Water Resour Manage (2010) 24:2057–2073
DOI 10.1007/s11269-009-9538-7

Multiobjective Multireservoir Operation in Fuzzy Environment

Sumant A. Choudhari · P. Anand Raj

Received: 15 January 2009 / Accepted: 23 November 2009 /
Published online: 11 December 2009

Abstract In this paper, the operation of a complex system of multireservoirs with multiple objectives is being demonstrated. The multireservoir system includes uncertainties of inflows, demands to the large extent. Fuzzy set theory has been proved as a robust theory where these kinds of uncertainties have major role. In this study, the fuzzy linear programming method is used to get the better policy for the system operation with the uncertainty in various parameters i.e. resources, technological coefficients, objective function coefficients. A four reservoir system (a compound parallel and series) is taken as a case study and the fuzzification of all parameters is tried on the system. The effect of fluctuations in irrigation demand and release, power demand and release and available storage volume are considered. The consequences on the objectives i.e. maximization of returns from irrigation release, maximization of returns from power releases are also examined. The operation policies thus evolved give a better understanding of the problem and the intricate complexities associated and there effects. With this the policy makers (a decision maker) can have a wide range of options at his disposal for suitable action.

Keywords Multireservoir · Fuzzy linear programming · Uncertainties · Optimization

Notations

FP	Firm power
ID	Irrigation demand
ID_{min}	Minimum irrigation demand
IN	Inflows into the reservoirs

S. A. Choudhari (✉) · P. Anand Raj
Water and Environment Division,
National Institute of Technology, Warangal, India
e-mail: sumantchoudhari@yahoo.co.in

P. Anand Raj
e-mail: a_raj_p@yahoo.co.in

RI	Releases into canals for irrigation
RP	Turbine releases
RWS	Release for water supply
SC	The storage capacity of the reservoirs
S	The storage in the reservoirs
S_{min}	Minimum storage capacity
TC	Flow through turbine capacities
OVF	Overflows from the reservoirs
$\alpha_1, \alpha_2, \alpha_3$	Constants
$\tilde{A}$	Fuzzy set A
A_α	Alpha cut of fuzzy set A
A_0	Area corresponds to dead storage
e_j	Evaporation loss rate
FCR	Feeder canal release
T_1, T_2, T_3	Turbine of the reservoirs R_1, R_2, R_3.
L_1, L_2, L_3, L_4	Losses from the reservoirs R_1, R_2, R_3, R_4.
$\alpha_c, \alpha_A, \alpha_b$	α levels on C_j, A_{ij}, B_i.

1 Introduction

The demand for water resources is ever increasing with the explosive population growth, advancement of civilization, increased agricultural and industrial productivity and other inter actual systems. Irrigated agriculture makes a major contribution to food security producing nearly 40% of food and agricultural commodities on 17% of agricultural land. Irrigated areas have almost doubled over the past decades and contributed much to the impressive growth in agricultural productivity over the last 50 years. More than 70% of the water withdrawn from the earth's rivers is presently used for irrigated agriculture; in developing countries the proportion is even higher. The management of water distribution in an irrigation system entails the solution of multiple problems. The planning of water distribution should be based on the concept of economic optimum.

Unlike all investigation methods, system analysis helps in solving the question properly and distinctly setting the objectives and correctly formulating the problem and task. System analysis converts a complicated problem into a simple one and it transforms the problem which is difficult to solve and understand into series of precise tasks with direct method of their solution. On the one hand, system analysis makes it possible to decompose a problem which is too complex for a direct solution into its components for definition and resolution of the specific tasks and on other hand keep them together as a single entity. Several new approaches are being demonstrated in last two decades in the applications of water resources.

Anand Raj and Nagesh Kumar (1998, 1999) introduce new method of fuzzy ranking with the concept of maximizing set and minimizing set. The method of RANking FUzzy Weights (RANFUW) is computationally simple and easy to implement. The proposed method was applied (RANFUW method) to a river basin planning and management problem. The method practiced on the Krishna River basin to find the most suitable planning of the reservoirs with their associated purposes.

Deterministic and stochastic optimizing models have been developed and applied to both static and dynamic planning problems by Jacoby and Loucks (1972). These optimization models were used to screen the set of possible plans and to select a small number worthy of simulation analysis. Alemany et al. (1973) gave an algorithm for solving a class of Linear Programming (LP) problems related to reservoir management and design. The algorithm gives the solution of problems related to the use of linear decision rule in reservoir design and management. The problem of noncommensurable multiobjective functions in water resources systems was discussed by Haimes and Hall (1974). The surrogate worth trade off method was developed for solving noncommensurable multiobjective functions. On the other hand a multiobjective optimization procedure using the constraint LP technique was developed by Louie et al. (1982). The main objectives of the study were water supply allocation, water quality control and prevention of undesirable overdraft of the ground water basin. Stedinger et al. (1983) developed a Chance Constrained (CC) model for a three reservoir system. Marino and Mohammadi (1983) developed a reliability programming model for the optimum operation of monthly releases from a multipurpose reservoir. The Chance Constraint Linear Programming (CCLP) and Dynamic Programming (DP) were used to formulate the model. The flood and draught reliabilities were included as chance constraints in CCLP, which were varied parametrically from minimum required levels to the maximum possible values. The DP model to the reservoir system was applied by Georgakakos and Yao (1993a, b). Umamahesh and Sudarsana Raju (2002) studied two phase DP–LP model for optimal irrigation planning under deficit water supply. Jairaj and Vedula (2000) formulated a fuzzy mathematical programming model to a multireservoir system with a number of upstream parallel reservoirs, and one downstream reservoir. The aim of the study was to minimize the sum deviations of the monthly irrigation withdrawals from their target demands over period of 1 year. Three reservoir system in the Upper Cauvery River Basin was taken as a case study. Timlant et al. (2002) showed that the classical Stochastic Dynamic Programming (SDP) gives the similar operation policies as the Fuzzy Stochastic Dynamic Programming (FSDP) formulation. The reservoir operation problem for the Mansour Eddahbi dam was taken for the demonstration. Raju and Duckstein (2003) formulated Multi-Objective Fuzzy Linear Programming (MOFLP) irrigation planning model for the evaluation of management strategy. Three conflicting objectives net benefits, agricultural production and labour employment were considered in the irrigation planning scenario. Dubrovin et al. (2002) constructed a fuzzy rule-based control model for multipurpose real-time reservoir operation. A new methodology for fuzzy inference named total fuzzy similarity was used and compared with the more traditional Sugeno-style method. Janga Reddy and Nagesh Kumar (2007) studied Multi Objective Differential Evolution (MODE) algorithm. The MODE was applied to the Hirakud reservoir system. The optimization problem takes minimization of flood risk, maximization of hydropower production and minimization of irrigation deficits. The optimal trade-offs are obtained with MODE and found that the MODE gives executable alternative after results were compared the results obtained from Genetic algorithm programming. Mujumdar (2002) gave a brief over view of some mathematical tools for irrigation system operation, crop water allocations and performance evaluation. Recent tools and techniques of fuzzy optimization and fuzzy interface systems that incorporate imprecision in management goals and constraints and address the interests of stakeholders were being discussed. Regulwar and Anand Raj (2008) developed a Multi objective, Multireservoir operation model for maximization of irrigation releases and maximization of hydropower production using Genetic Algorithm. These objectives were fuzzified and were simultaneously maximized by defining and then maximizing level of satisfaction (λ). A monthly Multi Objective Genetic Algorithm Fuzzy Optimization (MOGAFUOPT) model for the study was developed in 'C' Language. Afshar and Moeini (2008) presented a constrained formulation of the Ant Colony Optimization Algorithm (ACOA) for the optimization of large scale reservoir operation problem. The case study of Dez reservoir in Iran with two cases of simple and hydropower operation problems with storage volume as the decision variables of the problem was taken. Two different formulations i.e. partially constrained and fully constrained were examined with Max–Min Ant System for the solution of the reservoir operation problem. Bender and Simonovic (2000) analyzed the fuzzy compromise approach to decision analysis applied for the water resources systems planning under uncertainty. The comparative study between ELECTRE method and fuzzy compromise approach to demonstrate the welfares of acquiring a multicriteria decision analysis technique. Guangtao Fu (2008) presented a fuzzy optimization method based on the concept of ideal and anti-ideal points to solve multi-criteria decision making problems under fuzzy environment. The alternatives were represented as triangular fuzzy numbers with the quantitative criteria values. Linguistic terms were utilized to describe their qualitative counterparts and the weight of each criteria. For the optimal evaluation of the objective function the concept of fuzzy ideal and anti-ideal weight distances were used for each alternative. Afshar et al. (2008) studied lumped approach to multi-period–multi-reservoir cyclic storage system optimization. The LINGO solver was used to solve the mixed integer nonlinear programming model. The objective of the study was to minimize the total cost of operating cost over the planning period.

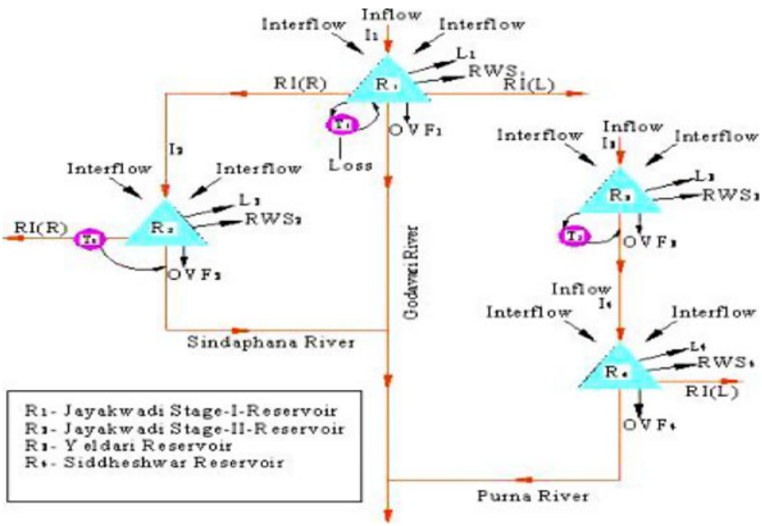


Fig. 1 Schematic representation of the physical system

Table 1 Maximum irrigation demands and inflows in reservoirs

Month	Jayakwadi stage-I (R_1)		Jayakwadi stage-II (R_2)		Yeldari (R_3)		Siddheshwar (R_4)	
	Irrigation demand Mm^3	Inflow Mm^3	Irrigation demand Mm^3	Inflow Mm^3	Irrigation demand Mm^3	Inflow Mm^3	Irrigation demand Mm^3	Inflow Mm^3
June	18.55	148.76	7.12	20.98	0	72.83	33.10	7.71
July	26.70	408.25	20.83	43.46	0	141.09	35.23	2.21
August	25.43	610.66	37.64	96.88	0	200.36	35.23	11.97
September	85.79	600.0	46.02	144.17	0	160.77	93.46	9.18
October	267.86	287.75	132.01	75.52	0	123.10	77.60	1.29
November	228.74	196.46	127.05	10.24	0	49.48	74.68	0.57
December	210.88	125.53	89.43	4.27	0	35.58	65.14	0.89
January	230.34	37.65	100.68	0.37	0	32.18	65.14	1.00
February	85.23	21.46	30.02	0.37	0	24.23	35.50	0.39
March	70.06	19.56	28.98	0.16	0	23.54	37.40	1.00
April	85.49	25.50	35.58	0.12	0	13.15	30.50	0.40
May	58.20	46.58	25.88	0.06	0	13.86	22.30	0.40
Total	1,393.2	2,528.17	681.24	396.60	0	890.17	605.2	37.01

1.1 System Description

River Godavari is one of the major peninsular rivers in Southern part of India spanning over three states namely, Maharashtra, Karnataka and Andhra Pradesh. The Physical System under investigation consists of four reservoirs which are connected in both parallel and serially. The schematic representation of the physical system is shown in Fig. 1. The Jayakwadi project stage-I (R_1) is one of the major projects built across river Godavari, Maharashtra State, India. The gross storage of reservoir is 2,909 Mm^3. Total installed capacity for power generation is 12.0 MW (Pumped Storage Plant). Irrigable command area is 1,416.40 km^2. The Jayakwadi project stage-II (R_2) is built across river Sindaphana, a tributary of river Godavari. The gross storage of reservoir is 453.64 Mm^3. Total installed capacity for power generation is 2.25 MW. Irrigable command area is 938.85 km^2. The Yeldari dam (R_3) is built across river Purna another major tributary to river Godavari. The gross storage of reservoir is 934.44 Mm^3. Total installed capacity for power generation is 15.0 MW and it is a pure Hydel Project. The Siddheshwar dam (R_4) is built across river Purna which is at downstream of R_3. The gross storage of reservoir is 250.85 Mm^3 and it is irrigation reservoir. Irrigable command area is 615.60 km^2. The monthly irrigation demands and inflows of the system are shown in Table 1.

2 Fuzzy Linear Programming

Definition 1 Let $\mathcal{F}$ denote a universal set. Then the membership function $\mu_{\tilde{A}}(x)$ by which a fuzzy set A is defined has the form

$$\mu_{\tilde{A}}(x): \quad \mathcal{F} \rightarrow [0, 1] \tag{1}$$

Where [0, 1] denotes the interval of real numbers between 0 to 1, both inclusive.

Among the various types of the fuzzy sets, of special significance are fuzzy sets that are defined on the real set $\mathcal{R}$ of real numbers. Membership functions of these sets, which have the form

$$\mu_{\tilde{A}}(x): \quad \mathcal{R} \to [0, 1] \tag{2}$$

Definition 2 A real fuzzy number described by ã is a fuzzy subset of the real line $\mathcal{R}$ represented as

$$\tilde{a} = (\alpha/\beta, \gamma/\delta) \tag{3}$$

where α, β, γ, δ are the real numbers and are the parameters of the fuzzy number ã.

Definition 3 Let the membership function $\mu_{\tilde{a}}(x)$ of the fuzzy number ã be given by

1. a continuous mapping from $\mathcal{R}$ to a closed interval $[0, v]$, $0 < v \leq 1$;
2. constant (zero) on $(-\infty, \alpha]$: $\mu_{\tilde{a}}(x) = 0$ for x when $-\infty < x \leq \alpha$;
3. strictly increasing in the interval $[\alpha, \beta]$;
4. a constant (v) in the interval $[\beta, \gamma]$: $\mu_{\tilde{a}}(x) = v$ for x when $\beta \leq x \leq \gamma$;
5. strictly decreasing in the interval $[\gamma, \delta]$; and
6. a constant (zero) in the interval $[\delta, \infty)$: $\mu_{\tilde{a}}(x) = 0$ for x when $\delta \leq x \leq \infty$

We call fuzzy number with such membership function a generalized triangular fuzzy number with trapezoidal membership function. This membership function can be represented as

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & x \leq \alpha \\ v(x-\alpha)/(\beta-\alpha) & \alpha \leq x \leq \beta \\ v & \beta \leq x \leq \gamma \\ v(\delta-x)/(\delta-\gamma) & \gamma \leq x \leq \delta \\ 0 & x \geq \delta \end{cases} \tag{4}$$

For triangular fuzzy number $\beta = \gamma$. The membership function $\mu_{\tilde{a}}$of the generalized fuzzy number is described by

$$\mu_{\tilde{a}} = \begin{cases} \mu_{\tilde{a}}^{L}(x) & \alpha \leq x \leq \beta \\ v & \beta \leq x \leq \gamma \\ \mu_{\tilde{a}}^{R}(x) & \gamma \leq x \leq \delta \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

For the normalized fuzzy number i.e. height of the fuzzy set is 1, the value of v is being equal to 1.The graphical representation of the membership function of a fuzzy number is shown in Fig. 2.

Definition 4 Given fuzzy set A on $\mathcal{F}$ and a number α in I, such that $0 < \alpha \leq 1$, we can associate a crisp set with A, denoted by A_α, and defined as

$$A_\alpha = \{x \in \mathcal{F} \,|\, A(x) \geq \alpha\} \tag{6}$$

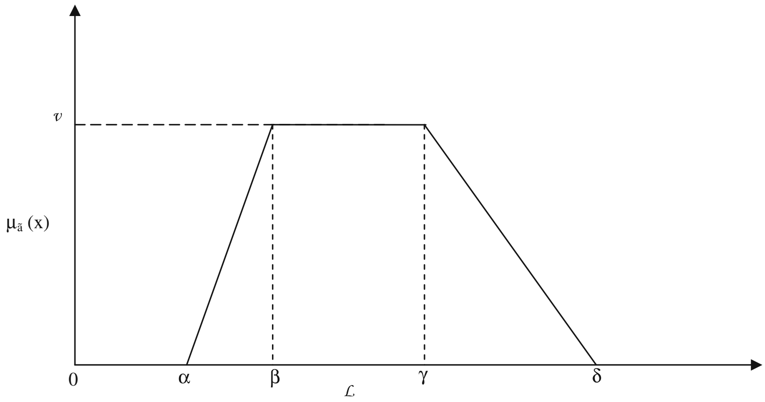


Fig. 2 Graphical representation of the membership function of a fuzzy number ã

A_α is called the *α-cut* of A. Thus, for each α, we can obtain an *α-cut* of A (Zimmermann 1996).

Fuzzy Parametric Programming (FPP), one of the fuzzy mathematical programming (FMP) approaches, is proposed by Carlsson and Korhonen (1986) for single objective problems. FPP can conceive all coefficients in the mathematical model as fuzzy giving resemblance to the real life. The general formulation of the FPP can be given as

$$\text{Max } z = \mathcal{c}\text{x} \tag{7}$$

$$\text{s.t.} \qquad \mathcal{A}\text{x} \leq \mathcal{b}, \qquad \{\text{x}|\, \text{x} \in \text{X}, \text{x} \geq 0\}, \tag{8}$$

where $\mathcal{c}$ is an n- vector, $\mathcal{A}$ is an (mxn) matrix and $\mathcal{b}$ is an m- vector. All these parameters in the real life are fuzzy (can not be known exactly), but they can be approximated with lower and higher values of the parameters. In other words, the better and worse predicted values with existing data or the subjective knowledge or the estimates given by the experts or the need of the decision maker. Along with this, the information of the feasibility (implementation) is most concern. The solution of the model must be optimal with highest feasibility level. The feasibility grades are defined by the fuzzy membership function for each parameter.

The intervals of the parameters with the knowledge obtained for the sources given above are decided. The possible values of fuzzy parameters are specified by user as $[\mathcal{c}^L, \mathcal{c}^U)$, $[\mathcal{A}^L, \mathcal{A}^U)$, $[\mathcal{b}^L, \mathcal{b}^U)$. The lower bound represents “risk-free” values giving the solution with highest feasibility in the sense that a solution most certainly should be implemented. The upper bound represents the parameter values which are certainly unrealistic, “unacceptable” and the solution gained by this formulation is not implementable.

The membership function for each parameter is defined. The monotonically decreasing function used for the all parameters values. As the values of the parameters changes form lower to higher the solution moves from “risk-free” to “unacceptable”. The fuzzy membership function of each parameter can be defined as (Fig. 3).

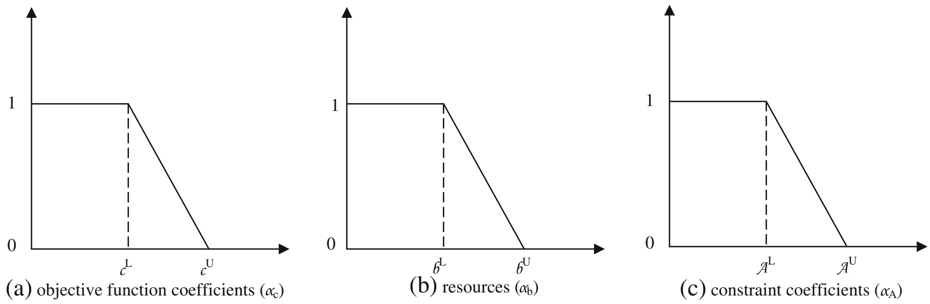


Fig. 3 Membership functions for various coefficients

If α is the feasibility level then feasibility of each parameter can be written as α_c, α_A, α_b. The feasibility of an optimal solution α_s is defined by the intersection of membership functions which belong to imprecise parameters.

$$\alpha_s = \left(\alpha_{cj} \cap \alpha_{Aij} \cap \alpha_{bi}\right), \; i = 1, 2, \ldots, \mathrm{m}. \quad j = 1, 2, \ldots, \mathrm{n}. \tag{9}$$

Each parameter can be transferred to its crisp form with the membership function defined earlier (Arikan and Gungor 2007).

$$\mathrm{c_j} = \quad c^{U} - \alpha_{cj} \times \left(c^{U} - c^{L}\right) \tag{10}$$

$$\mathrm{A_{ij}} = \mathcal{A}^{U} - \alpha_{Aij} \times \left(\mathcal{A}^{U} - \mathcal{A}^{L}\right) \tag{11}$$

$$\mathrm{b_i} = \quad b^{U} - \alpha_{bi} \times \left(b^{U} - b^{L}\right) \tag{12}$$

The optimization problem can be written as

$$\mathrm{Max}\ \mathcal{Z} = \left[c^{U} - \alpha_{cj} \times \left(c^{U} - c^{L}\right)\right] \times \ \mathrm{x}, \tag{13}$$

Subjected to

$$\left[\mathcal{A}^{U} - \alpha_{Aij} \times \left(\mathcal{A}^{U} - \mathcal{A}^{L}\right)\right] \times \mathrm{x} \le \left[b^{U} - \alpha_{bi} \times \left(b^{U} - b^{L}\right)\right] \tag{14}$$

$\mathrm{x} \ge 0$; where $\alpha_c = \alpha_A = \alpha_b = \alpha$

(Note: For the multiobjective problem formulation keeping the other things same the objective function can be transformed to

$$\mathrm{Max}\ \mathcal{Z}_k = \left[c_k^{U} - \alpha_{cj} \times \left(c_k^{U} - c_k^{L}\right)\right] \times \ \mathrm{x}, \qquad k = 1, 2, \ldots, \mathrm{K}.) \tag{15}$$

With the multiobjective formulation one objective at one time should be solved. The objective function value of that objective at each feasibility level i.e. $\alpha_c = \alpha_A = \alpha_b = \alpha$ is determined. The value of α ranges from 0 to 1 with the difference of 0.1 giving the eleven values of each objective function. The highest and lowest value of the objective is decided amongst these solutions. Following the same procedure for each objective the worst and best values are determined. These worst and best values give the entire solution space of each objective. Then the optimization model of multiple objectives is being transferred to single objective λ (satisfaction level). Each objective then fuzzified over the range which is obtained from the above procedure. The membership function of each objective fuzzification can be given as

$$\lambda = \begin{cases} 0, & Z_k \le Z_k^{\min} \\ \left(Z_k - Z_k^{\min}\right) / \left(Z_k^{\max} - Z_k^{\min}\right), & Z_k^{\min} < Z_k < Z_k^{\max} \\ 1, & Z_k \ge Z_k^{\max} \end{cases} \tag{16}$$

The new formulation can be given as

$$\text{Max} = \lambda \tag{17}$$

Subjected to

$$\left(Z_k - Z_k^{\min}\right) / \left(Z_k^{\max} - Z_k^{\min}\right) \ge \lambda \qquad k = 1, 2, \ldots, K. \tag{18}$$

$$\begin{gathered} \left[\mathcal{A}^U - \alpha_{Aij} \times \left(\mathcal{A}^U - \mathcal{A}^L\right)\right] \times \ x \le \left[\mathscr{b}^U - \alpha_{bi} \times \left(\mathscr{b}^U - \mathscr{b}^L\right)\right] \\ 0 \le \lambda \le 1 \end{gathered} \tag{19}$$

$x \ge 0$; where $\alpha_{Aij} = \alpha_{bi} = \alpha$.

The solution for each α value of the model is calculated using the fuzzy linear programming and presented to decision maker. Each feasibility level gives an alternative decision plan.

3 Model Development

A monthly Multi Objective Fuzzy Optimization (MOFUOPT) model is developed to derive an operation plan for the optimal utilization of the water resources available in the basin.

3.1 Objective Functions

The two objectives considered in this study are

1. Maximization of irrigation releases (i.e., RI)
2. Maximization of hydro-power releases (i.e., RP)

$$\text{Maximize } Z = \sum_{i=1}^{4} \sum_{j=1}^{12} (RI)_{ij} \tag{20}$$

$$\text{Maximize } Z = \sum_{i=1}^{4} \sum_{j=1}^{12} (RP)_{ij} \tag{21}$$

3.2 Constraints

3.2.1 Turbine Release-Capacity Constraints

The releases into turbines for power production, should be less than or equal to the flow through turbine capacities (TC) for all the months. Also, power production in each month should be greater than or equal to the releases for firm power (RFP). These constraints can be written as

$$\mathrm{RP}\left(\mathrm{i,j}\right) \leq \mathrm{TC}\,(\mathrm{i}) \qquad \forall \quad \mathrm{i} = 1, 2, \ldots 4. \tag{22}$$

$$\mathrm{RP}\left(\mathrm{i,j}\right) \geq \mathrm{FP}\,(\mathrm{i}) \quad \forall \quad \mathrm{j} = 1, 2, 3, \ldots\ldots..,12. \tag{23}$$

3.2.2 Irrigation Release-Demand Constraints

The releases into canals for irrigation (RI) should be less than or equal to the maximum irrigation demand (ID_{max}) on all reservoirs for all the months. Also, the releases into the canals for irrigation should be greater than or equal to the minimum irrigation demand (ID_{min}). A 30% minimum irrigation demand is taken for this study. R_2 is being a canal power house, whenever irrigation demand is more than power requirement, the extra requirements, over the turbine flows, is delivered through the canal bypassing the turbines. When irrigation demand is less than power requirements, the extra releases from turbine is released into the river. The irrigation release-demand constraint, can, therefore be written as

$$\mathrm{RI}\left(\mathrm{i,j}\right) \leq \mathrm{ID}\left(\mathrm{i,j}\right) \qquad \forall \quad \mathrm{i} = 1, 2, \ldots 4. \tag{24}$$

$$\mathrm{RI}\left(\mathrm{i,j}\right) \geq \mathrm{ID_{min}}\left(\mathrm{i,j}\right) \quad \forall \quad \mathrm{j} = 1, 2, 3, \ldots\ldots\ldots., 12. \tag{25}$$

3.2.3 Reservoir Storage-Capacity Constraints

The storage in the reservoirs (S) should be less than or equal to the maximum storage capacity (SC) and greater than or equal to the minimum storage capacity (S_{min}) for all months. The storages are taken at the beginning of the month. These constraints can be written as

$$\mathrm{S}\left(\mathrm{i,j}\right) \leq \mathrm{SC}\,(\mathrm{i}) \qquad \forall \quad \mathrm{i} = 1, 2, \ldots 4. \tag{26}$$

$$\mathrm{S}\left(\mathrm{i,j}\right) \geq \mathrm{S_{min}}\,(\mathrm{i}) \qquad \forall \quad \mathrm{j} = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, 12. \tag{27}$$

3.2.4 Hydrologic Continuity Constraints

These constraints relate to the turbine releases (RP), irrigation releases (RI), release for drinking water supply (RWS) which is taken as a constant, reservoir storage (S), inflows into the reservoirs (IN), Losses from the reservoirs for all months.

(i) Reservoir (R_1)

$$\begin{aligned}\left(1 + \mathrm{a_j}\left(1,\mathrm{j}\right)\right)\mathrm{S}\left(1,\mathrm{j}+1\right) = {} & \left(1 - \mathrm{a_j}\left(1,\mathrm{j}\right)\right)\mathrm{S}\left(1,\mathrm{j}\right) + \mathrm{IN}\left(1,\mathrm{j}\right) - \mathrm{RP}\left(1,\mathrm{j}\right) \\ & - \mathrm{RI}\left(1,\mathrm{j}\right) - \mathrm{OVF}\left(1,\mathrm{j}\right) - \mathrm{RWS}\left(1,\mathrm{j}\right) - \mathrm{FCR}\left(1,\mathrm{j}\right) \\ & + \alpha_1\mathrm{RP}\left(1,\mathrm{j}\right) - \mathrm{A_0 e_j}\left(1,\mathrm{j}\right) \\ \forall \mathrm{j} = {} & 1, 2, 3, \ldots\ldots\ldots., 12 \end{aligned} \tag{28}$$

(ii) Reservoir (R_2)

$$\begin{aligned}\left(1+a_j\left(2,j\right)\right)S\left(2,j+1\right)=&\left(1-a_j\left(2,j\right)\right)S\left(2,j\right)+IN\left(2,j\right)+FCR\left(1,j\right)\\&-RP\left(2,j\right)-RI\left(2,j\right)-OVF\left(2,j\right)\\&-RWS\left(2,j\right)-A_0e_j\left(2,j\right)\\&\forall j=1,2,3,\ldots\ldots\ldots,12\end{aligned}\tag{29}$$

(iii) Reservoir (R_3)

$$\begin{aligned}\left(1+a_j\left(3,j\right)\right)S\left(3,j+1\right)=&\left(1-a_j\left(3,j\right)\right)S\left(3,j\right)+IN\left(3,j\right)-RP\left(3,j\right)\\&-OVF\left(3,j\right)-RWS\left(3,j\right)-A_0e_j\left(3,j\right)\\&\forall j=1,2,3,\ldots\ldots\ldots,12\end{aligned}\tag{30}$$

(iv) Reservoir (R_4)

$$\begin{aligned}\left(1+a_j\left(4,j\right)\right)S\left(4,j+1\right)=&\left(1-a_j\left(4,j\right)\right)S\left(4,j\right)+IN\left(4,j\right)+\alpha_3OVF\left(3,j\right)\\&+\alpha_4RP\left(3,j\right)-RI\left(4,j\right)-RWS\left(4,j\right)\\&-OVF\left(4,j\right)-A_0e_j\left(4,j\right)\\&\forall j=1,2,3,\ldots\ldots\ldots,12\end{aligned}\tag{31}$$

Reservoir R_1 is a pumped storage scheme. The transition loss for pumping turbine releases back into the reservoir is taken as 10% of the turbine releases. Therefore α_1 in the constraint is 0.9 for reservoir R_1. Releases for water supply (RWS) is taken as constant for reservoir R_1 as 31.63 Mm^3, 3.55 Mm^3 for R_2 and 2.0 Mm^3 for R_3 and R_4 for all months. The transition loss for Feeder Canal Release (FCR) from R_1 to R_2 is taken as 10% of FCR. Therefore α_2 in the constraint is 0.9 for reservoir R_2. The transition loss for overflow (OVF) from R_3 to reach to R_4 is taken as 10% of OVF. Therefore α_3 in the constraint is 0.9 for reservoir R_4. The transition loss for turbine releases (RP) from R_3 to reach to R_4 is taken as 10% of RP. Therefore α_4 in the constraint is 0.9 for reservoir R_4.

4 Results and Discussions

In this study a monthly Multi Objective Fuzzy Optimization (MOFUOPT) model for the four reservoirs system is being proposed. A fuzzy linear programming model for all parameters being fuzzified i.e. resources (b_i), technological coefficients (A_{ij}), and objective function coefficients (C_j) was tried on the system for developing a suitable operating policy. Storage in the reservoir, demand for irrigation, and demand for hydropower were being fuzzified as resources fuzzification. The monitory values of per unit of water for hydropower and irrigation are considered. It is assumed that a unit of water released for irrigation benefits (net) twice that of a unit of water released for the hydropower and the fuzzification of these monitory values is also tried in this study. The developed model was run for each objective with different values of α (as described in methodology) giving the different values for each of the objectives. With these values the best and the worst values of each objective are decided. The objectives are then fuzzified over the best and worst values obtained

Table 2 Operation policy for maximization of level satisfaction i.e. $\lambda = 0.30507$ ($\alpha = 1$)

Month	Irrigation releases RI				Total	Turbine releases RP				Total
	R_1 Mm^3	R_2 Mm^3	R_3 Mm^3	R_4 Mm^3	RI Mm^3	R_1 Mm^3	R_2 Mm^3	R_3 Mm^3	R_4 Mm^3	RP Mm^3
June	16.43	2.14	0	29.79	48.36	29.65	24.00	86.75	0	140.41
July	24.03	9.57	0	31.71	65.31	29.65	24.00	107.48	0	161.14
August	22.89	33.88	0	31.71	88.47	29.65	24.00	142.03	0	195.68
September	77.21	41.42	0	84.11	202.74	29.65	24.00	142.03	0	195.68
October	241.07	53.61	0	69.84	364.52	29.65	24.00	107.48	0	161.14
November	118.44	38.12	0	67.21	223.77	29.65	24.00	107.48	0	161.14
December	63.23	26.83	0	58.63	148.69	29.65	24.00	86.75	0	140.41
January	69.55	30.20	0	58.63	158.38	29.65	24.00	86.75	0	140.41
February	25.51	9.01	0	31.95	66.47	25.79	24.00	69.48	0	119.27
March	21.02	8.69	0	33.66	63.37	22.67	24.00	67.26	0	113.93
April	25.63	10.67	0	27.45	63.75	22.67	24.00	22.00	0	68.67
May	17.44	7.76	0	20.07	45.27	22.67	24.00	22.00	0	68.67
Total	722.45	271.90	0	544.75	1,539.10	331.03	288.00	1,047.50	0	1,666.53

to transform the model into a single objective λ (overall satisfaction). Then this single objective problem was run for different feasibility levels (α values) giving the different satisfaction levels at each level. Then the worst λ value from highest α level (i.e. $\alpha = 1$) and the best λ level from the lowest α level (i.e. $\alpha = 0$) were obtained. The operating policies of the reservoir for different α value (i.e. α ranging from 0 to 1) are also determined.

As explained above the irrigation return maximization model run for different values of α (0 to1 with the step interval of 0.1) i.e. the eleven policies were obtained from the model. The best value of the irrigation return ($\alpha = 0$) and the policy obtained is named as IRR_RUN2. When $\alpha = 1$, the irrigation return model produces the worst power returns. The policy has given nomenclature as IRR_RUN1. The best value of irrigation return and the worst value of power return are found to be

Table 3 Operation policy for maximization of level satisfaction i.e. $\lambda = 0.96994$ ($\alpha = 0$)

Month	Irrigation releases RI				Total	Turbine releases RP				Total
	R_1 Mm^3	R_2 Mm^3	R_3 Mm^3	R_4 Mm^3	RI Mm^3	R_1 Mm^3	R_2 Mm^3	R_3 Mm^3	R_4 Mm^3	RP Mm^3
June	16.59	2.14	0	30.09	48.82	32.34	24.00	95.67	0	152.01
July	24.27	6.25	0	32.03	62.55	32.34	27.32	118.52	0	178.19
August	23.12	34.22	0	32.03	89.36	32.34	32.66	156.62	0	221.62
September	77.99	36.41	0	84.96	199.37	32.34	32.66	156.62	0	221.62
October	243.51	39.60	0	70.55	353.65	32.34	26.07	118.52	0	176.93
November	108.70	38.12	0	67.89	214.71	32.34	24.00	118.52	0	174.87
December	63.23	26.83	0	59.22	149.28	32.34	24.00	95.67	0	152.01
January	69.55	30.20	0	59.22	158.97	32.34	24.00	74.37	0	130.71
February	25.51	9.01	0	32.27	66.79	32.34	24.00	46.06	0	102.40
March	21.02	8.69	0	34.00	63.71	32.34	24.00	27.93	0	84.27
April	25.63	10.67	0	27.73	64.03	32.34	24.00	22.00	0	78.34
May	17.44	7.76	0	20.27	45.47	32.34	24.00	22.00	0	78.34
Total	716.55	249.90	0	550.25	1,516.70	388.10	310.70	1,052.49	0	1,751.30

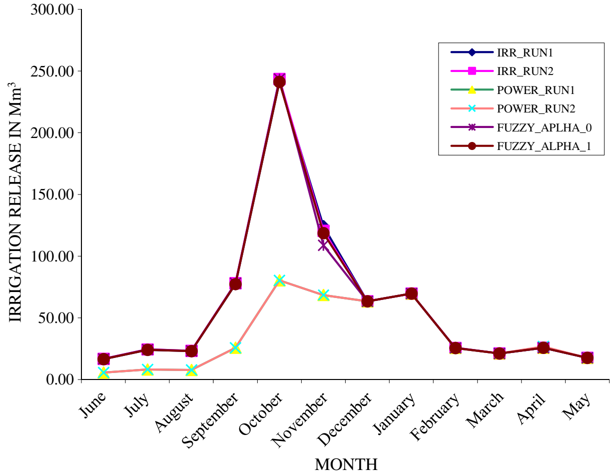


Fig. 4 Irrigation release for Jayakwadi stage-I (R_1)

6,201.967 and 824.04 Mm^3 respectively. The same procedure is followed to get the best value of power return and the worst value of irrigation return from the power return maximization model. The values obtained are 3,585.632 and 1,706.834 Mm^3 respectively from POWER_RUN1 and POWER_RUN2.

With these ranges of the irrigation returns and power returns, objectives were fuzzified as given in Eqs. 17, 18 and 19. The fuzzified model was again run for different values of α as the irrigation and power return maximization model. The operation policies for two values of α (i.e. 0 and 1) are given in Tables 2 and 3. The best objective function value (at $\alpha = 0$) of the modified problem (fuzzified problem or maximization of λ) is 0.96994 and the worst value (at $\alpha = 1$) is 0.30507. The Tables 2 and 3 give the operation policies i.e. the releases in each month not the returns. The yearly values of the returns when $\alpha = 0$ are 6,066.849 Mm^3 for irrigation return and

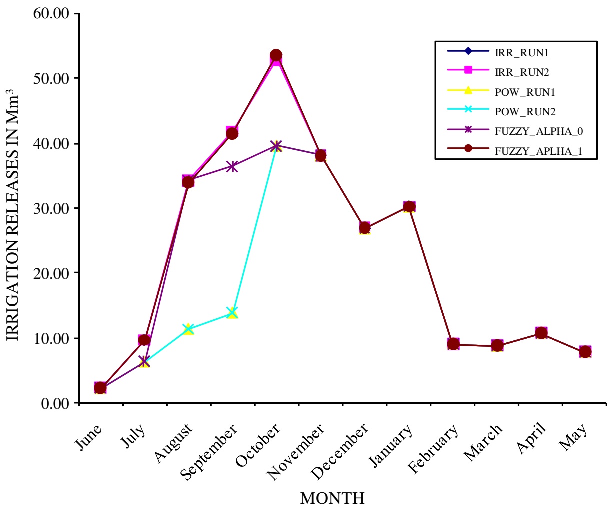


Fig. 5 Irrigation releases for Jayakwadi stage-II (R_2)

Fig. 6 Irrigation releases for Siddheshwar (R_4)

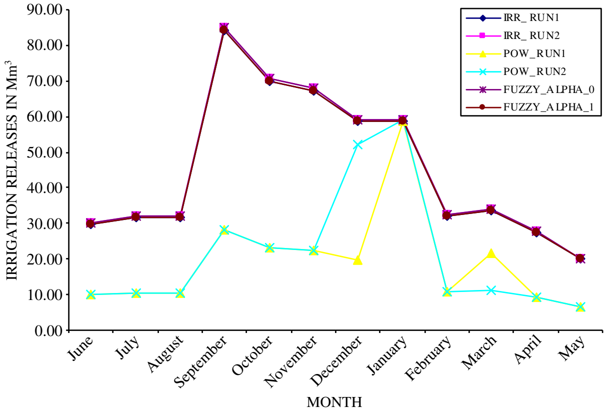


Fig. 7 Power releases for Jayakwadi stage-I (R_1)

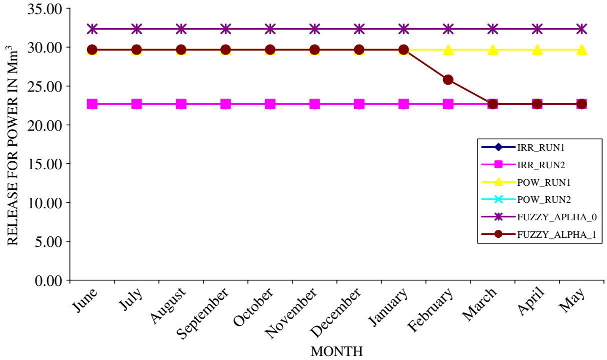
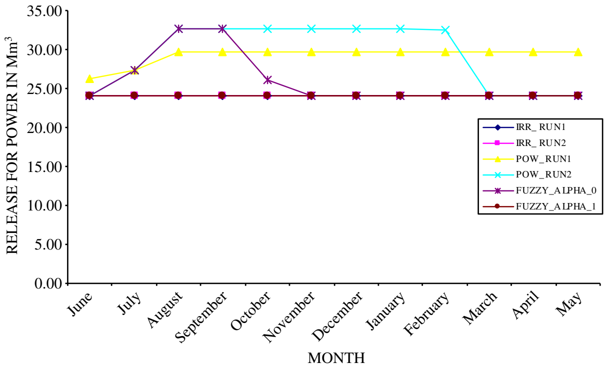


Fig. 8 Power releases for Jayakwadi stage-II (R_2)



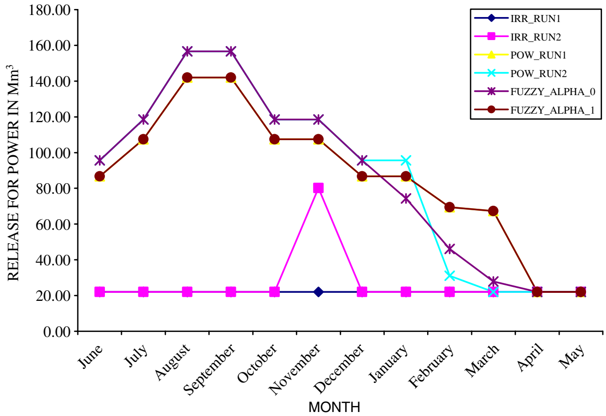


Fig. 9 Power releases for Yeldari (R_3)

that for power is 3,502.622. The irrigation return value is 3,078.193 Mm^3 and power return value is 1,666.537 when $\alpha = 1$.

Comparing Tables 2 and 3 it can be seen that the redistribution of releases, both for irrigation and hydropower is being occurred for the different values of α. The power releases are increased (i.e. from 1,666.53 to 1,751.30 Mm^3) when α value changes form 1 to 0 but along with a marginal reduction the irrigation releases (i.e. from 1,539.10 to 1,516.70 Mm^3) this is happened because the coefficient values in the objective function are fuzzified. It is clear form table that the power releases of reservoir R_1 are increased up its turbine capacity i.e. 32.34 Mm^3 ($\alpha = 0$), and for the reservoir R_3 the redistribution occurred in monthly releases resulting in a small change for the total at the end of the year (from 1,047.50 to 1,052.49 Mm^3). If the releases for irrigation are considered, it is clear that total irrigation releases for reservoir R_1 and R_2 are reduced. But the case is different for R_4 as its releases increased. The reason to validate this is that the R_3 and R_4 are in series and the power release for R_3 is considered to be an inflow to the reservoir R_4. As power release in the reservoir R_3 is increased, it has effect in irrigation releases on R_4. Thus policies for each model for different values of α are studied. The fluctuations in the releases are showed graphical form in Figs. 4, 5, 6, 7, 8 and 9. The graphs show that the irrigation releases in all reservoirs follow the almost same trends in all months. But the power releases in reservoir R_3 have different trend from other two reservoirs.

5 Conclusion

The results show that as the feasibility value goes on increasing, the corresponding satisfaction level goes on decreasing. When the feasibility is lowest i.e. $\alpha = 0$, the satisfaction level is highest i.e. $\lambda = 0.96994$ and for highest feasibility i.e. $\alpha = 1$ the satisfaction level is at its lowest level i.e. $\lambda = 0.30507$. With the above study, it can be said that the fuzzification of various parameters in the optimization problem will give the different operation policies giving more flexibility to the policy maker (or decision maker). The adaptation of a particular policy with the associated coefficients of the optimization model from out of all the feasible and non dominated solution is at the discretion of the DM to suit to his preference structure and values associated with various issues related to the problem. Thus it can be concluded that the multi-objective fuzzy optimization has robust applications in water resources engineering in general and reservoir operation, in particular. If the policies are developed using physical terms in stead of monitory values (i.e. giving equal weightages to all the competing demands) would lead to modified solutions which need to be justified before taking decisions.

Acknowledgements The authors are thankful to Command Area Development Authority, Aurangabad, Maharashtra State, India for providing necessary data for the analysis.

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