Acta Mechanica 168, 145–156 (2004)
DOI 10.1007/s00707-004-0084-3

Acta Mechanica
Printed in Austria

Combined radiation and mixed convection from a vertical wall with suction/injection in a non–Darcy porous medium

P. V. S. N. Murthy and **S. Mukherjee,** Kharagpur, and **D. Srinivasacharya** and
P. V. S. S. S. R. Krishna, Warangal, India

Received June 30, 2003; revised January 8, 2004

Summary. Mixed convection flow of an absorbing fluid up a uniform non–Darcy porous medium supported by a semi-infinite ideally transparent vertical flat plate due to solar radiation is considered. The external flow field is assumed to be uniform, the effect of the radiation parameter in the boundary layer adjacent to the vertical flat plate with fluid suction/injection through it is analyzed in both aiding and opposing flow situations. It is observed that the similarity solution is possible only when the fluid suction/injection velocity profile varies as $x^{-1/2}$. The velocity and temperature profiles in the boundary layer and the heat transfer coefficient are presented for selected values of the parameters. It is observed that the Nusselt number increases with the increase in the radiation parameter and also when the value of the surface mass flux parameter moves from the injection to the suction region.

Nomenclature

A	constant
C	empirical constant
C_P	specific heat at constant pressure
C_T	temperature ratio
g	acceleration due to gravity
K	permeability
k	thermal conductivity
k^*	mean absorption coefficient
Pe_x	Peclet number (defined in the text)
Pe_d	pore diameter dependent Peclet number (defined in the text)
q_y^r	radiation heat flux in the y-direction
Re_d	pore diameter dependent Reynolds number (defined in the text)
Ra_d	pore diameter dependent Rayleigh number (defined in the text)
R	conduction radiation parameter (defined in the text)
T	temperature
T_w	wall temperature
u, v	Darcian velocity components in x- and y-directions
U_∞	uniform free stream velocity

Greek symbols

ν	kinematic viscosity
α	thermal diffusivity constant
ρ	density
ψ	stream function
f	nondimensional stream function
η	similarity variable
σ	Stefan–Boltzmann constant
β	thermal expansion coefficient
θ	nondimensional temperature distribution

1 Introduction

The study of convection heat transfer from uniform surfaces embedded in a saturated porous medium has attracted many investigators over the last three decades due to their wide range of applications in Geophysics and Thermal Sciences. Cheng and Minkowycz [1] presented the mathematical analysis for the natural convection flow about a heated surface embedded in a fluid saturated porous medium. Cheng [2] and Merkin [3] studied the effect of fluid injection or withdrawal on a free convection boundary layer adjacent to a heated vertical wall. These studies are used in analyzing the convective movement of water discharged from a geothermal power plant into ground water of a different temperature and in the natural recharging of an aquifer by groundwater of a different temperature. It is observed that the fluid suction decreased the thermal boundary-layer thickness and enhanced the heat transfer coefficient in the medium. Plumb and Huenefeld [4] studied the fundamental problem of natural convection from a heated vertical wall in a non–Darcy saturated porous medium. Later Bejan and Poulikakos [5] studied the same problem by dividing the flow into intermediate and non–Darcy regions using the scale analysis argument.

The inertial and viscous effects on mixed convection about a vertical surface have been studied by Ranganathan and Viskanta [6]. Their results show that the effects of inertia and boundary friction are quite significant and cannot be ignored. Using the Forchheimer flow model, Nakayama and Pop [7], and Lai and Kulacki [8] analyzed the inertia effects on mixed convection along a vertical wall. A review of both natural and mixed convection boundary-layer flows in Darcian and non–Darcian fluid saturated porous media is given in Nield and Bejan [9].

Depending on the surface properties and solid geometry, radiative transport is often comparable with that of the convective heat transfer in many practical applications. It is therefore of great significance and interest to the researchers to investigate combined convective and radiative flow and heat transfer aspects. Many works have been reported on convection from a heated plate in clear fluids by considering convection flow due to solar radiation over a non absorbing plate with and without heat losses in view of their applications in solar collectors with direct solar collection using an absorbing fluid. Sparrow and Cess [10] give an account of the literature in this direction.

Radiation heat transfer in porous media has also been studied by many researchers. Whitaker [11] discussed radiative heat transfer in porous media. In Tong and Tien [12] two models were proposed, namely the two flux model and the linear anisotropic scattering model to predict the radiant heat flux in light weight fibrous insulation, and they discussed the conditions for the validity of these models. Raptis [13] analysed radiation and free convection flow through a porous medium using Rosseland approximation for the radiative heat flux. Chamkha [14] studied solar radiation assisted free convection in the boundary layer adjacent to a vertical flat plate in a uniform porous medium with a more general Darcy–Forchheimer–Brinkman flow model using a convection type boundary condition. A local similarity solution has been obtained, and boundary friction and heat transfer coefficients were discussed with various governing parameters. Chamkha et al. [15] extended the results in [14] to a variable porosity medium and observed that the boundary friction and Nusselt number are decreased as the porous medium parameter value is increased. In [14], [15] an exponential type of approximation has been used for incident solar radiation flux.

The thermal radiation effect on mixed convection from horizontal surfaces in saturated porous media was investigated by Bakier and Gorla [16]. Mansour [17] studied the effect of variable viscosity on the combined radiation and forced convection on the flow over a flat plate submerged in a porous medium of variable viscosity. It is observed that viscosity variation has a significant effect on the heat transfer coefficient. Very recently Bakier [18] studied the combined radiation and mixed convection from a vertical surface in a Darcian fluid saturated porous medium and reported that the radiation parameter has a considerable influence on the augmentation of the surface heat transfer rate.

In the present work, we aimed at analyzing the combined mixed convection-radiation heat transfer in the boundary layer arising from a vertical wall embedded in a non–Darcy (Forchheimer flow model) porous medium when buoyancy is aiding and opposing the uniform free stream. The radiation heat flux is approximated with the Rosseland approximation. The vertical wall is permeable so that it permits fluid suction or injection. It is observed that the similarity solution for the present problem is possible only when the suction/injection velocity is proportional to $x^{-1/2}$ on the isothermal vertical flat plate. The problem description, mathematical analysis and the results are presented in the following sections.

2 Governing equations

Consider the steady, laminar, two-dimensional, mixed convection due to a solar radiation boundary-layer flow over a semi-infinite vertical flat plate embedded in a saturated porous medium as shown in Fig. 1. The fluid is considered to be a gray, absorbing-emitting radiation but non-scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The coordinate system is chosen such that x measures the distance along the plate and y measures the distance normal to it. Far away from the plate surface, both the surroundings and the fluid are maintained at a constant temperature T_∞. The porous medium is assumed to be transparent and in thermal equilibrium with the fluid. Due to heating of the absorbing fluid and the plate by solar radiation, heat is transferred from the isothermal plate to the surroundings. Upon treating the fluid-saturated non–Darcian porous medium as a continuum and assuming that the Boussinesq approximation is valid, the boundary-layer form of the governing equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

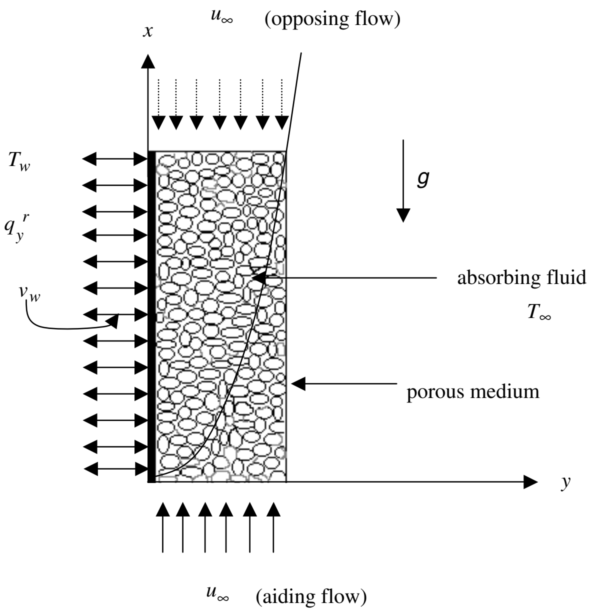

Fig. 1. Vertical flat plate in a fluid saturated porous medium

$$\frac{\partial u}{\partial y}+\frac{C\sqrt{K}}{v}\frac{\partial u^2}{\partial y}=\pm\frac{Kg\beta}{v}\frac{\partial T}{\partial y}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}-\frac{1}{\rho C_p}q_y^r\right), \tag{3}$$

and the boundary conditions may be written as

$$y=0; \quad v=Ax^{-1/2}, \quad T=T_w(const), \tag{4.1}$$

$$y\to\infty; \quad u=u_\infty(const), \quad T\to T_\infty. \tag{4.2}$$

Here u and v are the Darcian velocity components along the x- and y-directions, A is a constant, u_∞ is the uniform free stream velocity, T is the temperature, K is the permeability constant, C is an empirical constant, β is the coefficient of thermal expansion, v is the kinematic viscosity, ρ is the density, C_p is the specific heat at constant pressure, g is the acceleration due to gravity, and α is the thermal diffusivity constant. The radiative heat flux term q_y^r is written using the Rosseland approximation (Sparrow and Cess [10]; Raptis [13]) as

$$q_y^r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ and k^* are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. Defining the stream function ψ such that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, Eqs. (1)–(3) become

$$\frac{\partial^2\psi}{\partial y^2}+\frac{C\sqrt{K}}{v}\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)^2=\pm\left(\frac{Kg\beta}{v}\right)\frac{\partial T}{\partial y}, \tag{6}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}-\frac{1}{\rho C_p}q_y^r\right). \tag{7}$$

Using the following similarity transformation:

$$\eta = \frac{y}{x}\mathrm{Pe}_{\mathrm{x}}^{1/2}, \quad f(\eta) = \frac{\psi}{\alpha Pe_x^{1/2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{8}$$

the governing partial differential equations (6) and (7) transform to the ordinary differential equations as

$$f'' + 2\mathrm{Re}_d f'f'' = \pm\frac{Ra_d}{Pe_d}\theta', \tag{9}$$

$$\theta'' + \frac{1}{2}f\theta' + \frac{4}{3}R[(C_T + \theta)^3\theta']' = 0. \tag{10}$$

In the above equations, the nondimensional parameters are defined as: $\mathrm{Re}_d = u_\infty d/v$, the pore diameter dependent Reynolds number, $Ra_d = Kg\beta\theta_w d/\alpha v$, the pore diameter dependent Rayleigh number, $Pe_x = u_\infty x/\alpha$, the Peclet number, $Pe_d = u_\infty d/\alpha$, the pore diameter dependent Peclet number, $R = 4\sigma\theta_w^3/kk^*$, the conduction radiation parameter, and $C_T = T_\infty/T_w - T_\infty$, the temperature ratio. C_T assumes very small values by its definition as $(T_w - T_{\infty)}$ is very large compared to T_∞. In the present study, it is assigned the value 0.1. Finally, Ra_d/Pe_d is the mixed convection flow governing parameter, it is positive when the buoyancy is aiding the external flow (aiding flow) and is negative when the buoyancy is opposing the external flow. The suction/injection parameter in its nondimensional form is written as

$$f_w = -\frac{2x}{A\alpha}v_w(x)Pe_x^{-1/2}$$

and will be independent of x. Now the boundary conditions (4) will become

$$\eta = 0; \quad f = f_w, \quad \theta = 1,$$

$$\eta \to \infty; \quad f' = 1, \quad \theta \to 0. \tag{11}$$

3 Results and discussions

The resulting boundary value problem given by Eqs. (9) and (10) along with the boundary conditions (11) is integrated using the 4th-order Runge-Kutta method by giving appropriate initial guess values for $f'(0)$ and $\theta'(0)$, and the solution thus obtained is matched with the given values at $f'(\infty)$ and $\theta(\infty)$ using the Newton-Raphson method, making use of a NAG (D02HAF) routine. The results obtained are accurate up to the 4th decimal place. Extensive calculations have been performed to obtain the flow and temperature field with the parametric values $C_T = 0.1$, $0 \le \mathrm{Re}_d \le 5$, $-1.0 \le \frac{Ra_d}{Pe_d} \le 50$ and $0 \le R \le 1.0$. When the value of Re_d is close to zero, the present problem reduces to combined convection-radiation heat transfer from the vertical wall in a near–Darcy fluid saturated porous medium. The value of $f_w = -1$ represents the injection of fluid into the boundary layer through the vertical wall, and $f_w = 1$ represents the fluid suction from the thermal boundary layer.

Results in aiding flow: Typical profiles for the nondimensional velocity distribution $f'(\eta)$ inside the boundary layer in the near–Darcy ($\mathrm{Re}_d = 0.1$) and non–Darcy ($\mathrm{Re}_d = 1.0$) regions are plotted for the impermeable wall case ($f_w = 0$) in Figs. 2 and 3, respectively, for two values of the flow governing parameter with varying radiation parameter. In both regions, as expected, the velocity field is increased as the value of the flow governing parameter is increased.

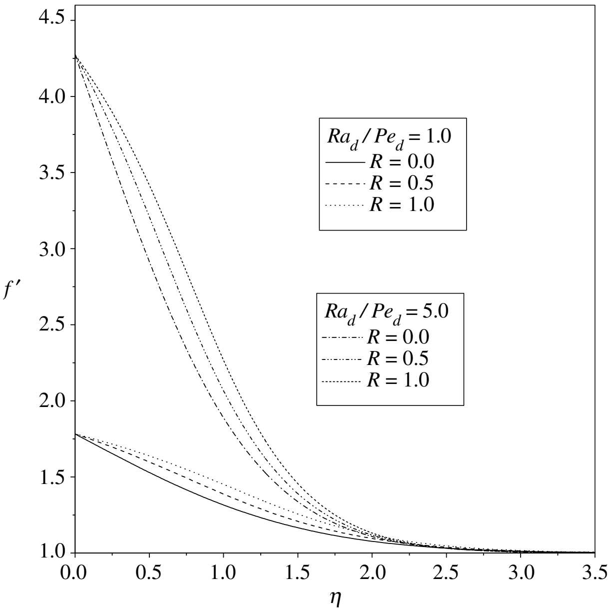

Fig. 2. $f'(\eta)$ vs. η for $f_W = 0$ and $\mathrm{Re}_d = 0.1$ (aiding flow)

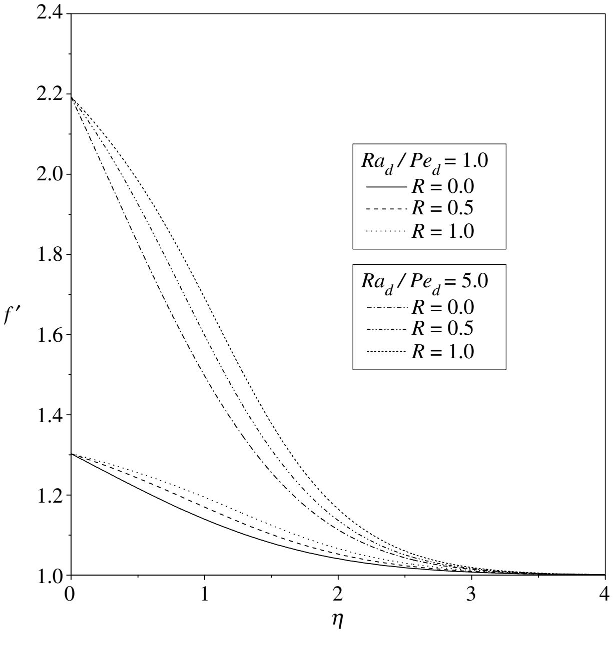

Fig. 3. $f'(\eta)$ vs. η for $f_W = 0$ and $\mathrm{Re}_d = 1.0$ (aiding flow)

Also increasing the value of the radiation parameter increased the fluid velocity further. Thus the effect of thermal radiation is to increase the convective moment in the boundary layer. It is also observed that in the non–Darcy region the fluid velocity is relatively low compared to that in the Darcy flow region as the inertia effect is to reduce the convective moment of the fluid particles.

The temperature distributions $\theta(\eta)$ in the near–Darcy and non–Darcy regions for the impermeable wall case are presented in Figs. 4 and 5, respectively. It is observed that the thermal boundary-layer thickness increased with an increase in the value of both the mixed

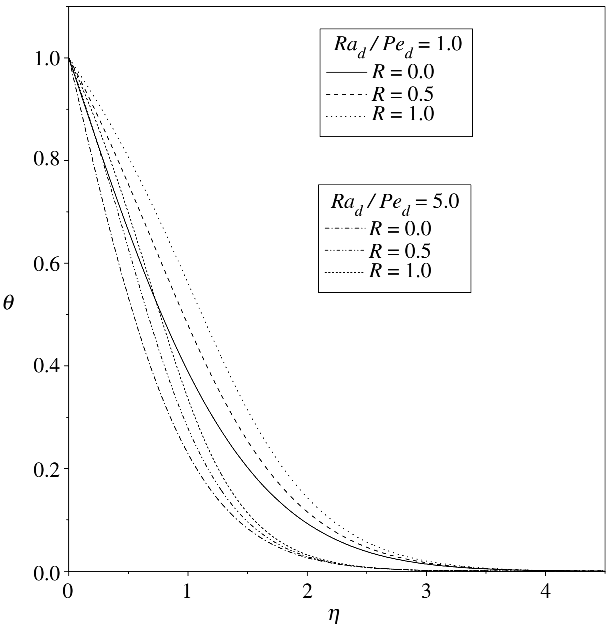

Fig. 4. $\theta(\eta)$ vs. η for $f_W = 0$ and $\text{Re}_d = 0.1$ (aiding flow)

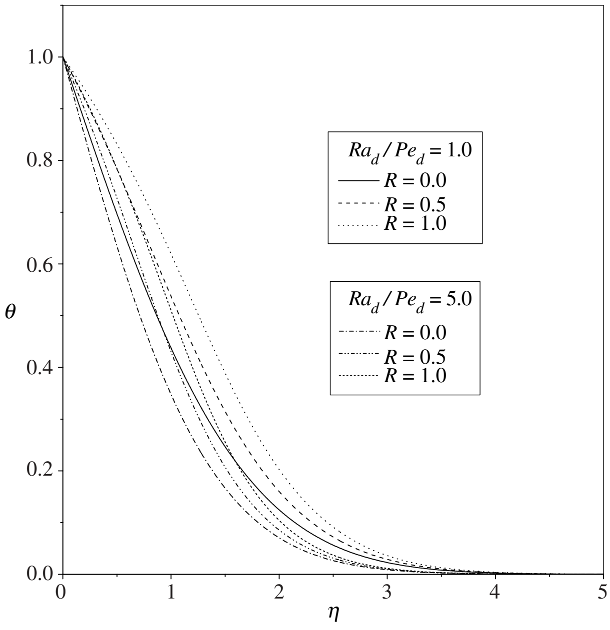

Fig. 5. $\theta(\eta)$ vs. η for $f_W = 0$ and $\text{Re}_d = 1.0$ (aiding flow)

convection parameter and radiation parameter. The temperature of the fluid in the medium increased due to the presence of thermal radiation.

The heat transfer coefficient in terms of the Nusselt number is given by

$$\frac{Nu_x}{Pe_x^{1/2}} = -\theta'(0)[1 + \frac{4R}{3}(C_T + \theta(0))^3] \tag{12}$$

and is shown in Fig. 6 against $\frac{Ra_d}{Pe_d}$ in both near–Darcy and non–Darcy flow regions for the impermeable vertical wall case. The heat transfer coefficient increased with the increasing values of the flow governing parameter. Also, the effect of radiation is to enhance the heat transfer

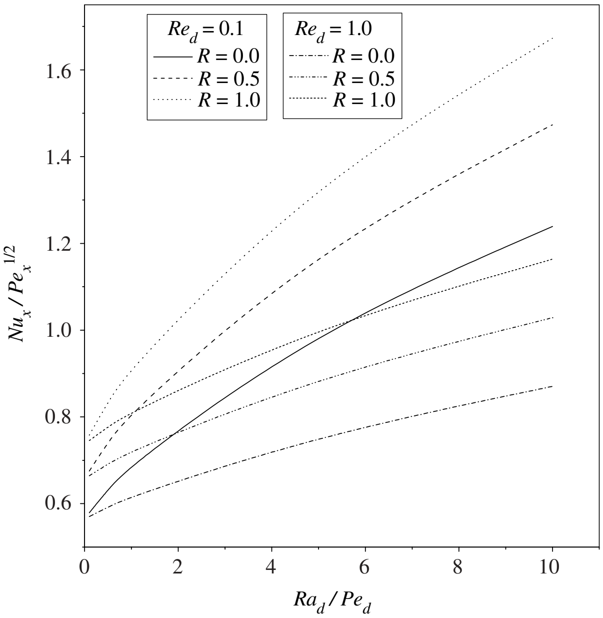

Fig. 6. Nusselt number vs. $\frac{Ra_d}{Pe_d}$ for $f_W = 0$ (aiding flow)

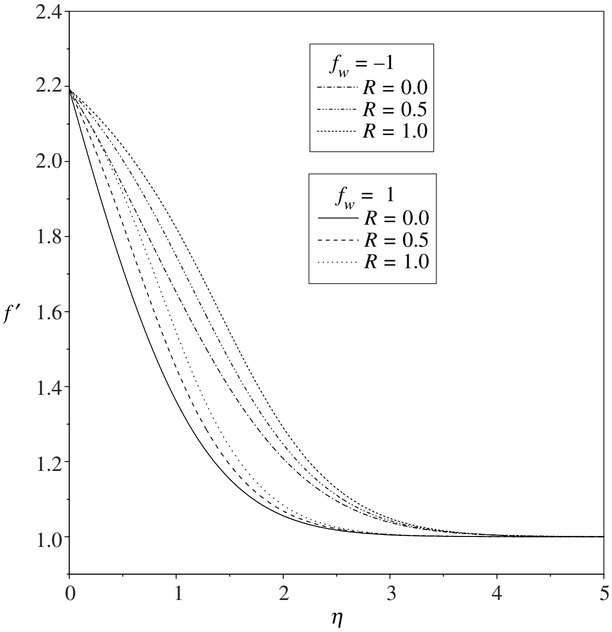

Fig. 7. $f'(\eta)$ vs. η for $\mathrm{Re}_d = 1.0$ and $\frac{Ra_d}{Pe_d} = 5.0$ (aiding flow)

coefficient in the medium. It is evident from this figure that the effect of radiation is more predominant in the near–Darcy region than in the non–Darcy region.

The effects of fluid suction and injection on the fluid flow, temperature distribution and the heat transfer coefficient have been analyzed and some of these results are presented in Figs. 7–9. In Fig. 7, $f'(\eta)$ is plotted against the similarity variable for suction ($f_w = 1$) and injection ($f_w = -1$) cases in the non–Darcy region for varying radiation parameter and by fixing other values of the parameters. Due to fluid injection, the velocity of the fluid is increased while

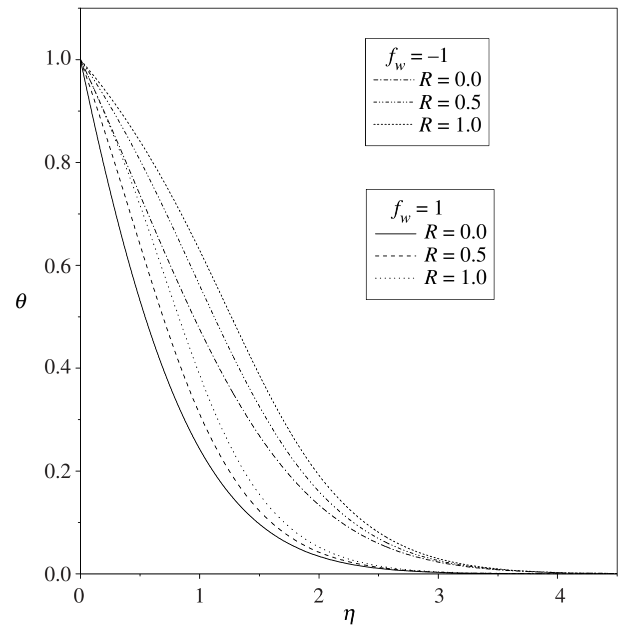

Fig. 8. $\theta(\eta)$ vs. η for $\mathrm{Re}_d = 1.0$ and $\frac{Ra_d}{Pe_d} = 5.0$ (aiding flow)

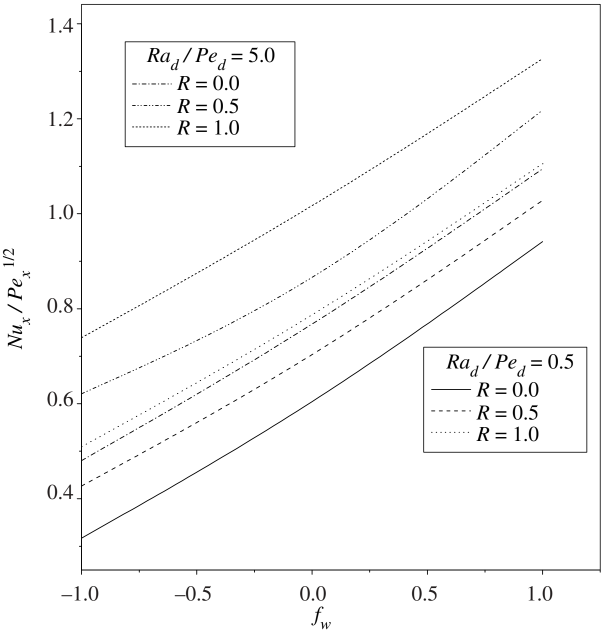

Fig. 9. Nusselt number vs. f_W for $\mathrm{Re}_d = 1.0$ (aiding flow)

suction decreased the same. Similarly the temperature distribution $\theta(\eta)$ has been presented in Fig. 8. From these figures, it is evident that the fluid suction decreased the thickness of the thermal boundary layer and enhanced the heat transfer into the medium whereas fluid injection increased the boundary-layer thickness and reduced heat transfer into the medium. The heat transfer coefficient plotted in Fig. 9 clearly indicated this feature. Thus the heat transfer coefficient increased as the parameter moves from the injection zone to the suction zone. Also, it can be seen clearly that the effect of radiation is to enhance the heat transfer coefficient in both the suction and injection regions.

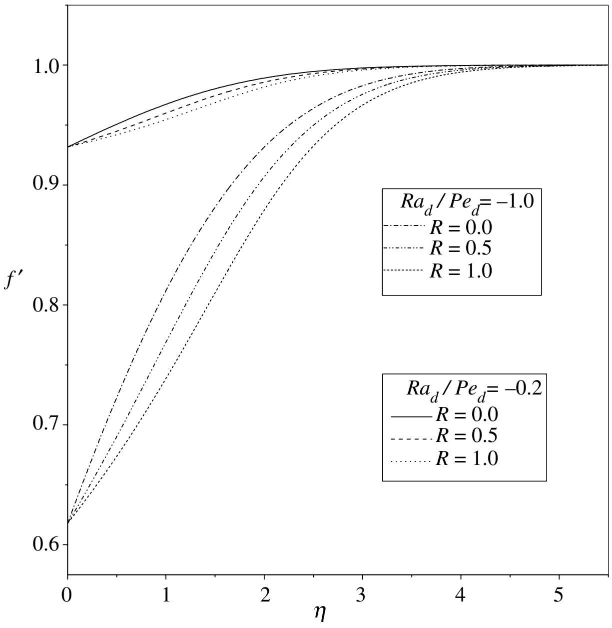

Fig. 10. $f'(\eta)$ vs. η for $f_W = 0$ and $\mathrm{Re}_d = 1.0$ (opposing flow)

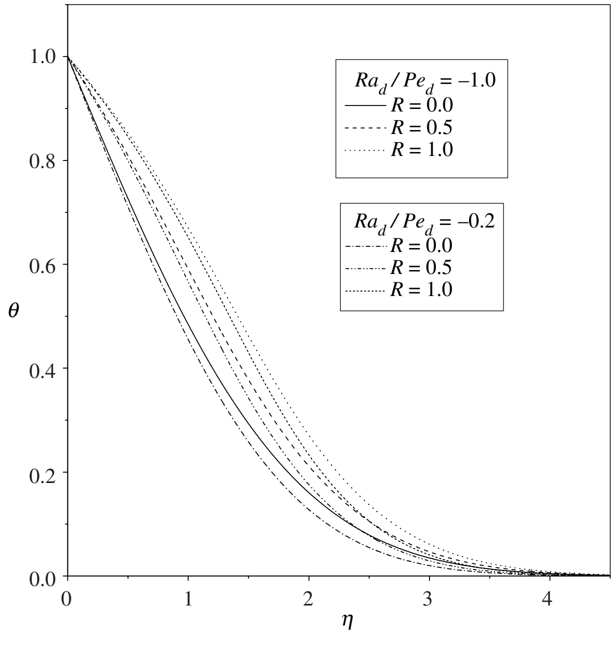

Fig. 11. $\theta(\eta)$ vs. η for $f_W = 0$ and $\mathrm{Re}_d = 1.0$ (opposing flow)

Results in opposing flow: The flow field becomes more complex when the buoyancy is opposing the free stream flow. In this case the value of the flow governing parameter is negative. Flow separation is observed to depend on the value of the flow governing parameter and also on Re_d. It is observed that the inertia effects delay the flow separation.

The typical nondimensional velocity and temperature distributions in the opposing flow are presented in Figs. 10 and 11 for the impermeable flat plate case for some selected values of the other parameters. Unlike in the aiding flow situation, an increase in the value of the radiation

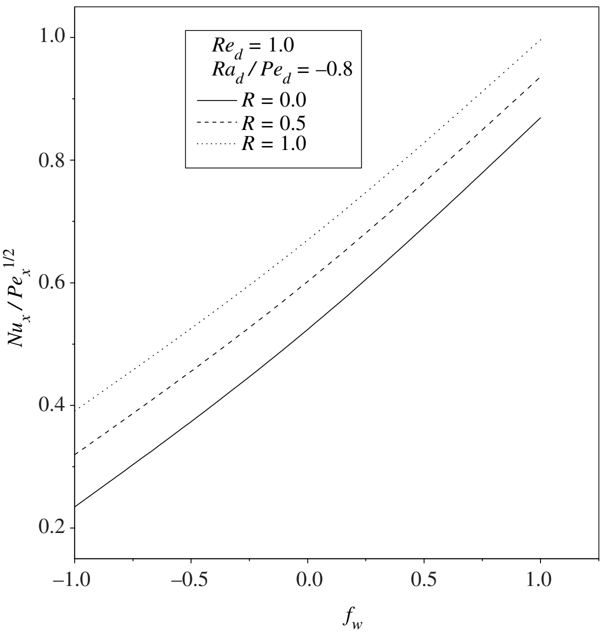

Fig. 12. Nusselt number vs. f_W for $\mathrm{Re}_d = 1.0$, $\frac{Ra_d}{Pe_d} = -0.8$ (opposing flow)

parameter decreased the fluid velocity, and it is evident from Fig. 10. The thermal boundary-layer thickness increased with increasing value of the radiation parameter thereby reducing the heat transfer into the medium. It is observed that the heat transfer coefficient increased with an increase in the value of the radiation parameter and also as the value of f_w moves from the injection region to the suction region. Also it is found that the heat transfer coefficient is more in the near–Darcy region than in the non–Darcy region in the opposing flow case. Some of these results are clear from the heat transfer coefficient versus the suction/injection parameter presented in Fig. 12 for fixed values of the other parameters.

4 Conclusions

Mixed convection flow of an absorbing fluid up a uniform non–Darcy porous medium supported by a semi-infinite ideally transparent vertical flat plate due to solar radiation is considered. The external flow field is assumed to be uniform, and the effect of the radiation parameter in the boundary layer adjacent to the vertical flat plate with fluid suction/injection through it is analyzed in both aiding and opposing flow situations. It is observed that the similarity solution is possible only when the fluid suction/injection velocity profile varies as $x^{-1/2}$. Extensive calculations have been performed to obtain the flow and temperature field with the parametric values $C_T = 0.1$, $0 \leq \mathrm{Re}_d \leq 5$, $-1.0 \leq \frac{Ra_d}{Pe_d} \leq 50$ and $0 \leq R \leq 1.0$. In the aiding flow, the effect of thermal radiation is to (i) increase the convective moment in the boundary layer, (ii) increase the thermal boundary-layer thickness with an increase in the value of the radiation parameter, (iii) enhance the heat transfer coefficient in the medium. Also the effect of radiation is more predominant in the near–Darcy region than in the non–Darcy region. Fluid suction decreased the thickness of the thermal boundary layer and enhanced the heat transfer into the medium whereas fluid injection increased the boundary-layer thickness and reduced the heat transfer into the medium, due to thermal radiation effects, and the heat transfer coefficient is increased in both the suction and injection regions. In the opposing flow, separation is seen to depend on the flow governing parameter and also on the inertial parameter Re_d. The thermal boundary-layer thickness increased with increasing value of the radiation parameter thereby reducing the heat transfer into the medium. It is observed that the heat transfer coefficient increased with an increase in the value of the radiation parameter and also as the value of f_w moves from the injection region to the suction region.

References

[1] Cheng, P., Minkowycz, W. J.: Free convection about a vertical plate embedded in a porous medium with application to heat transfer from a dike. J. Geophys. Res. **82,** 2040–2044 (1977).

[2] Cheng, P.: The influence of lateral mass flux on the free convection boundary layers in a saturated porous medium. Int. J. Heat Mass Transfer **20,** 201–205 (1977).

[3] Merkin, J. H.: Free convection with blowing and suction. Trans ASME J. Heat Transfer **15,** 989–999 (1972).

[4] Plumb, O., Huenefeld, J. C.: Non–Darcy natural convection from heated surfaces in saturated porous medium. Int. J. Heat Mass Transfer **24,** 765–768 (1981).

[5] Bejan, A., Poulikakos, D.: The non–Darcy regime for vertical boundary layer natural convection in a porous medium. Int. J. Heat Mass Transfer **27,** 717–722 (1984).

[6] Ranganathan, P., Viskanta, R.: Mixed convection boundary-layer flow along a vertical surface in a porous medium. Num. Heat Transfer **7,** 305–317 (1984).

[7] Nakayama, A., Pop, I.: A unified similarity transformation for free, forced, and mixed convection in Darcy and non–Darcy porous media. Int. J. Heat Mass Transfer **34,** 357–367 (1991).

[8] Lai, F. C., Kulacki, F. A.: Non–Darcy mixed convection along a vertical wall in saturated porous medium. Trans. ASME. J. Heat Transfer **113,** 252–255 (1991).

[9] Nield, D. A., Bejan, A.: Convection in porous media. New York: Springer 1992.

[10] Sparrow, E. M., Cess, R. D.: Radiation heat transfer. Washington: Hemisphere 1978.

[11] Whitaker, S.: Radiant energy transport in porous media. Int. Engng Chem. Fund. **19,** 210–218 (1980).

[12] Tong, T. W., Tien, C. L.: Radiative heat transfer in fibrous insulations. Trans. ASME. J. Heat Transfer **105,** 70–81 (1983).

[13] Raptis, A.: Radiation and free convection flow through a porous medium. Int. Comm. Heat Mass Transfer **25,** 289–295 (1998).

[14] Chamkha, A. J.: Solar radiation assisted convection in uniform porous medium supported by a vertical flat plate. Trans. ASME. J. Heat Transfer **119,** 89–96 (1997).

[15] Chamkha, A. J., Issa, C., Khanafer, K.: Natural convection due to solar radiation from a vertical plate embedded in a porous medium with variable porosity. J. Porous Media **4,** 69–77 (2001).

[16] Bakier, A. Y., Gorla, R. S. R.: Thermal radiation effect on mixed convection from horizontal surfaces in saturated porous media. Transport in Porous Media **23,** 357–363 (1996).

[17] Mansour, M. A.: Forced convection–radiation interaction heat transfer in boundary layer over a flat plate submerged in a porous medium. Appl. Mech. Engng **3,** 405–413 (1997).

[18] Bakier, A. Y.: Thermal radiation effect on mixed convection from vertical surfaces in saturated porous media. Int. Comm. Heat Mass Transfer **28,** 119–126 (2001).

Authors' addresses: P. V. S. N. Murthy and S. Mukherjee, Department of Mathematics, Indian Institute of Technology, Kharagpur 721302, India; D. Srinivasacharya and P. V. S. S. S. R. Krishna, Department of Mathematics and Humanities, National Institute of Technology, Warangal 506 004, India