Root finding techniques for economic dispatch problems

Conference Paper · August 2008
DOI: 10.1109/PES.2008.4596649 · Source: IEEE Xplore

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Root Finding Techniques for Economic Dispatch Problems

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Abstract-- **This paper describes the application of Secant method for solving Economic Dispatch (ED) problem with generator constraints and transmission losses. The proposed algorithm involves the selection of minimum and maximum incremental fuel costs (lambda values) and then the evaluation of optimal lambda is done by root finding techniques at required power demand. The proposed algorithm has been tested on a power system having 6, 15,26 and 40 generating units. The extensive studies have been made on the proposed method to solve the ED problem by taking 120 and 200 units with generator constraints. Simulation results of the proposed approach were compared in terms of solution quality, convergence characteristics and computation efficiency with conventional methods such as lambda iterative method, heuristic methods such as genetic algorithm and meta-heuristic methods such as particle swarm optimization. From the different examples, it is observed that the proposed method provides qualitative solution with less computational time compare to various methods available in the literature survey.**

Key words- **Secant method, Economic dispatch, Transmission losses and Quadratic fuel cost function.**

NOMINCLATURE

$F_i(P_i)$	Generator fuel cost of i^{th} generating unit
λ_i	Incremental fuel cost of i^{th} generating unit
F_t	Total Generation cost
P_i	Real power output of i^{th} generating unit
P_D	Power Demand
P_L	Transmission losses
ng	Number of generating units.
P_i^{min}	Minimum real output power of i^{th} generating unit
P_i^{max}	Maximum real output power of i^{th} generating unit
B_{ij}, B_{0i}, B_{00}	Loss coefficient matrices
$P_i^o =$	Output powers of generators at initial state
UR_i	Up ramp rate limit
DR_i	Down ramp rate limit

I. INTRODUCTION

The main objective of Economic Dispatch (ED) problem is to determine the optimal schedule of online generating units so as to meet the power demand at minimum operating cost under various system and operating constraints. This problem is a multi modal, discontinuous and highly non-linear problem due to the valve point loading, ramp rate limits and prohibited zones [1],[2],[3]. The fuel cost function of each generating unit is approximately represented by a quadratic function [4]. The major problem associated with the quadratic representation is that the model under estimates the fuel cost when the generation increases and overestimates the fuel cost when the generation decreases [5]. Accuracy of the fuel cost function is a great influence on the cost minimization because it is a major cost component. Reducing the fuel cost by little as 0.5% can result in saving of million dollars per year for large scale utilities. Therefore, the solution of the ED problem can be improved by introducing cubic cost function [6]. It is clearly mentioned in [6] that the cubic cost function gives less error than the quadratic function. In contrast, minimization of the fuel cost of the power generation depends on efficiency of the generating unit, fuel cost and minimization of transmission loss [7]. It is necessary to consider the incremental transmission loss for the optimal economic dispatch. The present day power systems are largely interconnected and operated in a deregulated environment. The main constraint in a deregulated environment is that the system is expected to operate with little reserve capacity.

Earlier, many conventional optimization techniques such as lambda iteration method, lambda projection method, base point and participation factors method and gradient methods [8] were used for solving ED problems. In these methods, the essential assumption is that the fuel cost curve is a quadratic function and the incremental fuel cost is monotonically increasing piecewise linear function. The main drawback of the above methods is the increase of computational time with increase of system size. In real time power system operation, the incremental fuel cost may not always monotonically increase. To overcome the above difficulty, dynamic programming [9] has been used for solving the ED problem with monotonically increased and decreased fuel cost functions as it will not impose any restrictions on the nature of the cost curve. However, these methods suffer from problem of increase of computational time with increased dimensionality. Thus, this method is not suitable for online application of ED problem. In order to get the qualitative solution for ED problem, artificial neural network techniques such as the back propagation (BP) algorithm based neural network [10] and the Hopfield neural network [11], [12] have been successfully employed for thermal units with piecewise quadratic function and prohibited zone constraints [13]. The BP algorithm takes more iterations due to improper selection of learning and momentum rates. Similarly the Hopfield model suffers from excessive iterations due to an unsuitable sigmoid function [14] as a result of which it takes more time to give optimal solution at required power demand. In the past decade, a global optimization technique known as a genetic algorithm has been used to solve the ED problem with a quadratic, piece wise quadratic fuel cost function and valve point loading [15]. It is a parallel search technique, which imitates natural genetic operation. Due to its high potential for global optimization, GA has received great attention in solving ED problems with a quadratic and piecewise quadratic cost function and valve point loading including network losses, ramp rates and prohibited zones [16]. But recent research has identified some deficiencies in GA performance as the cross over and mutation operations cannot ensure the better fitness of offspring because the chromosomes in the population have similar structures and their average fitness is high towards the end of the evolutionary process [17]. Recently, meta-heuristic techniques such as the evolutionary programming (EP) [19],particle swarm optimization (PSO) [20], ant colony searching algorithm (ACSA) [21] and the tabu search algorithm (TSA) [22] have been given much attention by many researchers due to their ability in getting the near global optimal solution. In these methods, the quality of solution depends on user defined factors. The improper selection of these factors may increase the computational time for getting optimal solution.

The main aim of the present paper is to develop a new method for solving the ED problem with generator constraints and transmission losses by employing root finding techniques. It is observed from the literature available for solution of ED problems that most of the existing methods have some limitations to get optimal solution with reduced computational burden. In the present approach the qualitative solution can be achieved with less computational time and it converges within a few iterations. Furthermore, the proposed method will provide a good qualitative solution than many existing algorithms including the evolutionary programming [18], the Hopfield neural network, the genetic algorithm, the lambda iterative method and the particle swarm optimization method. This paper presents root finding techniques to solve ED problem with the generator constraints and transmission losses. The proposed new algorithm has been implemented in MATLAB on a Pentium IV, 1.4 GHz personal computer with 256-MB RAM. The remaining paper is organized into three parts. Formulation of ED problem is introduced in Section II. The description of root finding techniques to solve ED problems is given in Section III. Case studies with various number of generator units are presented in Section IV. Conclusions are finally presented in the last section.

II. ECONOMIC DISPATCH PROBLEM

The ED problem is a non-linear programming optimization problem. The main objective is to minimize the total fuel cost at thermal plants subjected to the operating constraints of power system.

Considering the fuel cost function as a quadratic function of real power generation, ED problem can be formulated as follows,

$$F_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{1}$$

The objective function is

$$\text{Minimize } F_t = \sum_{i=1}^{ng} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{2}$$

subjected to

A) Equality constraint

It is given by the power balance equation

$$\sum_{i=1}^{ng} P_i = P_D + P_L \tag{3}$$

Where the total transmission loss is assumed as a quadratic function of the generator power outputs and given by

$$P_L = \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} P_i B_{ij} P_j + B_{i0} P_j + B_{00} \tag{4}$$

B) Inequality constraints

1) Generator constraints Generator constraints are given by minimum and maximum of output power of each generating unit.

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{5}$$

2) Prohibited zones Prohibited operating zones in the fuel cost is either due to vibrations in the shaft bearing caused by a steam valve or due to the associated auxiliary equipment such as a boiler, feed pumps. In practice, the determination of the shape of the fuel cost curve in the neighborhood of a prohibited zone is difficult. Therefore the best economy achieved by avoiding operation of generating units in these areas. The fuel cost curve with prohibited zones is shown in Fig 1.

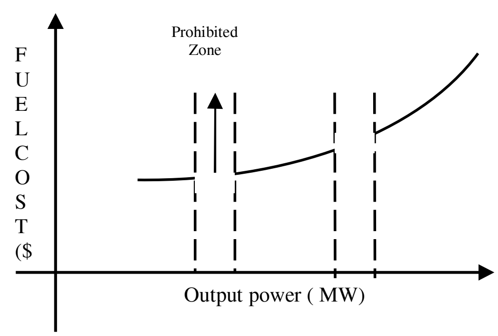

Fig.1 Fuel cost curve with prohibited operated zones

Prohibited zones divide the operating region between minimum and maximum generation limits into disjoint convex sub regions. The generation limits for units with prohibited zones are

$$P_i^{\min} \le P_i \le P_{i,1}^d$$

$$P_{i,j-1}^u \le P_i \le P_{i,j}^d \;,\; j = 2,3,....n_i$$

$$P_{i,n_i}^u \le P_i \le P_i^{\max} \tag{6}$$

3) Ramp rate limits The range of actual operation of online generating unit is restricted by its ramp rate limits. These limits can impact power system operation. The operational decision at the present hour may affect the operational decision at the later hour due to ramp rate limits. In actual operation, three possible situations exist due to variation in power demand from present hour to next hour. First, during the steady state operation, the operation of the online unit is in steady state condition. Second, the power demand increased, the power generation of the generating unit is also increases. Third, if the power demand decreased then the power generation of the generating unit also decreases. The ramp rate limits curves are shown in Fig.2.

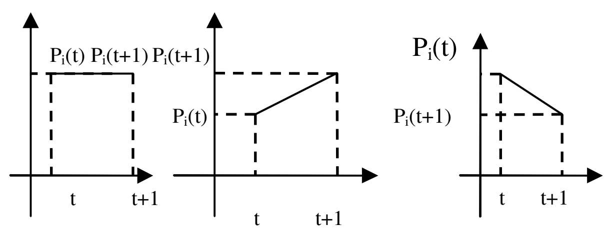

Fig 2 Ramp rate limits of the generating units

Inequality constraints due to ramp rate limits of generating units are given as

A. when generation increases

$$P_i - P_i^0 \le UR_i \quad (7)$$

B. when generation decreases

$$P_i^0 - P_i \le DR_i \quad (8)$$

Finally generator constraints can be modified as

$$\max(P_i^{\min}, P_{i,t}^o - DR_i) \le P_{i,t} \le \min(P_{i,t}^o + UR_i) \quad (9)$$

$i = 1,2,..\ to\ ng \quad$ t=1,2,...T

The formulation of Lagrange function for the ED problem and given by

$$\chi = F_T + \lambda \times \left(P_D + P_L - \sum_{i=1}^{ng} P_i \right) \quad (10)$$

The expressions of lambda and output power are

$$\lambda_i = \frac{\beta_i + (2 \times \gamma_i \times P_i)}{1 - \left(2 \times \sum_{i=1}^{ng} B_{ij} P_j + B_{i0} \right)} \quad (11)$$

$$P_i = \frac{\lambda_i \times \left(1 - B_{i0} - 2 \times \sum_{\substack{j=1 \\ i \ne j}}^{ng} B_{ij} P_j \right) - \beta_i}{2 \times (\gamma_i + \lambda_i B_{ii})} \quad (12)$$

Where

λ_i is incremental fuel cost of i^{th} generating unit

III. ROOT FINDING TECHNIQUES

In numerical methods, non linear equations with one variable are solved by root finding methods. A brief introduction has been presented in this section about root finding methods such as Secant, Muller and Brent method.

A. Secant method [24,25] It is a root finding algorithm that uses succession of roots of secant lines to better approximate the root of a function. This method assumes that the function is approximately linear in the local region of interest and uses the zero crossing over the line connecting the limits of the interval as the new reference point. The next iteration starts from evaluating the function at the new reference point and then forms another line. The process is repeated until the root is found. Geometrically, Newton method uses the tangent line and secant method approximates the tangent line by secant line. It has super linear convergence and also will converge within five iterations if the guess value is correct. The graphical representation of the secant method is shown in Fig.3.

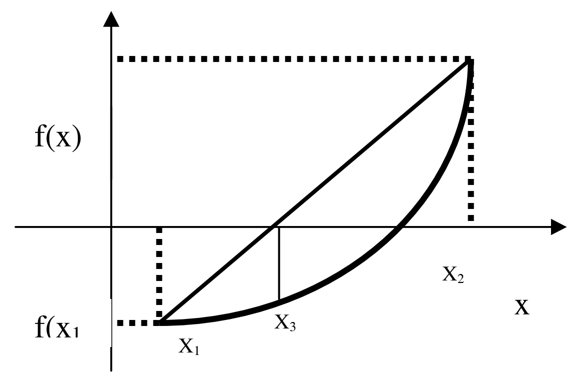

Fig.3 Graphical representation of the Secant method

In general, the guessed value is calculated from the two previous points $[x_{k-1}, f(x_{k-1})]$ and $[x_k, f(x_k)]$ as

$$x_{K+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k) \quad (13)$$

B. Muller method [26,27] It is a root finding algorithm for solving equation of the form of f(x) =0 where f(x) is a non linear function of x. It was presented by D.E. Muller in 1956. It is based on the secant method and used to find the root of the f(x) =0 when no information about the derivative exists. In this method, three points are used to find an interpolating quadratic polynomial. A parabola is constructed passing though these three points and then the quadratic formula is used to find a root of the quadratic for the next approximation. In the Muller method, higher order polynomial is approximated by a quadratic curve in the vicinity of a root. The roots of quadratic equation are then assumed to be approximately equal to be the roots of the equation f(x)=0. This method is iterative and converges almost quadratically. It has been proved that near a simple root Muller method converges faster than the secant and Newton methods. The graphical representation of the Muller method is shown in Fig. 4.

Let x_{i-2}, x_{i-1}, x_i are three distinct approximations to a root of $f(x) = 0$ and y_{i-2}, y_{i-1} *and* y_i are the corresponding values of $y = f(x)$. The relation between y and x can be represented by

$$y = A(x - x_i)^2 + B(x - x_i) + y_i \quad (14)$$

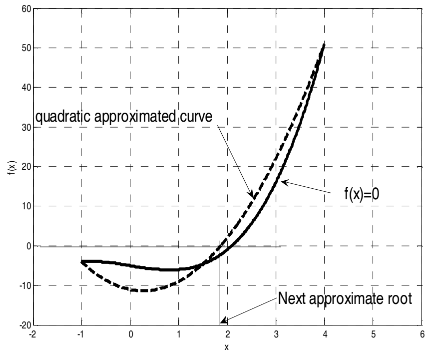

Fig. 4 Graphical representation of Muller method

Where

$$A = \frac{(x_{i-2} - x_{i-1})(y_{i-1} - y_i) - (x_{i-1} - x_i)(y_{i-2} - y_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)(x_{i-2} - x_i)} \tag{15}$$

$$B = \frac{(x_{i-2} - x_i)^2(y_{i-1} - y_i) - (x_{i-1} - x_i)^2(y_{i-2} - y_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)(x_{i-2} - x_i)} \tag{16}$$

$$x_{i-1}^{(1)} = x_{i-1}^{(0)} - \frac{2 \cdot y_i}{B \pm \sqrt{B^2 - 4Ay_i}} \tag{17}$$

The sign in the denominator should be chosen properly so as to make the denominator largest in magnitude. With this choice, equation (17) gives the next approximation to the root.

C. Brent method [28] It is a root finding method, which combines root bracketing, bisection and inverse quadratic interpolation. It uses a Lagrange interpolation polynomial of second degree. Brent claims that this method will always converge as long as the values of the function are computable within a given region containing a root.

From the three points x_1, x_2 and x_3, Brent method fits x as a quadratic function of y, then from the interpolation formula, the relation between the x and y are obtained as follows,

$$x = \frac{(y - f(x_1))(y - f(x_2))x_3}{(f(x_3) - f(x_1))(f(x_3) - f(x_2))} + \frac{(y - f(x_2))(y - f(x_3))x_1}{(f(x_1) - f(x_2))(f(x_1) - f(x_3))} + \frac{(y - f(x_3))(y - f(x_1))x_2}{(f(x_2) - f(x_3))(f(x_2) - f(x_1))} \tag{18}$$

Subsequent estimation of root is obtained by setting y=0

$$x = x_2 + \frac{P}{Q} \tag{19}$$

Where

$$P = S[T(R-T)(x_3 - x_2)] - [(1-R)(x_2 - x_1)] \tag{20}$$

$$Q = (T-1)(R-1)(S-1) \tag{21}$$

With

$$R = \frac{f(x_2)}{f(x_3)} \tag{22}$$

$$S = \frac{f(x_2)}{f(x_1)} \tag{23}$$

$$T = \frac{f(x_1)}{f(x_3)} \tag{24}$$

IV. IMPLEMENTATION OF THE PROPOOSED APPROACHES FOR ED PROBLEM

Two steps are involved in the proposed approaches.

A. Selection of the lambda values.

B. At required power demand, optimal lambda is evaluated by root finding techniques.

A. Selection of the Lambda values The selection of lambda values is as follows,

(i) From the equation (5), lambda values are evaluated at minimum and maximum output powers of all generating units.

(ii) All the lambda values are arranged in ascending order and then minimum and maximum values of lambda are selected.

B. Root finding techniques

In ED problem, optimal allocation of generating units is based on equal lambda criteria. The power balance equation can be written as a function of lambda. Therefore

$$f(\lambda) = \sum_{i=1}^{cu} P_i(\lambda) - P_D = 0 \tag{25}$$

From eqn(10), it is observed that $f(\lambda)$ is non linear in terms of lambda. Root finding methods available in numerical methods can solve non linear equation with single variable. Here three methods have been proposed for solving ED problem with cubic cost function.

1) Implementation of secant method

The values of $x_{k-1}, x_k, f(x_{k-1})$ and $f(x_k)$ are selected as follows

$$x_{k-1} = \lambda_{\min} \ \& \ f(x_{k-1}) = \sum_{i=1}^{ng} P_i(\lambda_{\min}) - PD_t \tag{26}$$

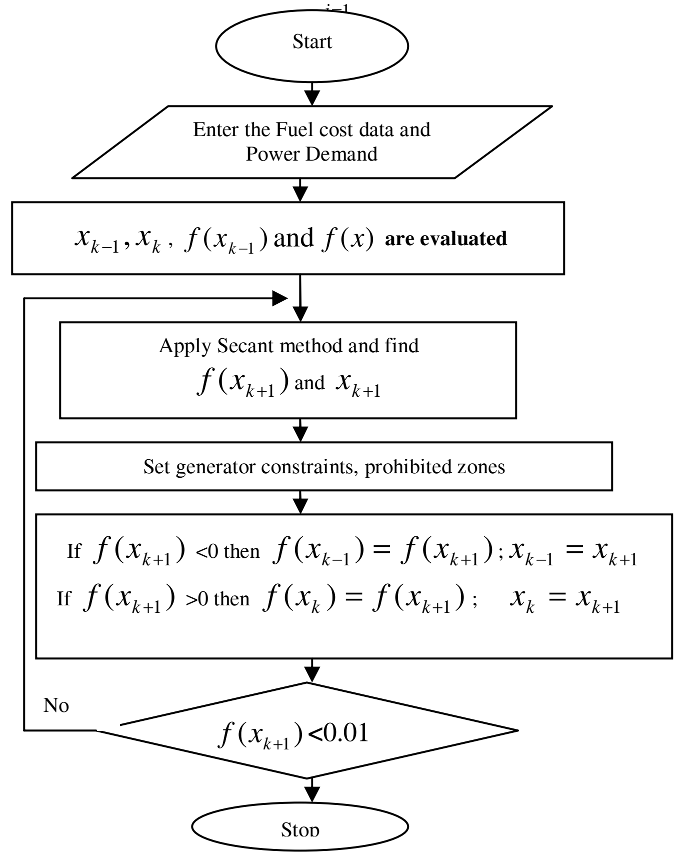

Fig.5 Flow chart of Secant method for solving ED problems

$$x_k = \lambda_{\max} \;\&\; f(x_k) = \sum_{i=1}^{ng} P_i(\lambda_{\max}) - PD_t \tag{27}$$

From (13), optimal lambda value is evaluated by secant method at required power demand.

2) Implementation of Muller method The values of $x_{k-2}, x_k, f(x_{k-2})$ and $f(x_k)$ are selected as follows

$$x_{k-2} = \lambda_{\min} \;\&\; f(x_{k-2}) = \sum_{i=1}^{ng} P_i(\lambda_{\min}) - P_D \tag{28}$$

$$x_k = \lambda_{\max} \;\&\; f(x_k) = \sum_{i=1}^{ng} P_i(\lambda_{\max}) - P_D \tag{29}$$

$$x_{i-1} = (x_{i-2} + x_i)/2$$

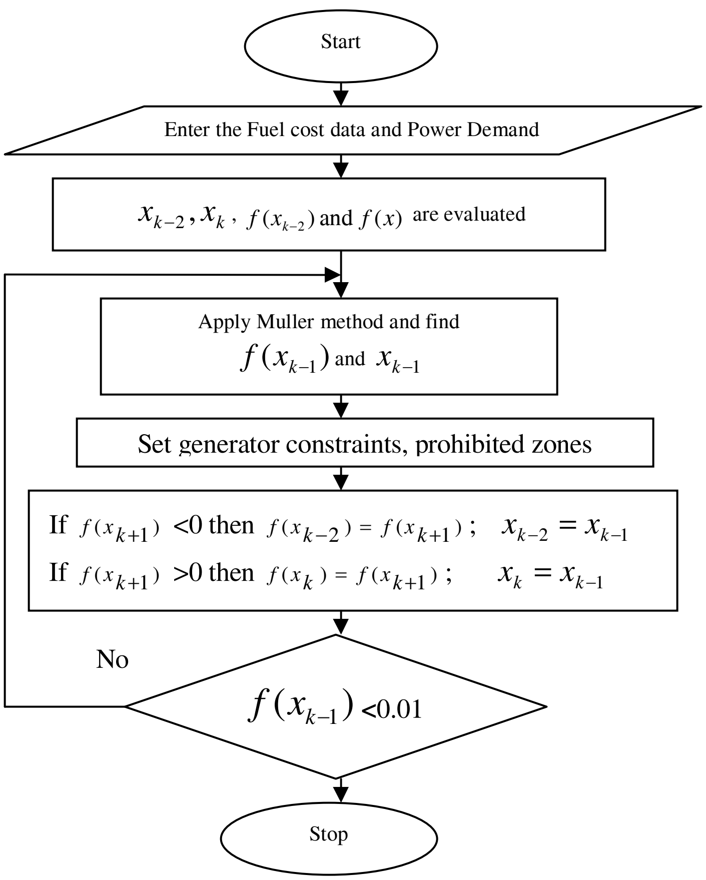

Fig.6 Flow chart of Muller method for ED problems

From (17), the optimal lambda is evaluated by an iterative approach.

3) Implementation of Brent method

In Brent method, at required power demand, the best two lambda values are obtained from PPD table.

PPD[9](Pre-prepared Power Demand) Table: The output powers are computed for all values of lambda. All lambda values, output powers, Sum of Output Powers(SOP) are formulated as a table. This table is called PPD table.

RPPD (Reduced Pre-prepared Power Demand) Table: At required power demand, the upper and lower rows of the PPD table are selected such that the power demand lies within the SOP limits and these two rows are formulated as a table and is known as Reduced PPD(RPPD) table.

At the required power demand,

$$x_1 = \lambda_j \text{ and } f(x_1) = SOP_j - \mathrm{P_D} \tag{30}$$

$$x_3 = \lambda_{j+1} \text{ and } f(x_3) = SOP_{j+1} - \mathrm{P_D} \tag{31}$$

$$x_2 = (\lambda_j + \lambda_{j+1})/2 \tag{32}$$

At x_2, $f(x_2)$ value is evaluated and finally from (19), the optimal lambda value is evaluated by an iterative approach.

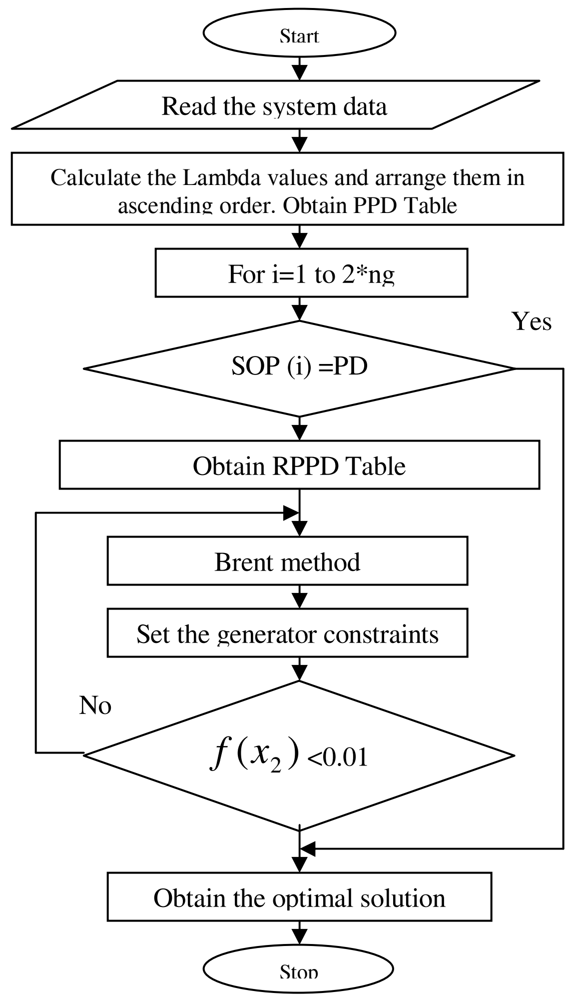

Fig.7 Flow chart of Brent method for ED problems

V. SIMULATION RESULTS

The proposed algorithms were implemented in MATLAB and executed on Pentium III, 550 MHz personal computer with 256 MB RAM to solve the ED problem of a power system having 6, 15,26 and 40 generating units with generator constraints and transmission losses. The simulation results obtained from the proposed method were compared in terms of the solution quality and computation efficiency with the conventional lambda iterative method, heuristic method such as genetic algorithm and meta-heuristic method such as particle swarm optimization.

During the execution of conventional lambda iterative method, the initial lambda is selected such that the lowest lambda value among all lambda values of the generating units at their minimum and maximum output powers. While executing the lambda iterative method and proposed methods, lambda was taken as control parameter.

A. *Examples*

*Example-**1*** In this example, the system contains six thermal units. The fuel cost data of six thermal units is extracted from [20] and given in TABLE-I.

TABLE I

FUEL COST DATA OF SIX UNITS SYSTEM

Unit	α_i (\$)	β_i (\$/MW)	γ_i (\$/MW^2)	$P_{\min}$ (MW)	$P_{\max}$ (MW)
1	240	7	0.007	100	500
2	200	10	0.0095	50	200
3	220	8.5	0.009	80	300
4	200	11	0.009	50	150
5	220	10.5	0.008	50	200
6	190	12	0.0075	50	120

In this example, the result of the proposed methods is compared with lambda iterative method. The procedure of the proposed methods for solving ED problem is as follows. Initially the values of lambda are computed for all generating units at their minimum and maximum output powers then arranged in ascending order and finally minimum and maximum lambda values are selected. The output powers are evaluated at these lambda values by incorporating the generating limits and also the values of $f(x_{k-1}), f(x)$ are computed at the given power demand. Finally the optimal lambda value is evaluated by root finding methods. The statistical results in terms of generation cost and computational time of lambda iterative and proposed methods are presented in TABLE II. It is clear from TABLE II that the proposed methods yield qualitative solution same as the lambda iterative method with less computational time.

TABLE II

THE OPTIMUM SOLUTION OF EACH UNIT BY LAMBDA ITERATIVE METHOD (LIM) AND PROPOSED METHOD

OUTPUT POWERS IN MW	LAMBDA ITERATIVE METHOD	PROPOSED METHOD
P1	446.7073	446.7073
P2	171.258	171.258
P3	264.105	264.105
P4	125.216	125.216
P5	172.118	172.118
P6	083.593	083.593
Fuel cost($)	15275.93	15275.93
Computational time (Sec)	0.031	0.016

Examples 2 In this example, the proposed methods are applied to solve the ED problem with transmission losses for 6 generating units system, which is presented in *example 1*. The incremental transmission loss is represented by quadratic form in terms of output powers. The B_{mn} coefficients are given as follows.

$$B_{ij} = 10^{-3} \begin{bmatrix} 1.7 & 1.2 & 0.7 & -0.1 & -0.5 & -2.0 \\ 1.2 & 1.4 & 0.9 & 0.1 & -0.6 & -0.1 \\ 0.7 & 0.9 & 3.1 & 0.0 & -1.0 & -0.6 \\ -0.1 & 0.1 & 0.0 & 0.24 & -0.6 & -0.8 \\ -0.5 & -0.6 & -0.1 & -0.6 & 12.9 & -0.2 \\ -2.0 & -1.0 & -0.6 & -0.8 & -0.2 & 15.0 \end{bmatrix}$$

$$B_{oi} = 10^{-3} \bullet \left[-0.3908 \;\; -1.297 \;\; 7.047 \;\; 0.591 \;\; 2.161 \;\; -6.635 \right]$$

$$B_{00} = 0.056$$

Power demand=**1263** MW.

TABLE III listed the statistical results that involved generation cost of various methods such as particle swarm optimization, genetic algorithm, lambda iterative method and proposed methods.

TABLE III

OPTIMAL SOLUTION OF 6 GENERATING UNIT SYSTEM BY VARIOUS METHODS

Power outputs (MW)	PSO [20]	GA [20]	Proposed methods
P1	447.497	474.806	447.3
P2	173.322	178.636	173.2182
P3	263.47	262.20	263.3
P4	139.059	134.282	138.9
P5	165.476	151.903	165.4
P6	87.1280	74.1812	87.05
Total Power (MW)	1276.01	1276.03	1275.
Loss (MW)	12.9584	13.0217	12.3935
Fuel cost ($/h)	15450	15459	15442.39

It is clear from TABLE III that the proposed approaches provide qualitative solution than modern heuristic methods such as particle swarm optimization and genetic algorithm reported in [20].

Examples 3 In this example, the proposed methods have been tested on 15 units system with transmission losses, ramp rate limits and prohibited zones. The data was obtained from [20].

TABLE IV

THE OPTIMUM SOLUTION OF EACH UNIT BY VARIOUS METHODS

Output Powers (MW)	GA	PSO	Proposed method
P1	415.3108	439.1162	455
P2	359.7206	407.9727	455
P3	104.425	119.6327	130
P4	74.9853	129.9925	130
P5	380.2844	151.0681	305
P6	426.7902	459.9978	287.1
P7	341.3164	425.5601	366.51
P8	124.7867	98.5699	162.88
P9	133.1445	113.4936	25
P10	89.2567	101.1142	146.48
P11	60.0572	33.9116	80
P12	49.9998	79.9583	69.656
P13	38.7713	25.0042	25
P14	41.9425	41.414	15
P15	22.6445	35.614	15
Total Power (MW)	2668.2782	2662.443	2667.6245
P Loss	38.2782	32.4306	37.6245
Fuel cost ($/Hr)	33113	32858	32857.44
Computational time (Sec)	-	-	0.26

For this test case, conventional lambda iterative method and proposed methods are executed for solving ED problem by considering all equality and inequality constraints. Here, the main aim is to enlighten the effectiveness of the proposed method compare to other methods in terms of the solution quality, convergence characteristics and computational efficiency to solve large scale ED problems with all constraints with mixed generating units system. The simulation results in terms of output powers, fuel cost and computational time of all methods were presented in TABLE IV.

*Example **4*** In this case, the proposed algorithms and Lambda iterative algorithm were applied for 40 generating units [16] system and the results are presented in TABLE V. Also the proposed algorithms were applied to solve the ED problem for 120 generating units systems. The fuel cost data of 80 units system was extracted from 40 units system by considering two times of the fuel cost data of 40 generating units system.

TABLE V
SIMULATION RESULTS OF LAMBDA ITERATIVE METHOD AND SECANT METHOD

Units	40		120	
Power Demand (MW)	10500		31500	
Methods	Lambda Iterative	Propos ed	Lambda Iterative	Propos ed
Optimal lambda($/MW)	16.257	16.257	16.257399	16.257
Fuel cost ($/hr)	143926	143926	431804.00	431804
Time(Sec)	0.35	0.05	0.981	0.07

*Example **5*** In this example, the system contains 26 thermal units. The fuel cost data was taken [29] and is given in TABLE VI. Power Demand is 2500 **MW.**

TABLE VI
FUEL COST DATA OF 26 GENERATING UNIT S

Unit	a_i	b_i	c_i	d_i	$P_{i\min}$	$P_{i\max}$
1	24.38	25.54	0.025	5.08e-009	2.4	12
2	24.41	25.67	0.026	-1.01e-08	2.4	12
3	24.63	25.8	0.028	1.01e-008	2.4	12
4	24.76	25.93	0.028	-5.08e-09	2.4	12
5	24.88	26.06	0.028	-5.72e-16	2.4	12
6	117.75	37.55	0.011	8.31e-008	4	20
7	118.1	37.66	0.012	8.56e-008	4	20
8	118.45	37.77	0.013	8.15e-008	4	20
9	118.82	37.88	0.014	8.29e-008	4	20
10	81.13	13.32	0.008	-5.8e-010	15.2	76
11	81.29	13.35	0.008	-5.47e-10	15.2	76
12	81.46	13.38	0.009	-5.49e-10	15.2	76
13	81.62	13.4	0.009	-5.5e-010	15.2	76
14	217.89	18	0.006	1.25e-018	25	100
15	218.33	18.09	0.006	-1.19e-18	25	100
16	218.77	18.2	0.005	2.44e-018	25	100
17	142.73	10.69	0.004	1.11e-010	54.25	155
18	143.02	10.71	0.004	1.03e-010	54.25	155
19	143.31	10.73	0.004	1.03e-010	54.25	155
20	143.59	10.75	0.004	1.03e-010	54.25	155
21	259.3	23	0.002	1.07e-010	68.95	197
22	259.64	23.1	0.002	1.04e-010	68.95	197
23	260.17	23.2	0.002	1e-010	68.95	197
24	177.05	10.86	0.001	-4.42e-19	140	350
25	310	7.49	0.001	-1.1e-019	100	400
26	311.91	7.5	0.001	-3.55e-20	100	400

In this example also the simulations results of the proposed methods have been compared with the conventional lambda iterative method in terms of solution quality and computational time. Individual output powers of the generating units are tabulated in TABLE VII.

TABLE VII
OPTIMAL SOLUTION BY PROPOSED METHODS FOR EXAMPLE 5

unit	output power(MW)	unit	output power(MW)
1	2.4	14	68.464
2	2.4	15	60.74
3	2.4	16	61.971
4	2.4	17	155
5	2.4	18	155
6	4	19	155
7	4	20	155
8	4	21	68.95
9	4	22	68.95
10	76	23	68.95
11	76	24	350
12	76	25	400
13	76	26	400

The simulation results in terms of fuel cost and computational time of the proposed method and lambda iterative method were given in TABLE VIII. It is clear from the Table 8 that the proposed methods have better convergence and computational characteristics than the lambda iterative method. These two characteristics are important in real time operation of power system. The proposed methods have been tested on large scale systems by considering 52, 78 and 104 units. The fuel costs and power demands are two, three and four times of the fuel cost of example 5. Fuel costs and computational times of various methods are tabulated in TABLES IX,X. It is observed from Table 9,10 that the lambda iterative method takes more computational time when the dimensionality of problem increases but the proposed methods can offer optimal solution in almost fixed time

TABLE VIII
SIMULATION RESULTS BY VARIOUS METHODS FOR 26 UNITS

	Methods			
	Lambda Iterative	Muller	Secant	Brent
Optimal Lambda ($/MW)	18.8195	18.819632	18.81959	18.81976
Fuel Cost ($)	34506.1	34506.1	34506.1	34506.1
No of Iterations	248	8	9	1
Computational Time in Sec	0.331	0.07	0.06	0.06

TABLE IX
FUEL COSTS FOR 52, 78 AND 104 UNITS BY VARIOUS METHODS

	Methods			
Units	Iterative	Muller	Secant	Brent
52	69012.238	69012.238	69012.24	69012.238
78	103518.357	103518.357	103518.4	103518.357
104	138024.477	138024.477	138024.5	138024.477

TABLE X
COMPUTATIONAL TIME FOR 52,78 AND 104 UNITS BY VARIOUS METHODS

	Methods			
Units	Iterative	Muller	Secant	Brent
52	0.35	0.14	0.09	0.09
78	0.47	0.16	0.01	0.09
104	0.611	0.22	0.01	0.11

V. CONCLUSIONS

This paper suggests the solution of economic dispatch problem with generator constraints and transmission losses by root finding methods such as Secant, Muller and Brent methods. The proposed methods can provide qualitative solution with less computational time at the required power demand compare to other methods presented in the case studies. The chief advantages of the proposed methods are listed below.

(i) Computational time is less for getting optimal solution.

(ii) These algorithms will not depend on any user defined parameters.

(iii) The system constraints such as generator constraints, ramp rate limits and prohibited zones can be easily incorporated.

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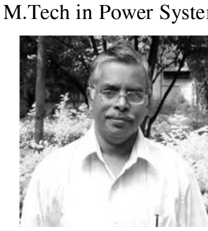

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