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Short Communication

An effective non-iterative "λ-logic based" algorithm for economic dispatch of generators with cubic fuel cost function

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ABSTRACT

This paper presents a very fast and effective non-iterative "λ-logic based" algorithm for economic dispatch (ED) with cubic cost function of thermal units. Representation of generator fuel cost curves by polynomials in real-time economic dispatch is standard practice in the industry and it shows a great influence on the accuracy of the economic dispatch solution. As the proposed approach is a direct or non-iterative method, it does not demand any initial guess values for ED of units for given demand (P_D). Many of the existing conventional methods fail to impose *p*-limits at violating units and find difficult in dealing with cubic cost function that reflect nonlinearity of the actual generator response. The proposed method is very efficient for both small and large size of generating units with cubic fuel cost function in the problem and it remarkably reduces the computation time. The proposed algorithm has been tested successfully on three units, five units and 26 units and results are reported in the paper.

1. Introduction

Economic dispatch (ED) is the scheduling of generators to minimize the total operating cost depending on equality and inequality constraints. Many techniques have been applied to ED to obtain better solutions, which includes λ-iteration, gradient, Newton's method and base-point participation factor method [1–3]. In these methods, the fuel cost function is chosen to be of quadratic form. However, the fuel cost function becomes more non-linear when the actual generator response is considered [8–15]. Cubic cost function models more accurately the actual response of thermal generators [4–8]. The rough approximation of the generator cost function makes the ED solution deviated from the optimality. ED solution can be improved by introducing higher order generator cost functions. Unlike traditional algorithms, dynamic programming (DP) imposes no restrictions on the nature of the cost curves and therefore it can solve ED problems with inherently non-linear and discontinuous cost curves [5]. This method, however suffers from the "curse of dimensionality" or local optimality [11]. In this respect stochastic search algorithms such as genetic algorithm, evolution strategies, evolutionary programming and simulated annealing may prove to be very effective in solving non-linear ED problems without any restriction on the shape of the cost curves. Although these heuristic methods do not always guarantee discovering the globally optimal solution in finite time, they often provide a fast and reasonable solution (sub optimal near-globally optimal) [3]. The proposed method is very fast and effective for the third order polynomial cost function due to the 'pre-prepared data' as a big support for obtaining the solutions for any specified P_D in the range of $(P_D)_{\min}$ and $(P_D)_{\max}$ [4]. Since the unit incremental cost of cubic fuel cost function is quadratic in nature, quadratic interpolation is applied for critical cases to get an optimal solution. The proposed method produces very accurate results for cubic cost function for different cases. The method yields a true optimal solution for large-scale systems.

2. Problem formulation

The economic dispatch problem minimizes the following cost function associated with dispatchable units.

Minimize:

$$F_T = F_1(P_1) + F_2(P_2) + \cdots + F_k(P_k) \quad (1)$$

At *i*th unit

$$F_i(P_i) = a_i + b_iP_i + c_iP_i^2 + d_iP_i^3 \ \$/\text{h} \quad (2)$$

where F_T is the total fuel cost (\$/h), $F_i(P_i)$ the fuel cost of unit *i*, where $i = 1,2,\ldots,k$, a_i, b_i, c_i, d_i *the* fuel cost coefficients of unit *i*, and P_i *is the* power generation of unit *i*(MW).

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Subject to the following constraints:

(i) Equality constraints:

$$P_1 + P_2 + \cdots + P_k = P_D \quad (3)$$

where P_D power demand.

(ii) Inequality constraints:

$$P_{imin} \leqslant P_i \leqslant P_{imax} \quad (4)$$

where P_{imin} and P_{imax} is minimum and maximum, Power output of ith unit.

3. Proposed method

The proposed method for ED consists of two stages i.e., (i) pre-prepared power demand data (PPD) using λ-logic and (ii) calculation of solution $P_1, P_2, \ldots, P_k$ for specified power demand (P_D). The first step involves a systematic approach with fixed number of steps and offers unique PPD (Pre-prepared power demand data) for K-units. This PPD acts as a big source in reducing the computational burden of ED of K-units. This PPD remains unaltered for all values of P_D variations (ranging from P_{Dmin} and P_{Dmax}).

The second step consists of calculation of $P_1, P_2, \ldots, P_k$ for given P_D.

This above two steps are illustrated for $k = 5$ units case and the test results of $k = 3$ units and 26 units are also presented.

For economic dispatch of k thermal units of plant, the ED condition is

$$dF_1/dP_1 = dF_2/dP_2 = \cdots = dF_k/dP_k = \lambda \quad (5)$$

$$F_i/dP_i = \lambda_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \,\$/\text{MWh} \quad (6)$$

where λ_i is incremental fuel cost \$/MWh, $\alpha_i = b_i$, $\beta_i = 2^* c_i$, $\gamma_i = 3^* d_i$ coefficients of incremental cost function of unit i.

This Eq. (6) gives the value of incremental fuel cost with respect to output power P_i. Then P_i is calculated by using

$$P_i = (-\beta_i \pm \sqrt{(\beta_i^2 - 4^*(\gamma_i)(\alpha_i - \lambda)))}/(2^*\gamma_i) \quad (7)$$

Data of five units [3]:
Fuel cost functions in \$/h

$$F_1(P_1) = 749.55 + 6.950P_1 + 9.680E - 4P_1^2 + 1.270E - 7P_1^3$$

$$F_2(P_2) = 1285.0 + 7.051P_2 + 7.375E - 4P_2^2 + 6.453E - 8P_2^3$$

$$F_3(P_3) = 1531.0 + 6.531P_3 + 1.040E - 3P_3^2 + 9.980E - 8P_3^3$$

$$F_4(P_4) = 749.55 + 6.950P_4 + 9.680E - 4P_4^2 + 1.270E - 7P_4^3$$

$$F_5(P_5) = 1285.0 + 7.051P_5 + 7.375E - 4P_5^2 + 6.453E - 8P_5^3$$

$$\text{dF}_1/\text{dP}_1 = 6.950 + 1.936E - 4P_1 + 3.81E - 7P_1^2 \ \$/\text{MWh}$$
$$= \alpha_1 + \beta_1 P_1 + \gamma_1 P_1^2 \ 320 \leqslant P_1 \leqslant 800 \text{ MW}$$

$$\text{dF}_2/\text{dP}_2 = 7.051 + 1.475P2 + 1.9359E - 7P_2^2 \ \$/\text{MWh}$$
$$= \alpha_2 + \beta_2 P_2 + \gamma_2 P_2^2 \ 300 \leqslant P + 2 \leqslant 1200 \text{ MW}$$

$$\text{dF}_3/\text{dP}_3 = 6.531 + 2.08E - 3P_3 + 2.994E - 7P_3^2 \ \$/\text{MWh}$$
$$= \alpha_3 + \beta_3 P_3 + \gamma_3 P_3^2 \ 275 \leqslant P_3 \leqslant 1100 \text{ MW}$$

$$\text{dF}_4/\text{dP}_4 = 6.950 + 1.936E - 4P4 + 3.81E - 7P_4^2 \ \$/\text{MWh}$$
$$= \alpha_4 + \beta_4 P_4 + \gamma_4 P_4^2 \ 320 \leqslant P_4 \leqslant 800 \text{ MW}$$

$$\text{dF}_5/\text{dP}_5 = 7.051 + 1.475P_5 + 1.9359E - 7P_5^2 \ \$/\text{MWh}$$
$$= \alpha_5 + \beta_5 P_5 + \gamma_5 P_5^2 \ 300 \leqslant P_5 \leqslant 1200 \text{ MW}$$

Steps for pre-prepared power demand data (PPD):
Step (a) Calculate

$$\lambda_{imin} = (dF_i/dP_i) \quad \text{at } P_i = P_{imin} \quad (8)$$

$$\lambda_{imax} = (dF_i/\text{d}P_i) \quad \text{at } P_i = P_{imax} \quad (9)$$

$i = 1$–$5(=k)$ units

3.1. Incremental cost for five units data

Unit = i	P_{limits}	λ_i
1	$P_{1min} = 320$	7.608534
1	$P_{1max} = 800$	8.74264
2	$P_{2min} = 300$	7.510923
2	$P_{2max} = 1200$	9.09977
3	$P_{3min} = 275$	7.125642
3	$P_{3max} = 1100$	9.181274
4	$P_{4min} = 320$	7.608534
4	$P_{4max} = 800$	8.74264
5	$P_{5min} = 300$	7.510923
5	$P_{5max} = 1200$	9.09977

Step (b) Arrange λ-vector in ascending order as below

3.2. Pre-prepared power demand data (PPD)

J = S. No.	$\lambda_{asc} = \lambda$-vector	Unit index vector	Total power = PPD (i) = $\sum P_i$ (MW)
1.	7.125642	3	1515.00
2.	7.510923	2	1682.883231
3.	7.510923	5	1682.883231
4.	7.608534	1	1846.076284
5.	7.608534	4	1846.076284
6.	8.74264	1	4561.654519
7.	8.74264	5	4561.654519
8.	9.09977	4	5070.142071
9.	9.09977	2	5070.142071
10.	9.181274	3	5100.00

Step (c) Now calculate total power demand at each λ_j of the λ_{asc} – vector. For this, the following fundamental observation of ED condition is used.

$$\text{If } \lambda_J \leqslant \lambda_{imin} \quad \text{then, } P_i = P_{imin} \quad (10)$$

$$\text{If } \lambda_J \geq \lambda_{imax} \quad \text{then, } P_i = P_{imax} \quad (11)$$

and for $\lambda_{imin} < \lambda_J < \lambda_{imax}$

$$\text{the } P_i = (-\beta_i \pm \sqrt{(\beta_i^2 - 4 * (\gamma_i)(\alpha_{i-}\lambda_J)))}/(2 * \gamma_i) \quad (12)$$

Thus the pre-prepared power demand data is formed and these calculations of the PPD(J) are performed only once and tabulated against corresponding values of incremental fuel cost. This information can be treated as static system data and can be supplied as input data along with the coefficients of fuel cost functions.

A close observation of λ_{asc} vector of step-b clearly indicates that unit-1 is having lower incremental cost compared to units two and three. Thus, unit-1 governs the λ-value till the demand reaches to 1682.883231 MW from 1515.00 MW. This implies that the graph between any two entries of λ_{asc} and PPD(J) is assumed to be a quadratic nature. We can calculate the slope between any two intervals and the λ_{new} corresponding P_{Dnew} which lies between PPD(J) and PPD($J + 1$) is given by

$$\lambda_{new} = (m)(\Delta P_D) + \lambda_{asc}(J) \quad (13)$$

Table 1
Cost function coefficients of three units system [1].

Unit no.	1	2	3
a_i	749.55	1285	1531
b_i	6.95	7.05	6.531
c_i	9.68e−4	7.375e−4	1.04e−3
d_i	1.27e−7	6.453e−8	9.98e−8
P_{imin}	320	300	275
P_{imax}	800	1200	1100

Table 2
Results for three units system.

Power demand	P_1	P_2	P_3	Computation time
P_D = 1100, λ = 7.54633	320.00	322.10	457.89	0.012
P_D = 1300.00, λ = 7.70505	361.21	416.56	522.28	0.012
P_D = 3000, λ = 9.02169	800.00	1159.04	1040.90	0.014

Table 3
Cost function coeffiecients of five units system [3].

Unit no.	1	2	3	4	5
a_i	749.55	1285	1531	749.55	1285
b_i	6.95	7.05	6.531	6.95	7.05
c_i	9.68e−4	7.375e−4	1.04e−3	9.68e−4	7.375e−4
d_i	1.27e−7	6.453e−8	9.98e−8	1.27e−7	6.453e−8
P_{imin}	320	300	275	320	300
P_{imax}	800	1200	1100	800	1200

Table 4
Results for five units system.

Power demand	P_D = 5000, λ = 9.05050	P_D = 2500, λ = 7.881632	P_D = 1800, λ = 7.88163
P_1	800	436.27159	320
P_2	1159.04	518.085815	343.708776
P_3	1040.96	591.938266	472.583547
P_4	800	436.27159	320
P_5	1174.14712	518.085815	343.708776
Computation time (s)	0.016	0.016	0.015

The steps a–c completes the first step of the proposed λ-logic method, i.e., pre-prepared power demand data (=PPD(J)). This part can be done off-line and $\lambda_{asc}(J)$, PPD(J) and slope $m(J)$ vectors would remain as static data [4].

Step (ii) Estimation of $P_1, P_2, \ldots, P_k$ for specified P_D

Say P_D = 1800 MW = P_{Dnew}
Scan PPD(J) and identify the interval J and J + 1 corresponding to P_{Dnew}.

$$J = 3;\quad J+1 = 4;$$
$$\lambda_{asc}(J) = 7.510923,\quad \mathrm{PPD}(J) = 1682.883231\mathrm{MW},\quad P_{Dnew} = 1800\mathrm{MW}.$$
$$\Delta P_D = 1800 - 1682.883231 = 117.11677;$$
$$m = (\lambda(j+1) - \lambda(J))/(\mathrm{PPD}(J+1) - \mathrm{PPD}(J)) = 5.9813e-04;$$

Using this λ_{new} from (13) and equations (10)–(12), the values of P_1, P_2, P_3, P_4 and P_5 are P_1 = 320.00 MW, P_2 = 343.708776 MW, P_3 = 472.583547 MW, P_4 = 320.00 MW, P_5 = 343.708776 MW, P_D = 1800.001 MW. If any mismatch is observed between Pgen and P_D, then repeat similar step with quadratic interpolation.

Table 5
Cost function coefficients of 26 units system [2].

Unit	*a*	*b*	*c*	*d*
1	24.38	25.54	0.025	5.08e−9
2	24.41	25.67	0.026	−1.01e−8
3	24.63	25.80	0.028	1.01e−8
4	24.76	25.93	0.028	−5.08e−9
5	24.88	26.06	0.028	−5.72e−16
6	117.75	37.55	0.011	8.31e−8
7	118.10	37.66	0.012	8.56e−8
8	118.45	37.77	0.013	8.15e−8
9	118.82	37.88	0.014	8.29e−8
10	81.13	13.32	0.008	−5.80e−10
11	81.29	13.35	0.008	−5.47e−10
12	81.46	13.38	0.009	−5.49e−10
13	81.62	13.40	0.009	−5.50e−10
14	217.89	18.00	0.006	1.25e−18
15	218.33	18.09	0.006	−1.19e−18
16	218.77	18.20	0.005	2.44e−18
17	142.73	10.69	0.004	1.11e−10
18	143.02	10.71	0.004	1.03e−10
19	143.31	10.73	0.004	1.03e−10
20	143.59	10.75	0.004	1.03e−10
21	259.13	23.00	0.002	1.07e−10
22	259.64	23.10	0.002	1.04e−10
23	260.17	23.20	0.002	1.00e−10
24	177.05	10.86	0.001	−4.42e−19
25	310.00	7.49	0.001	−1.10e−19
26	311.91	7.50	0.001	−3.55e−20

Table 6
Minimum and maximum limits of 26 units power and Incremental cost.

P_{min}	P_{max}	λ_{min}	λ_{max}
2.4	12.0	25.668784	26.155122
2.4	12.0	25.802452	26.311056
2.4	12.0	25.937148	26.474944
2.4	12.0	26.068216	26.613878
2.4	12.0	26.19814	26.7463
4	20.0	37.646924	38.0307
4	20.0	37.764584	38.168203
4	20.0	37.885724	38.320698
4	20.0	38.004244	38.462899
15.2	76	13.593504	14.65871
15.2	76	13.62588	14.714191
15.2	76	13.65714	14.76369
15.2	76	13.690628	14.82393
25	100	18.3115	19.246
25	100	18.406	19.324
25	100	18.499	19.396
54.25	155	11.196356	12.129308
54.25	155	11.228606	12.181707
54.25	155	11.258586	12.227807
54.25	155	11.286696	12.268007
68.95	197	23.357163	24.020472
68.95	197	23.458541	24.124412
68.95	197	23.562678	24.236232
140	350	11.29	11.9326
100	400	7.8801	9.0441
100	400	7.8931	9.0631

Similarly for any other P_D in the range of $1515 \leqslant P_D \leqslant 5100$, P_1, P_2, P_3, P_4 and P_5 can be calculated very easily and quickly with very easily and quickly with very low computation burden.

4. Results

The above algorithm is tested on three units [1], five units [3] and 26 units [2] test system data. The results of three units, five units and 26 units is tabulated in Tables 1–7

Table 7
Results for 26 units system with various demands.

P_D = 2600, λ = 19.307086	P_D = 2400, λ = 18.45908	P_D = 2200, λ = 13.90887	P_D = 2000, λ = 11.895186
2.4	2.4	2.4	2.4
2.4	2.4	2.4	2.4
2.4	2.4	2.4	2.4
2.4	2.4	2.4	2.4
2.4	2.4	2.4	2.4
4	4	4	4
4	4	4	4
4	4	4	4
4	4	4	4
76	76	33.200466	15.2
76	76	31.009599	15.2
76	76	29.031409	15.2
76	76	26.908339	15.2
100	36.776138	25	25
98.641139	29.397975	25	25
92.542284	25	25	25
155	155	155	129.705623
155	155	155	124.70136
155	155	155	120.413222
155	155	155	116.71206
68.95	68.95	68.95	68.95
68.95	68.95	68.95	68.95
68.95	68.95	68.95	68.95
350	350	350	337.688912
400	400	400	400
400	400	400	400
Computation time (s)			
0.021	0.020	0.018	0.018

5. Conclusion

The proposed λ-logic method offers a best contribution in the area of economic dispatch. In contrast to other conventional methods, this approach gives a promising value of power for providing improved economic dispatch. In this paper, it has two stages. At first stage, PPD is to be prepared only once and at second stage, power generation of each unit is calculated for specified P_D from Eq. (13). Since the Incremental cost curve is quadratic in nature, quadratic interpolation is used to determine the optimal generation for special cases. It reduces the computational burden and is superior to many of available techniques of economic dispatch. The method yields a true optimal solution for large-scale systems.

As power systems are usually large-scale systems, λ-logic method may be suggested for the solution for ED problems. It offers a better convergence rate, minimum cost to be achieved, and for better solutions since the PPD helps the Starting Location of Incremental Cost whereas the other techniques have to find iteratively from an unknown starting point.

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