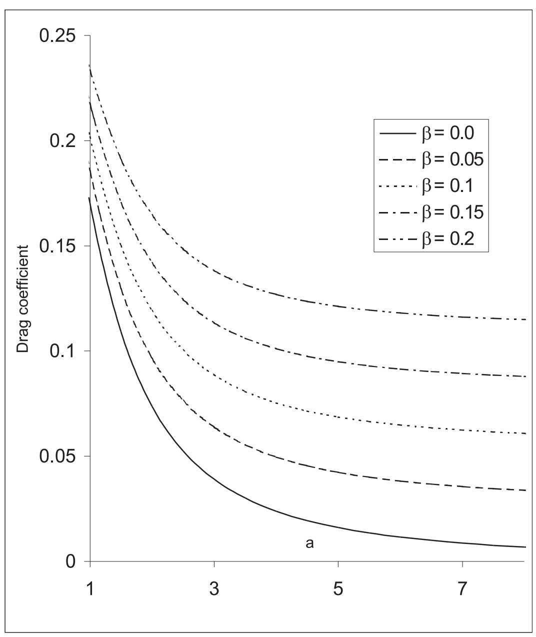

Figure 3. Variation of Drag Coefficient with α for various values of β; $\sigma = 0.75$, $\gamma = 0.1$ and $\eta = 0.6$

$$- 6\pi\mu U a^3(a\beta + 2)[a^3(\beta b^2 + 3b + 30k\beta) - b^4(b\beta + 3)]/[2\beta(\beta b^2 + 3b + 30k\beta)a^6$$
$$+ 6(\beta b^2 + 3b + 30k\beta)a^5 + 3k\beta(\beta b^2 + 3b + 30k\beta)a^4 + (-2\beta^2 b^5 - 6\beta b^4 + 6k\beta b^2$$
$$+ 18kb + 180k^2\beta)a^3 - 6b^4(b\beta + 3)a^2 + 12b^4k(b\beta + 3) \quad (51)$$

and is same as (50) with η = b/a, $\alpha^2 = a^2/k$, $\sigma = \beta\sqrt{k}$. Thus the expression for drag in Jones [8] is recoverable from (50) as a special case.

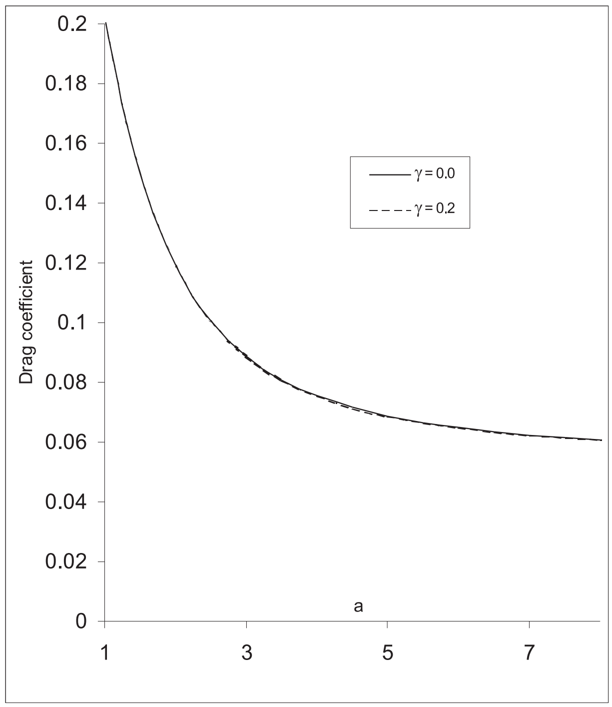

Figure 4. Variation of Drag Coefficient with α for various values of γ; $\sigma = 0.75$, $\beta = 0.1$ and $\eta = 0.6$

Since$\sqrt{k}$ is of the order of the size of the pore, we take $\sqrt{k}$ to be much smaller than ‘a’ (i.e. α to be greater than one) when Darcy’s model is used in the porous region. The variation of drag coefficient $D_N = - D / (4\pi\mu Ua)$ for various values of α, $\beta_2 = \beta_4 = \beta$ with $\sigma = 0.75$, $\eta = 0.6$, $\gamma_2 = \gamma_4 = \gamma = 0.0$ is shown in Fig. 2 and for $\gamma = 0.1$ is shown in Fig. 3. From these two figures it can be observed that the drag coefficient is decreasing as the permeability parameter (α) is increasing. There is increase in the drag coefficient as the deformation parameter of the outer sphere (β) is increasing. It is interesting to note that the drag coefficient on the spherical shell is less than that of the drag on the approximate spherical shell. The

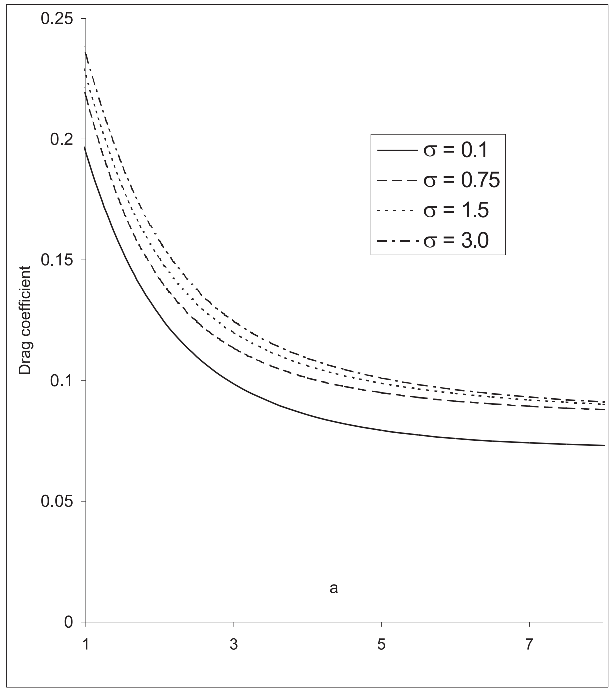

Figure 5. Variation of Drag Coefficient with α for various values of σ; $\eta = 0.6$, $\gamma = 0.1$ and $\beta = 0.15$

variation of drag coefficient D_N for various values of α and γ with $\beta = 0.1$, $\sigma = 0.75$, $\eta = 0.6$ is shown in Fig. 4. There is no effect of deformation parameter of the inner sphere (γ) on the drag coefficient. The effect the parameter σ on drag coefficient D_N for fixed values $\gamma = 0.1$, $\beta = 0.15$, $\eta = 0.75$ is shown in Fig. 5. As the parameter σ increases, the drag coefficient is increasing.

Conclusions

An exact solution for the problem of the creeping flow of an incompressible viscous liquid past a porous approximate spherical shell is obtained by considering the Darcy's law in the porous region and Stokes equations in the liquid region. At the porous-liquid interface Beavers and Joseph slip boundary condition, continuity of the normal velocity and continuity of the pressure have been used. An expression for the drag on the porous approximate spherical shell is obtained. It is observed that the drag coefficient on the spherical shell is less than that of the drag on the approximate spherical shell and there is increase in the drag coefficient as the parameter σ is increasing. The drag coefficient increases as the deformation parameter of the outer sphere is increasing and the effect of deformation parameter of the inner sphere on the drag coefficient is negligible.

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