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Creeping flow of micropolar fluid past a porous sphere

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Abstract

The creeping flow of incompressible micropolar fluid past a porous sphere with permeability k is studied assuming uniform flow far away from the body. The stream function is determined by matching the solutions of Stokes equations for the flow outside the sphere with that of the Brinkman equations inside the porous sphere. The drag force experienced by the sphere is determined. The variation of drag and the streamline pattern for different values of the permeability parameter (η), the coupling number (N) and the micropolar parameter (m) is studied numerically. It is observed that the drag on the porous sphere, when the fluid is micropolar is more than that of the Newtonian fluid case. The flow pattern depends on the permeability k as in the case of Newtonian fluid and on the coupling number (N) but not on the micropolar parameter (m).

Keywords: Micropolar fluid; Porous medium; Sphere; Drag

1. Introduction

The study of uniform flow of fluids around porous bodies involving variety of geometries has been considered by many researchers using various analytical and numerical methods, in view of their applications in industry and engineering. Several studies of the flow past and within porous bodies are limited mainly to low Reynolds numbers. Joseph and Tao [1] examined the flow of an

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incompressible viscous fluid past a porous spherical particle by employing Darcy's law in the porous region and no-slip condition at the surface of the sphere. They found that the drag on the porous sphere is same as that of a rigid sphere with reduced radius. The flow past a porous circular cylinder using Darcy's law for the fluid inside the porous region with Saffman's boundary conditions at the surface of the cylinder was studied by Palaniappan et al. [2]. Recently, the flow past and within permeable spheroid was considered by Vainshtein et al. [3] using Darcy's law and the continuity of the tangential velocity component at the boundary of the spheroid. They have examined the flow past permeable circular disk and elongated rods as limiting cases. To model the flows with high porosity and large shear rates, Brinkman [4] and Debye and Beuche [5] independently suggested a modification to Darcy's model which is known as Brinkman model. Using this Brinkman model for the flow inside the porous sphere, Qin and Kaloni [6] obtained a cartesian tensor solution for the flow of incompressible viscous fluid past a porous sphere. Hidgdon and Kojima [7] have studied Stokes flow past porous particles using Brinkman's equations for the flow inside. They derived some asymptotic results for small and large permeability by using Green's function formulation of the Brinkman's equation. Zlatonovski [8] has considered the axisymmetric Stokes flow of an incompressible viscous fluid past a porous prolate spheroidal particle using the Brinkman model for the flow inside the spheroidal particle. Srinivasacharya [9] has studied the flow of viscous fluid past and within a porous approximate sphere and obtained the cases of flow past a porous sphere and spheroid as special cases.

The model of micropolar fluid introduced by Eringen [10] represents fluids consisting of rigid, randomly oriented bar like elements or dumbell shaped molecules and each volume element has microrotation about its centroid, in addition to its translatory motion in an average sense. Micropolar fluids exhibit some microscopic effects arising from the local structure and micromotion of the fluid elements and they can sustain couple stresses. The Stokes flow of micropolar fluid past a rigid sphere, spheroid and approximate sphere were considered by Lakshmana Rao and Bhujanga Rao [11], Lakshmana Rao and Iyengar [12], and Iyengar and Srinivasacharya [13] respectively. Ramkissoon [14] has studied the flow of micropolar fluid past a Newtonian sphere. The mathematical theory of equations of Micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media are dealt in a recent book by Lukaszewicz [15].

In the present paper, we consider the creeping flow of an incompressible micropolar fluid past a porous sphere. We have used the Brinkman's equation for the flow inside the porous region and Stokes equation for the free flow region in their stream function formulation. As boundary conditions, continuity of the velocity, pressure and tangential stresses across the interface and no spin condition for the microrotation components in both the regions are employed. The stream function and pressure, both for the flow inside and outside the sphere are calculated. The drag force experienced by the sphere is determined. The flow pattern for various values of micropolar parameters and permeability parameter is studied.

2. Formulation of the problem

Let (r, θ, ϕ) denote a spherical polar coordinate system with $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$ as the corresponding unit base vectors and $h_1 = 1$, $h_2 = r$ and $h_3 = r\sin\theta$ as the scale factors. Consider a steady incompressible micropolar fluid flow past a porous sphere of radius a with a uniform velocity U far away from the body along the axis of symmetry $\theta = 0$. We assume that the flow outside the porous sphere to be Stokesian and inside to be governed by Brinkman model.

The equations of motion for the region outside the sphere are the equations governing the steady flow of an incompressible micropolar fluid under Stokesian assumption with the absence of body force and body couple and are given by

$$\nabla \cdot \vec{q}^{(1)} = 0, \tag{1}$$

$$-\nabla p^{(1)} + \kappa \nabla \times \vec{\omega}^{(1)} - (\mu + \kappa)\nabla \times \nabla \times \vec{q}^{(1)} = 0, \tag{2}$$

$$-2\kappa\vec{\omega}^{(1)} + \kappa\nabla \times \vec{q}^{(1)} - \gamma\nabla \times \nabla \times \vec{\omega}^{(1)} + (\alpha + \beta + \gamma)\nabla(\nabla \cdot \vec{\omega}^{(1)}) = 0. \tag{3}$$

For the region inside the sphere the equations of the motion are the equations of motion of the fluid in steady state in the porous medium based on Brinkman's model and are given by

$$\nabla \cdot \vec{q}^{(2)} = 0, \tag{4}$$

$$\frac{\mu}{k}\vec{q}^{(2)} + \nabla p^{(2)} - \kappa\nabla \times \vec{\omega}^{(2)} + (\mu + \kappa)\nabla \times \nabla \times \vec{q}^{(2)} = 0, \tag{5}$$

$$-2\kappa\vec{\omega}^{(2)} + \kappa\nabla \times \vec{q}^{(2)} - \gamma\nabla \times \nabla \times \vec{\omega}^{(2)} + (\alpha + \beta + \gamma)\nabla(\nabla \cdot \vec{\omega}^{(2)}) = 0, \tag{6}$$

where $\vec{q}^{(1)}$ is the velocity vector, $\vec{\omega}^{(1)}$ is the microrotation vector and $p^{(1)}$ is the fluid pressure outside the sphere, and k is the permeability of the porous medium, $\vec{q}^{(2)}$ is the velocity vector of the flow, $\vec{\omega}^{(2)}$ is the microrotation vector and $p^{(2)}$ is the pressure inside the sphere. Further, the material constants μ, κ, α, β, and γ satisfy the following inequalities [10]

$$2\mu + \kappa \geqslant 0, \quad \kappa \geqslant 0, \qquad 3\alpha + \beta + \gamma \geqslant 0 \quad \gamma \geqslant |\beta|. \tag{7}$$

Since the flow generated is axially symmetric, all the flow functions are independent of ϕ. Hence, for this flow we choose the velocity and microrotation vectors as

$$\vec{q}^{(i)} = u^{(i)}(r,\theta)\vec{e}_r + v^{(i)}(r,\theta)\vec{e}_\theta, \quad \vec{\omega}^{(i)} = v_\phi^{(i)}(r,\theta)\vec{e}_\phi, \qquad i = 1,2. \tag{8}$$

Introducing the stream function through

$$u^{(i)} = -\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta}, \qquad v^{(i)} = \frac{1}{r\sin\theta}\frac{\partial\psi^{(i)}}{\partial r} \tag{9}$$

the following nondimensional variables

$$r = a\tilde{r}, \qquad \psi^{(i)} = Ua^2\psi^{(i)}, \qquad p^{(i)} = \frac{\mu U}{a}\tilde{p}^{(i)}, \qquad v_\phi^{(i)} = \frac{U}{a}\tilde{v}_\phi^{(i)} \tag{10}$$

into the Eqs. (1)–(6) and dropping the tildes, we get the equations for the region outside the sphere as

$$-\frac{\partial p^{(1)}}{\partial r} + \left(\frac{N}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin\theta v_\phi^{(1)}\right) - \left(\frac{1}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(E^2\psi^{(1)}) = 0, \tag{11}$$

$$-\frac{1}{r}\frac{\partial p^{(1)}}{\partial\theta} - \left(\frac{N}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(r\sin\theta v_\phi^{(1)}\right) + \left(\frac{1}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}(E^2\psi^{(1)}) = 0, \tag{12}$$

$$-2v_\phi^{(1)} + \frac{1}{r\sin\theta}\frac{\partial}{\partial r}(E^2\psi^{(1)}) + \frac{2-N}{m^2}\left[\nabla^2 - \frac{1}{r^2\sin\theta}\right]v_\phi^{(1)} = 0 \tag{13}$$

and the equations for the region inside the sphere as

$$-\frac{\partial p^{(2)}}{\partial r} + \left(\frac{N}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin\theta v_\phi^{(2)}\right) - \left(\frac{1}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(E^2\psi^{(2)}) + \eta^2\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(2)}}{\partial\theta} = 0, \tag{14}$$

$$-\frac{1}{r}\frac{\partial p^{(2)}}{\partial\theta} - \left(\frac{N}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(r\sin\theta v_\phi^{(2)}\right) + \left(\frac{1}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}(E^2\psi^{(2)}) - \eta^2\frac{1}{r\sin\theta}\frac{\partial\psi^{(2)}}{\partial r} = 0, \tag{15}$$

$$-2v_\phi^{(2)} + \frac{1}{r\sin\theta}\frac{\partial}{\partial r}(E^2\psi^{(2)}) + \frac{2-N}{m^2}\left[\nabla^2 - \frac{1}{r^2\sin\theta}\right]v_\phi^{(2)} = 0, \tag{16}$$

where

$$E^2 = \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}\right] \tag{17}$$

is the Stokes stream function operator, $\eta^2 = a^2/k$, $N = \kappa/(\mu+\kappa)$ is the coupling number ($0 \leqslant N < 1$) and

$$m^2 = \frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)} a^2$$

is the micropolar parameter. As $\kappa \to 0$ and $\gamma \to 0$ (i.e. $N \to 0$ and $m \to \infty$), the Eqs. (11)–(13) reduce to Navier–Stokes equations and the Eqs. (14)–(16) reduce to Brinkman equations.

The corresponding boundary conditions (dimensional) [6,7]

1. Continuity of velocity components i.e., $u^{(1)}(r,\theta) = u^{(2)}(r,\theta)$ and $v^{(1)}(r,\theta) = v^{(2)}(r,\theta)$ on the boundary $r = a$.
2. Continuity of pressure i.e., $p^{(1)}(r,\theta) = p^{(2)}(r,\theta)$ on the boundary $r = a$.
3. Continuity of tangential stresses components i.e., $\tau_{r\theta}^{(1)}(r,\theta) = \tau_{r\theta}^{(2)}(r,\theta)$ on the boundary $r = a$.
4. No-Spin condition on the microrotation i.e., $v_\phi^{(1)} = 0$ and $v_\phi^{(2)} = 0$ on the boundary $r = a$.

Additionally, we have the regularity conditions at infinity and the condition that velocity and pressure must be nonsingular everywhere in the flow field.

The equivalent nondimensional conditions on the boundary $r = 1$ in terms of the stream functions are

$$\begin{aligned} &\psi^{(1)}(r,\theta) = \psi^{(2)}(r,\theta), \qquad \psi_r^{(1)}(r,\theta) = \psi_r^{(2)}(r,\theta), \\ &p^{(1)}(r,\theta) = p^{(2)}(r,\theta), \qquad \frac{\partial^2 \psi^{(1)}(r,\theta)}{\partial r^2} = \frac{\partial^2 \psi^{(2)}(r,\theta)}{\partial r^2}, \\ &v_\phi^{(1)}(r,\theta) = 0, \qquad v_\phi^{(2)}(r,\theta) = 0 \end{aligned} \tag{18}$$

together with $\psi^{(1)} \to (1/2) r^2 \sin^2\theta$ as $r \to \infty$ and $\psi^{(2)}$ is finite at $r = 0$.

3. Solution of the problem

3.1. Solution for the region outside the sphere

Eliminating pressure from (11) and (12), we get

$$E^4 \psi^{(1)} - N E^2 \left(r \sin\theta\, v_\phi^{(1)} \right) = 0. \tag{19}$$

Substituting it in (13)

$$v_\phi^{(1)} = \frac{1}{2r\sin\theta} \left[E^2 \psi^{(1)} + \frac{2-N}{N m^2} E^4 \psi^{(1)} \right]. \tag{20}$$

From (19) and (20)

$$E^4\left(E^2 - m^2\right)\psi^{(1)} = 0. \tag{21}$$

Using the separation of variables, the general solution of (21) is

$$\begin{aligned}\psi^{(1)} = \sum_{n=0}^{\infty}\Big\{ &\big[A_n^{(1)} r^n + B_n^{(1)} r^{-n+1} + C_n^{(1)} r^{n+2} + D_n^{(1)} r^{-n+3} \\ &+ E_n^{(1)}\sqrt{r}K_{n-1/2}(mr) + F_n^{(1)}\sqrt{r}I_{n-1/2}(mr)\big]\vartheta_n(\zeta) \\ &+ \Big[\widetilde{A}_n^{(1)} r^n + \widetilde{B}_n^{(1)} r^{-n+1} + \widetilde{C}_n^{(1)} r^{n+2} + \widetilde{D}_n^{(1)} r^{-n+3} \\ &+ \widetilde{E}_n^{(1)}\sqrt{r}K_{n-1/2}(mr) + \widetilde{F}_n^{(1)}\sqrt{r}I_{n-1/2}(mr)\Big] H_n(\zeta)\Big\},\end{aligned} \tag{22}$$

where $\zeta = \cos\theta$, $K_{n-1/2}(mr)$ and $I_{n-1/2}(mr)$ are modified Bessel functions of the first and second kind and $\vartheta_n(\zeta)$ and $H_n(\zeta)$ are Gegenbauer functions of the first and second kinds.

If we retain the terms which are multiplied by $\vartheta_0(\zeta)$ and $\vartheta_1(\zeta)$ in (22), then the velocities will be irregular at the axis. Also, $H_n(\zeta)$ are irregular on the axis for all n. Hence we ignore the terms which are multiplied by $\vartheta_0(\zeta)$, $\vartheta_1(\zeta)$ and $H_n(\zeta)$ for all n. Using the regularity condition at infinity, we notice that the terms involving $I_{n-1/2}(mr)$ are to be dropped and the terms r^n and r^{n+2} must also be absent with the exception of the term involving r^2. Hence the solution for the region outside the sphere contain only the terms of order $n = 2$ of the general solution (22). Therefore, the stream function is given by

$$\psi^{(1)} = \left[r^2 + B_2^{(1)} r^{-1} + D_2^{(1)} r + E_2^{(1)}\sqrt{r}K_{3/2}(mr)\right]\vartheta_2(\zeta). \tag{23}$$

Substituting this in (20), we get the microroration component as

$$v_\phi^{(1)} = \frac{1}{r\sin\theta}\left[-D_2^{(1)} r^{-1} + \frac{m^2}{N}E_2^{(1)}\sqrt{r}K_{3/2}(mr)\right]\vartheta_2(\zeta). \tag{24}$$

Using the expressions for velocity and microrotation in (11) and (12), we get the expression for the pressure as

$$p^{(1)} = -\frac{2-N}{2(1-N)} D_2^{(1)} r^{-2} P_1(\zeta), \tag{25}$$

where $P_1(\zeta)$ is the Legendre polynomial.

3.2. Solution for the region inside the sphere

Eliminating pressure from (14) and (15), and substituting (16) in the resulting equation, we get the microrotation component in terms of stream function as

$$v_{\phi}^{(2)} = \frac{1}{2r\sin\theta}\left[E^2\psi^{(2)} + \frac{2-N}{Nm^2}\left(E^4\psi^{(2)} - \eta^2(1-N)E^2\psi^{(2)}\right)\right]. \tag{26}$$

From (16) and (26)

$$E^2\left(E^2-\alpha^2\right)\left(E^2-\beta^2\right)\psi^{(2)} = 0, \tag{27}$$

where

$$\alpha^2+\beta^2 = \eta^2(1-N)+m^2 \quad \text{and} \quad \alpha^2\beta^2 = \frac{2(1-N)}{2-N}\eta^2 m^2.$$

The general solution of (27) is

$$\begin{aligned}\psi^{(2)} = \sum_{n=0}^{\infty}\Big\{&\big[A_n^{(2)}r^n + B_n^{(2)}r^{-n+1} + C_n^{(2)}\sqrt{r}K_{n-1/2}(\alpha r)\\ &+ D_n^{(2)}\sqrt{r}I_{n-1/2}(\alpha r) + E_n^{(2)}\sqrt{r}K_{n-1/2}(\beta r) + F_n^{(2)}\sqrt{r}I_{n-1/2}(\beta r)\big]\vartheta_n(\zeta)\\ &+\Big[\tilde{A}_n^{(2)}r^n + \tilde{B}_n^{(2)}r^{-n+1} + \widetilde{C}_n^{(2)}\sqrt{r}K_{n-1/2}(\alpha r) + \widetilde{D}_n^{(2)}\sqrt{r}I_{n-1/2}(\alpha r)\\ &+ \widetilde{E}_n^{(2)}\sqrt{r}K_{n-1/2}(\beta r) + \widetilde{F}_n^{(2)}\sqrt{r}I_{n-1/2}(\beta r)\Big]H_n(\zeta)\Big\}.\end{aligned} \tag{28}$$

Since the velocities are nonsingular everywhere in the flow region, we neglect the terms which are multiplied by $\vartheta_0(\zeta)$, $\vartheta_1(\zeta)$ and $H_n(\zeta)$ for all n, as in the case of solution for outside the sphere. Further, the modified Bessel functions $K_{n-1/2}(\alpha r)$ and $K_{n-1/2}(\beta r)$, for all n and the terms involving r^{-n+1}, for $n \geqslant 2$ are irregular at $r=0$, hence we take $C_n^{(2)}=0$ and $E_n^{(2)}=0$, for all n and $B_n^{(2)}=0$ for $n \geqslant 2$. Therefore, the general solution (28) reduce only to the terms multiplied by $\vartheta_2(\zeta)$ and is given by

$$\psi^{(2)} = \left[A_2^{(2)}r^2 + D_2^{(2)}\sqrt{r}I_{3/2}(\alpha r) + F_2^{(2)}\sqrt{r}I_{3/2}(\beta r)\right]\vartheta_2(\zeta). \tag{29}$$

Hence, the microrotation component and pressure distribution inside the sphere are given by

$$v_{\phi}^{(2)} = \frac{1}{r\sin\theta}\left[D_2^{(2)}A_{\alpha}\sqrt{r}I_{3/2}(\alpha r) + F_2^{(2)}A_{\beta}\sqrt{r}I_{3/2}(\beta r)\right]\vartheta_2(\zeta) \tag{30}$$

and

$$p^{(1)} = \frac{2-N}{(1-N)}\frac{\alpha^2\beta^2}{2m^2}A_2^{(2)}rP_1(\zeta), \tag{31}$$

where

$$A_\alpha = \frac{[Nm^2 - (2-N)(1-N)\eta^2]\alpha^2 + (2-N)\alpha^4}{2Nm^2},$$
$$A_\beta = \frac{[Nm^2 - (2-N)(1-N)\eta^2]\beta^2 + (2-N)\beta^4}{2Nm^2}. \tag{32}$$

4. Results and discussion

The drag force acting on the porous sphere can be obtained by integrating the stresses on the surface of the sphere and is found to be

$$-4\pi(2\mu+\kappa)UD_2^{(1)}. \tag{33}$$

Using the boundary conditions (18), we get linear system of equations in $B_2^{(1)}$, $D_2^{(1)}$, $E_2^{(1)}$, $A_2^{(2)}$, $D_2^{(2)}$ and $F_2^{(2)}$. To solve this system of equations, the computer code was generated in MATHEMATICA and the expression for $D_2^{(1)}$ was obtained as

$$\begin{aligned} D_2^{(1)} = {} & -3m\alpha^2\beta^2K_{3/2}(m)I_{3/2}(\alpha)I_{3/2}(\beta)(\alpha^2A_\beta - \beta^2A_\alpha) \\ & /\{(\alpha^2A_\beta - \beta^2A_\alpha)\left[m(3m^2+2\alpha^2\beta^2)K_{3/2}(m) - \alpha^2\beta^2NK_{1/2}(m)\right]I_{3/2}(\alpha)I_{3/2}(\beta) \\ & + m\alpha^2\beta^2(N-2)K_{3/2}(m)\left[\beta A_\alpha I_{3/2}(\alpha)I_{1/2}(\beta) - \alpha A_\beta I_{1/2}(\alpha)I_{3/2}(\beta)\right]\}. \end{aligned} \tag{34}$$

Hence, the nondimensional drag $D_N = D/(4\pi\mu U)$is given by

$$\begin{aligned} D_N = {} & -\left[3(m+1)\alpha^2\beta^2(\alpha\cosh(\alpha) - \sinh(\alpha))(\beta\cosh(\beta) \right. \\ & \left. - \sinh(\beta))(\alpha^2A_\beta - \beta^2A_\alpha)\right] / \left[\alpha^2\{(\alpha(3(m+1)m^2 \right. \\ & + (2m-n+2)\alpha^2\beta^2)\cosh(\alpha) - m(n\alpha^2\beta^2 + 3m(m+1))\sinh(\alpha))(\beta\cosh(\beta) \\ & - \sinh(\beta))\}A_\beta - \beta^2\{(\alpha\cosh(\alpha) - \sinh(\alpha))(\beta(3(m+1)m^2 \\ & \left. + (2m-n+2)\alpha^2\beta^2)\cosh(\beta) - m(n\alpha^2\beta^2 + 3m(m+1))\sinh(\beta))\}A_\alpha\right]. \end{aligned} \tag{35}$$

As the micropolar parameter $m \to \infty$ and $N \to 0$, (then $a^2 \to \eta^2$, $\beta^2 \to \infty$, $A_\alpha \to \eta^2/2$ and $A_\beta \to \infty$) this drag simplifies to

$$\frac{\eta^2(\sinh\eta - \eta\cosh\eta)}{\eta(3+2\eta^2)\cosh\eta - 3\sinh\eta} \tag{36}$$

which agrees with the drag on the porous sphere derived by Qin and Kaloni [6], when the fluid is Newtonian.

The variation of drag D_N with η^2 for $m = 20$ and for various values of N is shown in Fig. 1. From Fig. 1 it can be observed that the drag is decreasing as the permeability parameter (η^2) is increasing. Also, there is decrease in the drag

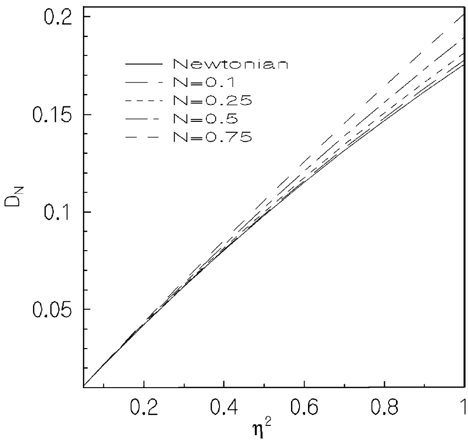

Fig. 1. Variation of drag coefficient with η^2 for $m = 20$.

as the coupling number N is decreasing. It is interesting to note that the drag on the sphere, when the fluid is micropolar, is more than that of the Newtonian fluid case. Fig. 2 shows the variation of drag D_N with η^2 for $N = 0.5$ and for various values of m. It can be observed from this figure that the drag is decreasing as the permeability parameter (η^2) is increasing. Also, there is decrease in the drag as the coupling number m is decreasing.

The stream line pattern has been plotted for different values of the permeability parameter (η), the coupling number (N) and the micropolar parameter (m). Fig. 3 illustrates the streamline pattern for different values of η with

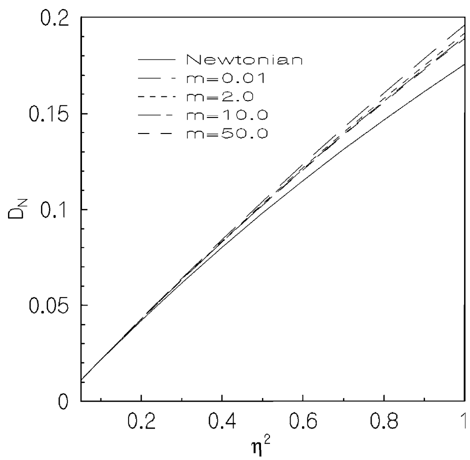

Fig. 2. Variation of drag coefficient with η^2 for $N = 0.5$.

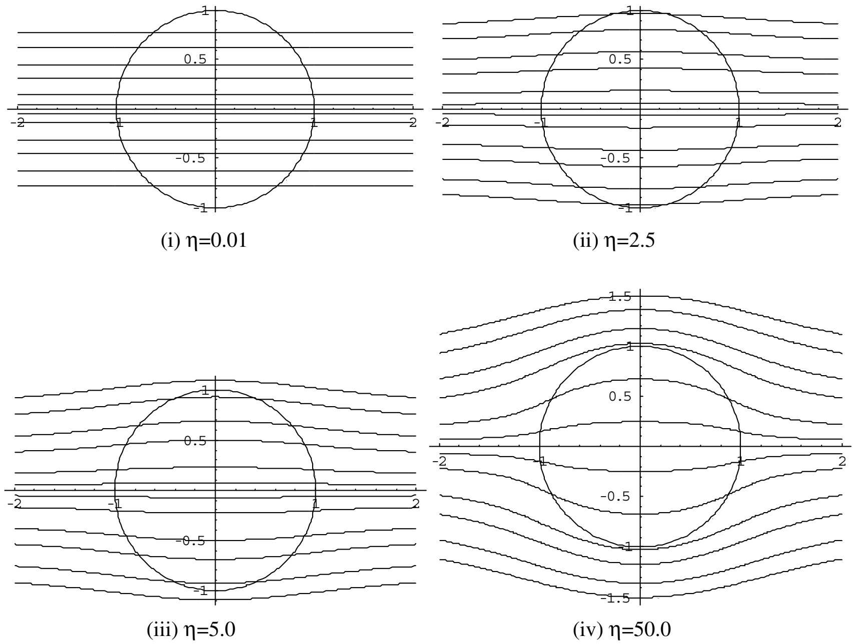

Fig. 3. Stream lines for $m = 5.0$ and $N = 0.25$.

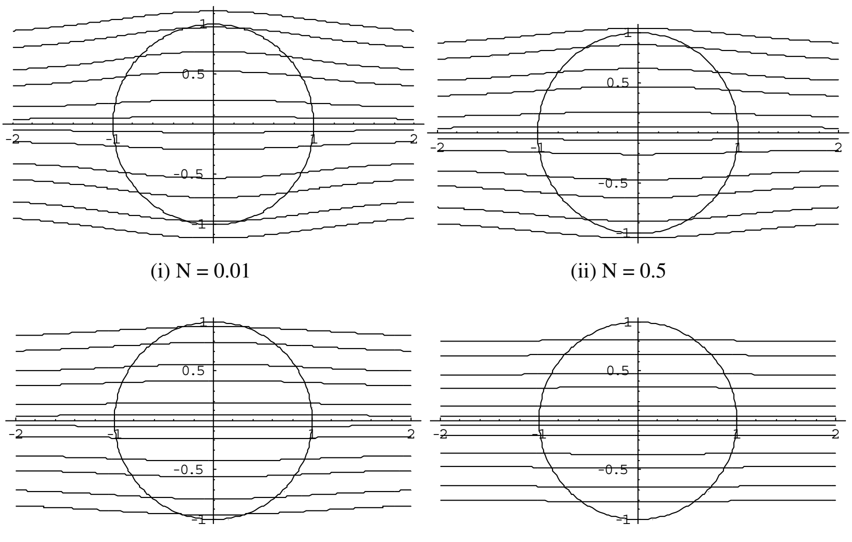

Fig. 4. Stream lines for $m = 10$ and $\eta = 5.0$.

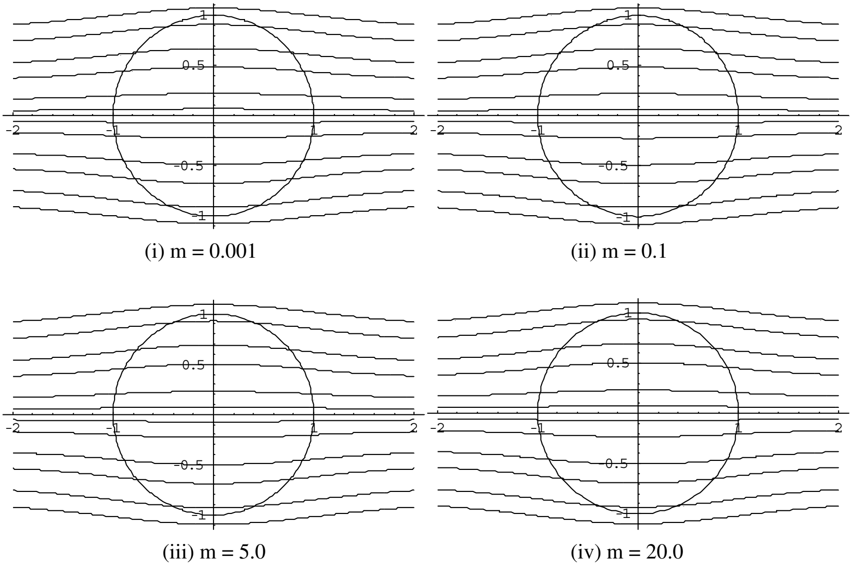

Fig. 5. Stream lines for $N = 0.25$ and $\eta = 5.0$.

$N = 0.25$ and $m = 5.0$. It is observed that increase of permeability k (i.e. decrease of permeability parameter η) flattens the streamlines, as is to be expected. For large permeability k, the flow is almost uniform. However, for smaller permeability (i.e. large η), the flow is greatly perturbated. The effect of coupling number N on the flow for fixed values of $m = 10$ and $\eta = 5.0$ is shown through the stream line pattern in Fig. 4. It can be seen from Fig. 4, that the flow becomes almost uniform as the coupling number N is increasing. The variation of stream line pattern with the micropolar parameter m for fixed values of $N = 0.75$ and $\eta = 0.5$ is shown in Fig. 5. It is interesting to note from Fig. 5 that the flow pattern is almost independent of m.

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