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Unsteady stokes flow of micropolar fluid between two parallel porous plates

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Abstract

In this paper, we consider the unsteady flow of incompressible micropolar fluid between two parallel porous plates when there is a periodic suction at the lower plate and injection at the upper plate. Stream function for the flow is obtained and the effects of microrotation parameter and frequency parameter on skin friction at the lower and upper plates are numerically studied. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In the theory of micropolar fluids, originated by Eringen [1], both the effect of couple stresses and the microscopic effects arising from local structure and microrotation of the fluid element are simultaneously taken into account. The fluids containing certain additives, some polymeric fluids and animal blood are examples of micropolar fluids. The mathematical theory of equations of micropolar fluids and application of these fluids in the theory of lubrication and in the theory of porous media is presented in [2].

The problem of steady flow of an incompressible viscous fluid through a porous channel was considered by Berman [3]. He obtained a perturbation solution assuming normal wall velocities to be equal. The analysis of Berman was extended by Sellars [4], Terrill [5] and Yuan [6] for various values of suction and injection Reynolds numbers. Terrill and Shrestha [7] have examined the same problem, assuming different normal velocities at the walls. The steady flow of an incom-

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pressible micropolar fluid between two parallel porous plates was studied by Teeka Rao and Moizuddin [8]. They obtained the solution by the method of parameter perturbation, with parameter being the suction Reynolds number. Sastry and Ramamohan Rao [9] have obtained the solution to the problem of micropolar fluid flow in a channel with porous walls by numerical method based up on quasilinearisation, parametric differentiation and extrapolation. Several researchers [10–12] have considered the problems of unsteady flow of classical viscous fluid through a channel. Das and Sanyal [13] have considered the unsteady flow of a micropolar fluid through rectangular channel under periodic pressure gradient.

This paper concerns the unsteady flow of an incompressible micropolar fluid between two parallel plates, assuming periodic suction at the lower plate and injection at the upper plate. The velocities are expressed in terms of a stream function and expressions for velocities and microrotation component are derived by introducing the similarity variable. The variation of skin friction at the upper and lower plates is studied with respect to fluid parameters.

Assuming the flow to be stokesian, neglecting the inertial and gyroinertial terms and Ignoring the body force and body couple, the field equations of the micropolar fluid dynamics are

$$\operatorname{div} \vec{q} = 0, \tag{1}$$

$$\rho \frac{\partial \vec{q}}{\partial t} = -\operatorname{grad} P - k \operatorname{curl} \vec{v} - (\mu + k) \operatorname{curl} \operatorname{curl} \vec{q}, \tag{2}$$

$$\rho j \frac{\partial \vec{v}}{\partial t} = 2k\vec{v} + k \operatorname{curl} \vec{q} - \gamma \operatorname{curl} \operatorname{curl} \vec{v} + (\alpha + \beta + \gamma) \operatorname{grad} \ (\operatorname{div} \vec{v}), \tag{3}$$

where $\vec{q}$ is the velocity vector, $\vec{v}$ is the microrotation vector and the P is the fluid pressure, ρ and j are the fluid density and microgyration parameter, and $\{\mu, k\}$ and $\{\alpha, \beta, \gamma\}$ are viscosity and gyroviscosity coefficients.

The stress tensor t_{ij} and the couple stress tensor m_{ij} are given by

$$t_{ij} = -P\delta_{ij} + (2\mu + k)e_{ij} + k\epsilon_{ijm}(\omega_m - v_m), \tag{4}$$

$$m_{ij} = \alpha v_{k,k}\delta_{ij} + \beta v_{i,j} + \gamma v_{j,i}, \tag{5}$$

where $\vec{\omega}$ is the vorticity vector and δ_{ij} is the Kronecker delta and ϵ_{ijm} is the alternating symbol.

2. Formulation of the problem

Consider the flow of micropolar fluid through two porous parallel plates $y = 0$ and $y = h$ along the direction of X-axis. since the flow is along X-direction, all the variables are independent of Z. Assume that there is a periodic suction of velocity $v_1 e^{i\omega t}$ at the lower plate and periodic injection of velocity $v_2 e^{i\omega t}$ at the upper plate. Hence we choose the velocity vector ($\vec{q}$), microrotation vector ($\vec{v}$) and the pressure in the form

$$\vec{q} = \left[u(x,y)\vec{i} + v(x,y)\vec{j}\right]e^{i\omega t}, \quad \vec{v} = C(x,y)\vec{k}e^{i\omega t} \quad \text{and} \quad P(x,y) = p(x,y)e^{i\omega t}. \tag{6}$$

In view of the continuity equation (1), we introduce the stream function $\psi(x,y)$ through

$$u(x,y) = \frac{\partial \psi}{\partial y}, \quad v(x,y) = -\frac{\partial \psi}{\partial x}. \tag{7}$$

Eqs. (2) and (3) give rise to:

$$i\rho\omega \frac{\partial \psi}{\partial y} = -\frac{\partial p}{\partial x} + k\frac{\partial C}{\partial y} + \frac{\partial \nabla^2 \psi}{\partial y}, \tag{8}$$

$$i\rho\omega \frac{\partial \psi}{\partial x} = \frac{\partial p}{\partial y} + k\frac{\partial C}{\partial x} + \frac{\partial \nabla^2 \psi}{\partial x}, \tag{9}$$

$$i\rho\omega jC = -2kC - k\nabla^2\psi + \gamma\nabla^2 C. \tag{10}$$

Eliminating pressure p from Eqs. (8) and (9), we get

$$\nabla^2(\nabla^2 - \alpha^2)(\nabla^2 - \beta^2)\psi = 0 \tag{11}$$

and

$$k(i\rho j\omega + 2k)C = -\gamma(\mu + k)\nabla^4\psi + (i\gamma\rho\omega - k^2)\nabla^2\psi, \tag{12}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian operator and

$$\alpha^2 + \beta^2 = \frac{k(2\mu + k) + i\rho\omega[\gamma + j(\mu + k)]}{\gamma(\mu + k)}, \tag{13}$$

$$\alpha^2\beta^2 = \frac{i\rho\omega(2k + i\rho j\omega)}{\gamma(\mu + k)}. \tag{14}$$

Eq. (11) is to be solved with the boundary conditions:

$$u(x,h) = u(x,0) = 0, \quad v(x,0) = v_1 \quad \text{and} \quad v(x,h) = v_2, \quad C(x,0) = C(x,h) = 0. \tag{15}$$

Following Terrill [9], we introduce $f(\eta)$ and $g(\eta)$ as follows:

$$\psi = h\left[\frac{U_0}{a} - \frac{v_2 x}{h}\right]f(\eta), \quad C = \frac{1}{h}\left[\frac{U_0}{a} - \frac{v_2 x}{h}\right]g(\eta), \tag{16}$$

where $\eta = y/h$, $a = 1 - (v_1/v_2)$, $0 \leqslant |v_1| \leqslant |v_2|$ and U_0 is the average entrance velocity.

Now Eqs. (11) and (12) become

$$D^2\left(D^2-\alpha^2h^2\right)\left(D^2-\beta^2h^2\right)f(\eta)=0, \tag{17}$$

where $D^2=\mathrm{d}^2/\mathrm{d}\eta^2$ and

$$kh^2(\mathrm{i}\rho j\omega+2k)g(\eta)=-\gamma(\mu+k)D^4f(\eta)+h^2(\mathrm{i}\gamma\rho\omega-k^2)D^2f(\eta). \tag{18}$$

The boundary conditions on $f(\eta)$ and $g(\eta)$ are now given by

$$f(0)=1-a,\quad f(1)=1,\quad f'(0)=f'(1)=0\quad\text{and}\quad g(0)=g(1)=0. \tag{19}$$

Solving (17), we get

$$f(\eta)=c_1+c_2\eta+c_3\mathrm{e}^{\alpha h\eta}+c_4\mathrm{e}^{-\alpha h\eta}+c_5\mathrm{e}^{\beta h\eta}+c_6\mathrm{e}^{-\beta h\eta} \tag{20}$$

and hence

$$g(\eta)=A_\alpha\left(c_3\mathrm{e}^{\alpha h\eta}+c_4\mathrm{e}^{-\alpha h\eta}\right)+A_\beta\left(c_5\mathrm{e}^{\beta h\eta}+c_6\mathrm{e}^{-\beta h\eta}\right), \tag{21}$$

where

$$\begin{aligned}A_\alpha&=\frac{-\gamma(\mu+k)\alpha^4h^4+(-k^2+\mathrm{i}\rho\omega\gamma)\alpha^2h^2}{k(2k+\mathrm{i}\rho\omega j)},\\ A_\beta&=\frac{-\gamma(\mu+k)\beta^4h^4+(-k^2+\mathrm{i}\rho\omega\gamma)\beta^2h^2}{k(2k+\mathrm{i}\rho\omega j)}.\end{aligned} \tag{22}$$

Using the six boundary conditions in (19), the six constants in (20) can be found. Substituting $f(\eta)$ and $g(\eta)$ in (16) the stream function and the microrotation component can be calculated. Using the expression for ψ and Eq. (7), the velocity components are determined.

2.1. Pressure distribution

From Eqs. (8), (9) and (16), we have

$$\left[\frac{U_0}{a}-\frac{v_2x}{h}\right]\mathrm{i}\rho\omega f'(\eta)=-\frac{\partial p}{\partial x}+\frac{k}{h^2}\left[\frac{U_0}{a}-\frac{v_2x}{h}\right]g'(\eta)+\frac{(\mu+k)}{h^2}\left[\frac{U_0}{a}-\frac{v_2x}{h}\right]f'''(\eta) \tag{23}$$

and

$$\mathrm{i}\rho\omega v_2f(\eta)=-\frac{1}{h}\frac{\partial p}{\partial\eta}+\frac{kv_2}{h^2}g(\eta)+\frac{(\mu+k)}{h^2}v_2f''(\eta). \tag{24}$$

From (23) and (25), we have

$$\frac{k}{h^2}g'(\eta)+\frac{(\mu+k)}{h^2}f'''(\eta)-\mathrm{i}\rho\omega f'(\eta)=c_7. \tag{25}$$

Hence c_7 can be obtained as

$$c_7 = \frac{k}{h^2} g'(0) + \frac{(\mu + k)}{h^2} f'''(0). \tag{26}$$

Hence the pressure drop is given by

$$p(x,\eta) - p(0,0) = c_7 \left[\frac{U_0}{a} - \frac{v_2 x}{h} \right] x + v_2 \frac{(\mu + k)}{h^2} f'(\eta) + v_2 \int_0^\eta \left[\frac{k}{h^2} g(\eta) - \mathrm{i}\rho\omega f(\eta) \right] \mathrm{d}\eta. \tag{27}$$

2.2. Skin friction

From Eq. (4), we get

$$t_{yx} = \frac{k}{h} \left[\frac{U_0}{a} - \frac{v_2 x}{h} \right] \left[\frac{(\mu + k)}{h^2} f''(\eta) + g(\eta) \right]. \tag{28}$$

Hence coefficient of skin friction on the lower and upper plates is given by

$$c_\mathrm{f} = \frac{2t_{yx}}{\rho U_0^2} \quad \text{at } \eta = 0 \text{ and } \eta = 1. \tag{29}$$

3. Numerical calculations and discussion

The variation of skin friction coefficient $c_\mathrm{f}' = paf''(\eta) + g(\eta)$ with $pa = (\mu + k)/k$ for the frequency parameter $(pt = \rho\omega h^2/(\mu + k))$, micropolarity parameter $(pl = k(2\mu + k)h^2/(\gamma(\mu + k)))$ and the microrotation parameter $(pj = (j(\mu + k))/\gamma)$ is shown in Figs. 1–4.

From Eqs. (13) and (14), $\alpha^2 h^2$ and $\beta^2 h^2$ can be obtained as the roots of equation $x^2 + m_1 x + m_2 = 0$, where

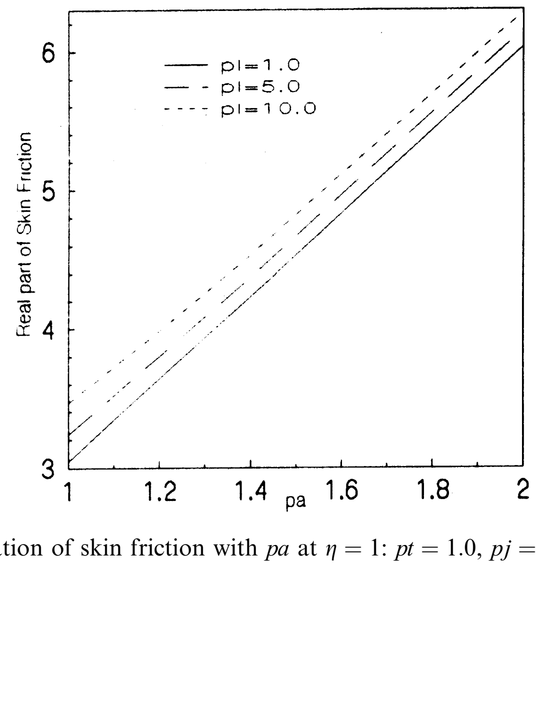

Fig. 1. Variation of skin friction with pa at $\eta = 1$: $pt = 1.0$, $pj = 0.25$, $a = 0.5$.

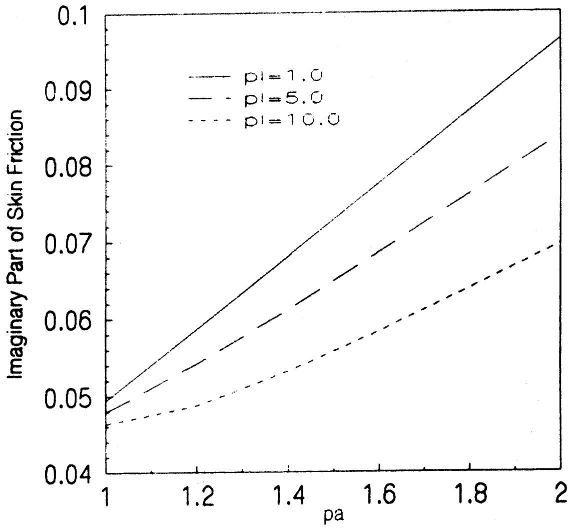

Fig. 2. Variation of skin friction with *pa* at $\eta = 1$: $pt = 1.0$, $pj = 0.25$, $a = 0.5$.

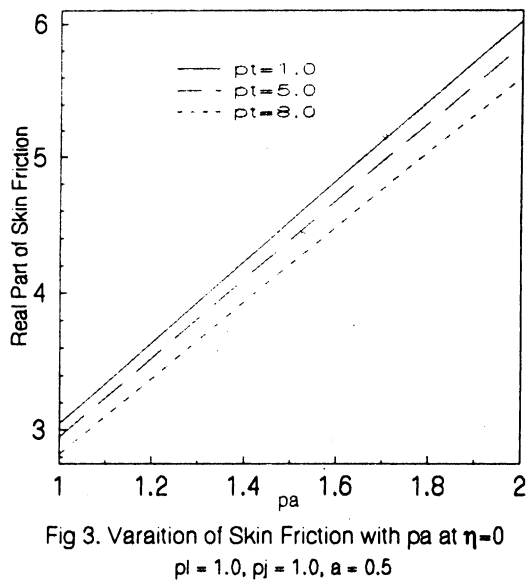

Fig. 3. Variation of skin friction with *pa* at $\eta = 0$: $pt = 1.0$, $pj = 1.0$, $a = 0.5$.

$$m_1 = pl + \mathrm{i}pt(1 + pj) \quad \text{and} \quad m_2 = -pt[pt \cdot pj + 2\mathrm{i}\left[\frac{pl \cdot pa}{(1 - 2pa)}\right]. \tag{30}$$

By giving different values to the parameters pa, pj, pt and pl, the values of m_1 and m_2 are obtained and then αh and βh can be determined. Then using the boundary conditions (19), the constants $c_1, c_2, \ldots, c_6$ and hence $f(\eta)$ and $g(\eta)$ can be calculated.

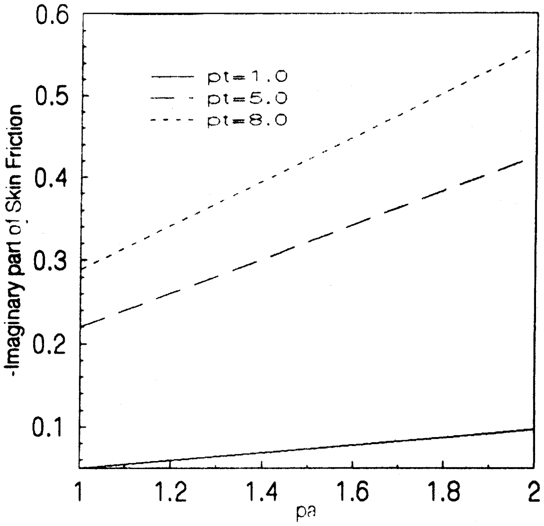

Fig. 4. Variation of skin friction with *pa* at $\eta = 0$: $pt = 1.0$, $pj = 1.0$, $q1 = 0.5$.

It is observed that as the frequency of suction and injection rate increases i.e. as *pt* increases the real part of the skin friction coefficient (c_f') decreases and imaginary part of c_f' increases. An increase in micropolarity parameter *pl* increases the real part of c_f' and decreases the imaginary part of c_f'. An increment in microrotation parameter *pj* causes a reduction of real part of c_f' and increase of imaginary part of c_f'. It is noted that an increase in suction injection ratio $a = v_1/v_2$ increases numerically both the real and imaginary parts of skin friction c_f'. The skin friction at the lower plate is numerically same as the skin friction at the upper plate. This is due to the equal suction and injection frequencies at the upper and lower plates. In the case of steady flow [8], the skin friction at the upper plate is more than that of the lower plate.

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