EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 4, 111–118 (1975)

DUCTILITY OF REINFORCED CONCRETE SECTIONS WITH CONFINED COMPRESSION ZONES

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SUMMARY

Confinement of concrete in circular spiral steel binders imparts to it considerable ductility and also some increase in strength. This property can be utilized in designing concrete structures to withstand seismic forces where the members are required to possess not only strength but also energy absorbing capacity. Assuming the stress–strain behaviour of confined concrete as elastic–plastic, the ductility factor for strain and the strength factor (denoting increase in strength) have been determined for concrete confined to different degrees. Similarly, assuming the moment–curvature behaviour of reinforced concrete sections with confined compression concrete to be elastic–plastic, the ductility factor for curvature has been determined for such beams. The computed moment–curvature plots have been found to compare satisfactorily with tests on 18 beams. Ductility factors for curvature of singly and doubly reinforced concrete sections with compression concrete confined to different degrees have been determined and presented for certain typical cases. Such plots would be of use in designing reinforced concrete beam sections for required ductility.

INTRODUCTION

Reinforced concrete structures, unlike steel structures, tend to fail in a relatively brittle manner as the deformation capacity of concrete is limited. For resisting dynamic forces concrete structures are not quite suitable because they cannot absorb strain energy as efficiently as steel. When concrete structures are subjected to earthquake forces, the critical sections of the structural members must be able to absorb strain energy, if sudden failures are to be avoided. This is possible only if the material is capable of withstanding considerable deformation without a reduction in its load-carrying capacity. Economy and safety in design dictate that structures which must resist dynamic loadings must be designed for energy absorption capacity in addition to its strength. Hence for concrete structures located in seismic zones the improvement of ductility is of paramount importance.

It is possible to improve the ductility of concrete by confining it in steel binders (spirals or stirrups). Tests on confined concrete in compression and on reinforced and prestressed concrete beams having confined compression concrete have shown considerable increase in ductility and ultimate strength. Blume[1] recognized that the ability of a structure to absorb earthquake energy is vastly increased with 'ductile concrete', *viz.* a structural concrete in which the design is so made that 'in flexural members shear failure and compression failure cannot occur prior to stretching of tensile bars and in compression members shear failure cannot occur and any concrete that fails in compression will be confined'. He assumed different stress–strain behaviours (shown in Figure 1) for unconfined and confined concretes and presented the variation of ductility ratio for curvature when a reinforced concrete column section has unconfined or confined concrete. The attempt seems to be to take all precautions in providing the reinforcement in such a way that the necessary confinement is available for concrete under compression.

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Received 16 August 1974

In a project undertaken at the Indian Institute of Science, attempts were made to define quantitatively the confinement given by steel circular spiral binders to concrete through a confinement index and to determine the influence of confinement on the stress–strain behaviour of concrete. From this, the stress-blocks applicable

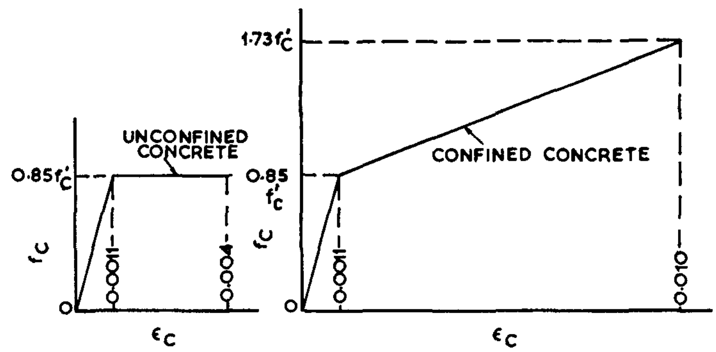

Figure 1. Stress–strain curves of unconfined and confined concretes assumed by Blume (Reference 1)

to the flexural analysis of reinforced concrete sections with confined compression zones were determined and an analytical and experimental study of determinate and indeterminate reinforced concrete beams and frames with concrete confined at critical zones was made. Some of these results have already been reported.[2–6] The present paper gives the results on the ductility factor for strain, a factor which is taken as a measure of the improved plastic strain capability and the influence of confinement on the same. Also, the moment–curvature diagrams of reinforced concrete sections, with and without confinement, are idealized to elastic–plastic plots and compared with test results of 18 beams. Based on such computations, ductility factors for curvature of reinforced concrete sections with confined compression concrete are determined for different ratios of tension reinforcement, compression reinforcement and the confinement index.

DUCTILITY OF CONFINED CONCRETE IN COMPRESSION

Figure 2 shows typical stress–strain curves of concrete with and without confinement. The confinement is found to give some increase in strength and considerably greater increase in strain capability to concrete.

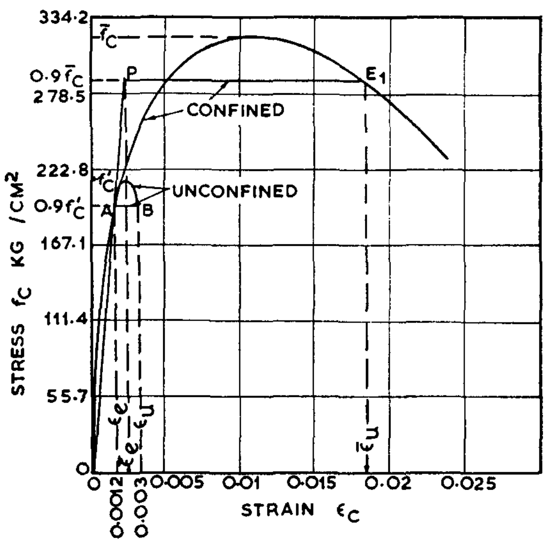

Figure 2. Typical stress–strain curves of unconfined and confined concrete with elastic–plastic approximations

The curves may be approximated to elastic–plastic plots such as OAB for unconfined concretes and OPE_1 for confined concrete where OP is line OA extended up to P. The ductility factor for strain (D_{fc}) may then be defined as the ratio $\varepsilon_u/\varepsilon_e$ ($\bar{\varepsilon}_u/\bar{\varepsilon}_e$) for unconfined (confined) concrete. The increase in strength realized due to confinement may be defined by a strength factor (S_f) and defined as the ratio $\bar{f}_c/f'_c$.

The confinement offered by the binders can be quantitatively represented by a parameter called 'confinement index' (C_i) defined by[2]

$$C_i = (p_b - \bar{p}_b)(f_y/f'_c) \tag{1}$$

The ultimate compressive strength of concrete confined in circular spiral steel binders is given by[2]

$$\bar{f}_c = f'_c(1 + 2{\cdot}30C_i) \tag{2a}$$

and thus the strength factor S_f resulting from confinement is given by

$$S_f = (1 + 2{\cdot}30C_i) \tag{2b}$$

The strain at the point $E_1(\bar{\varepsilon}_u)$ of Figure 2 is related to the stress according to the equation of the line BE_1 in Figure 4 and hence can be related to the confinement index through equation (2a), resulting in

$$\bar{\varepsilon}_u = 0{\cdot}30 + 8{\cdot}42C_i \tag{3}$$

Noting that OP is line OA extended and using equation (2a), $\bar{\varepsilon}_e$ may be obtained as

$$\bar{\varepsilon}_e = 0{\cdot}12 + 0{\cdot}276C_i \tag{4}$$

In equations (3) and (4), $\bar{\varepsilon}_u$ and $\bar{\varepsilon}_e$ are in per cent and C_i is to be expressed as a decimal.

The ductility factor for strain may then be obtained by dividing (3) by (4) as

$$D_{fs} = \frac{\bar{\varepsilon}_u}{\bar{\varepsilon}_e} = \frac{0{\cdot}30 + 8{\cdot}42C_i}{0{\cdot}12 + 0{\cdot}276C_i} \tag{5}$$

The magnitudes of S_f and D_{fs} have been determined for different values of C_i and Figure 3 shows the variation of S_f and D_{fs} with the variation C_i. It may be noted that for an increase of C_i from 0·0 to 0·7, D_{fs} increases from 2·5 to 19·75 whereas the increase in S_f is only from 1 to 2·61.

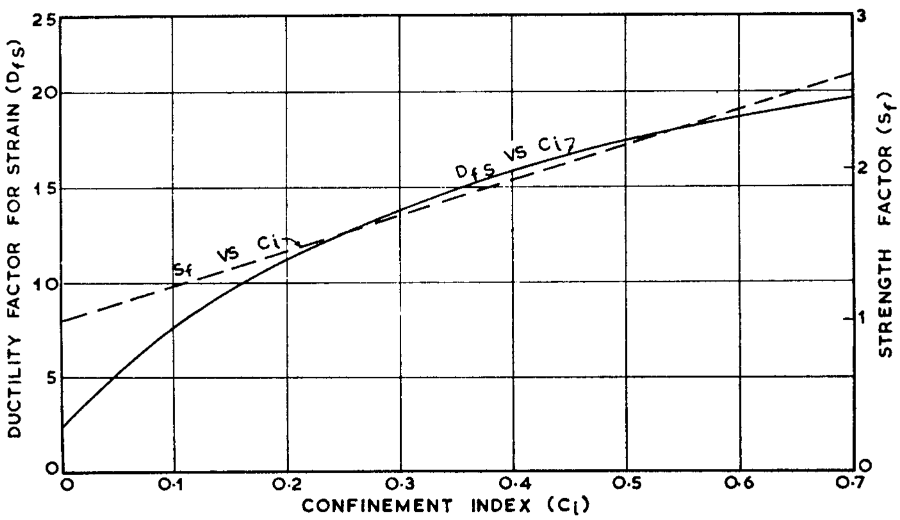

Figure 3. Variation of ductility factor for strain and strength factor with confinement index for concrete confined in circular spiral binders and tested in compression

DUCTILITY OF REINFORCED CONCRETE SECTIONS

In reinforced concrete sections subjected to bending, in addition to confinement index, the usual other factors, namely, the type and amount of tension steel and compression steel, also influence the ductility. Figure 4 gives the diagram from which the stress-block applicable to the flexural analysis of reinforced con-

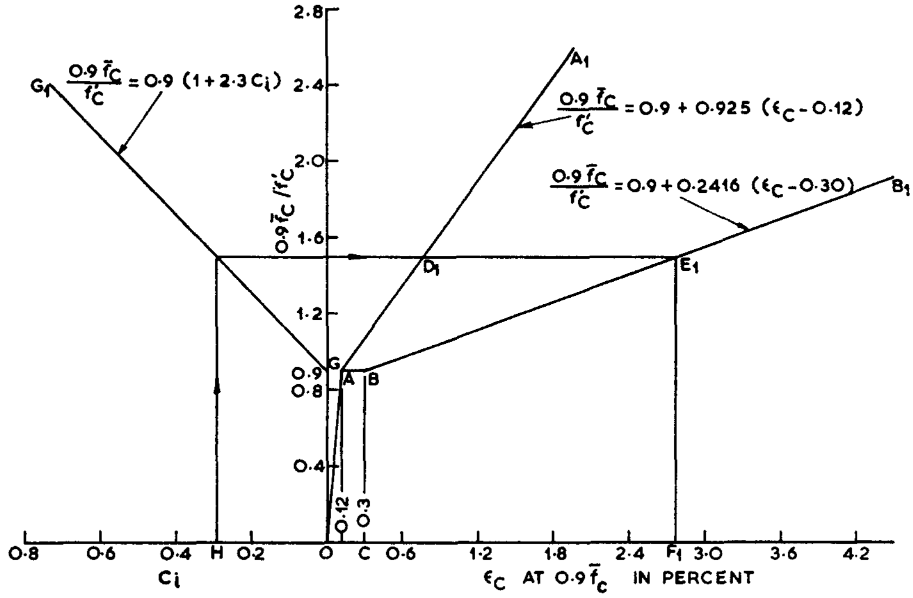

Figure 4. Stress-blocks for unconfined and confined concretes. $OABC$—stress-block for unconfined concrete, $OAD_1E_1F_1$—stress-block for concrete confined in circular spiral binder giving a confinement of $C_i = OH$

crete sections with a confined compression zone can be obtained and the equations arising out of using the block have also been presented.[3,4]

Figure 5 shows typical moment–curvature diagrams of confined and unconfined sections which are otherwise similar. Approximating the M–χ diagrams to the elastic–plastic plots, OLL_1 and ONN_1 are the plots corresponding to the unconfined and confined sections. The ordinates to the line OLN may be obtained from

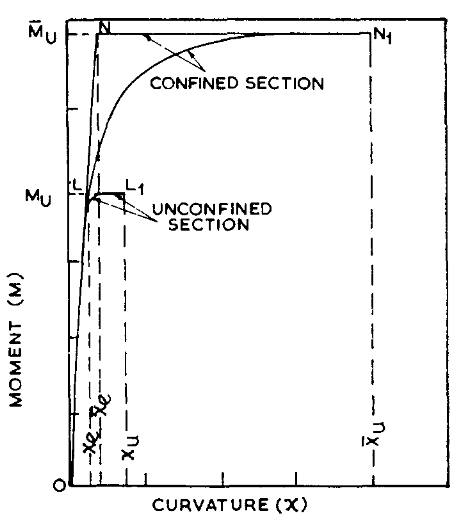

Figure 5. Typical moment–curvature diagrams of unconfined and confined reinforced concrete sections with elastic–plastic approximations

the usual elastic analysis, *viz*

$$\chi = M/E_c I_{cr} \tag{6}$$

where M is the bending moment, at which χ is being calculated, E_c is the modulus of elasticity of concrete and I_{cr} is the moment of inertia taken as that of the cracked transformed section (or I_g if $I_g < I_{cr}$). M_u ($\bar{M}_u$) and χ_u ($\bar{\chi}_u$) are obtained in the ultimate strength analysis using the proposed stress-block[3,4] and $\bar{\chi}_u$ may sometimes be controlled by the maximum strain in steel which is limited to a safe value of 15 per cent.

Referring to Figure 5, the ductility factor for curvature (D_{fc}) may now be defined as the ratio of χ_u/χ_e ($\bar{\chi}_u/\bar{\chi}_e$) for unconfined (confined) reinforced concrete sections subjected to bending.

COMPARISON WITH TEST RESULTS

The proposed elastic–plastic moment–curvature diagrams have been compared with those obtained in tests of 18 reinforced concrete beams, nine of which had confinement. The details of the experimental work on these beams have already been reported[3] and Figures 6 and 7 show how the experimental and computed

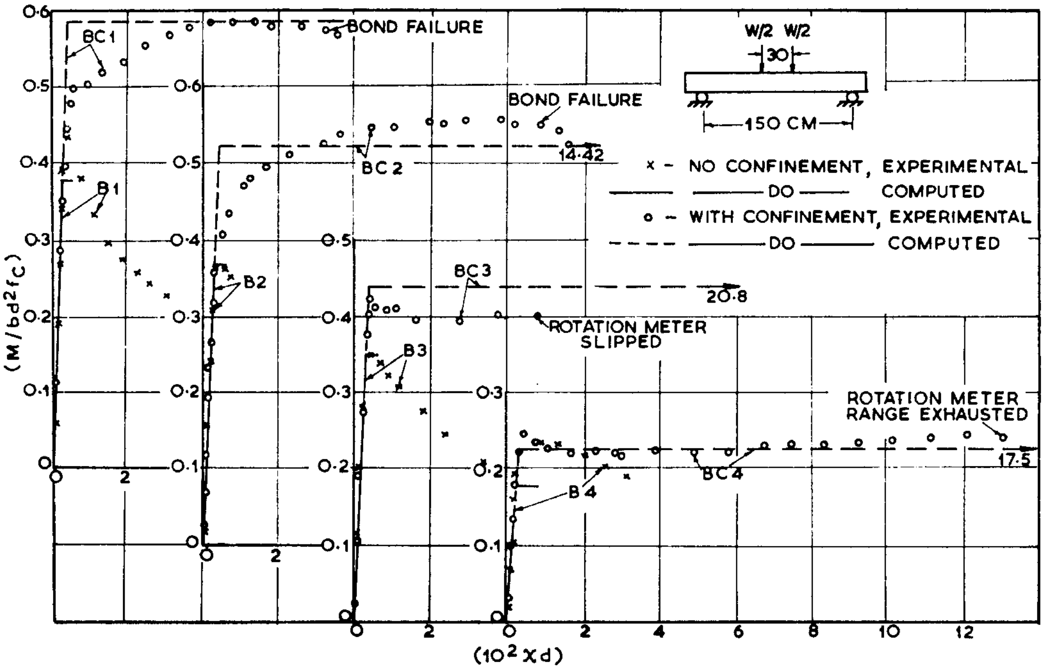

Figure 6. Experimental and computed moment–curvature diagrams of beam series 1, 2, 3 and 4

M–χ diagrams compare. (For these beams $E_c = 57{,}000\sqrt{(f'_c)}$ (in psi with f'_c in psi) was used for determining the modulus of elasticity of concrete.) The comparison is fairly satisfactory except in a few confined beams where the ultimate curvature is smaller than the computed one—the reason for this is that during testing, when the beams started having large plastic rotations, the rotation meter sometimes slipped or the dial gauges were sometimes removed to avoid damage and so the full plastic curvature could not be recorded.

DUCTILITY FACTOR FOR CURVATURE

Using the proposed method of computing χ_e ($\bar{\chi}_e$) and χ_u ($\bar{\chi}_u$), the ductility factor for curvature has been determined for sections with the following parameters:

$$C_i = 0{\cdot}0,\ 0{\cdot}2,\ 0{\cdot}4 \text{ and } 0{\cdot}6$$

$$p = 0{\cdot}01,\ 0{\cdot}02,\ 0{\cdot}03,\ 0{\cdot}04,\ 0{\cdot}05 \text{ and } 0{\cdot}06$$

$$p' = 0,\ 0{\cdot}25p \text{ and } 0{\cdot}5p$$

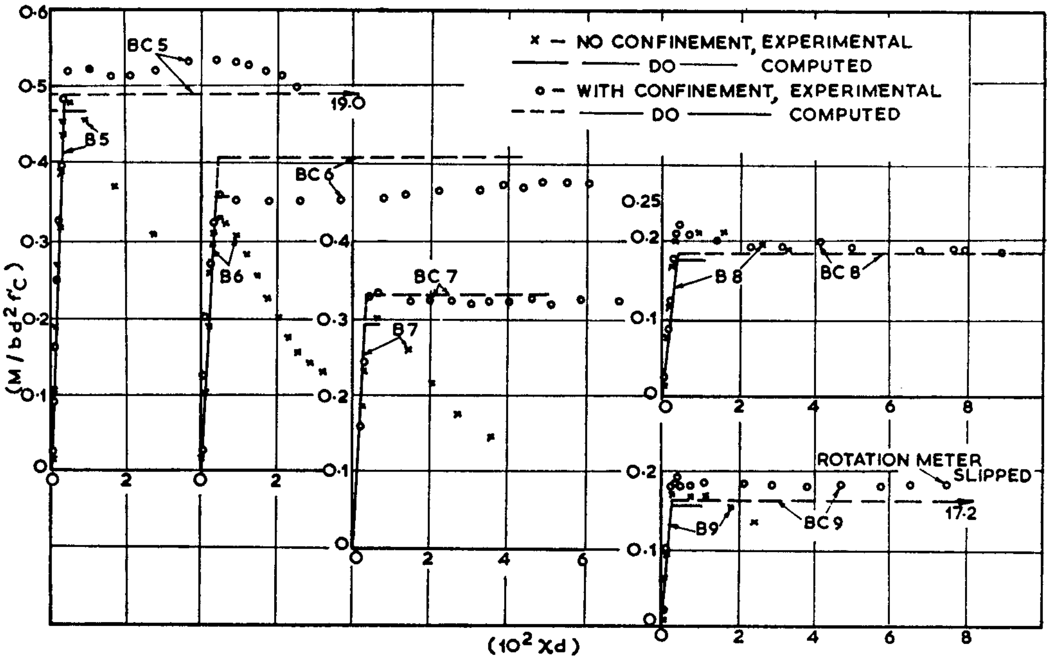

Figure 7. Experimental and computed moment–curvature diagrams of beam series 5, 6, 7, 8 and 9

Figures 8, 9 and 10 show the variation of D_{f_0} with respect to p, for various values of p' and C_i. In these computations, in order to work with suitable non-dimensional parameters, E_c is taken equal to $1000f'_c$. Also the results are illustrative for concretes with a modular ratio of 8 and mild steels with a yield strength of 2,820 kg/cm². Similar plots can be worked out for other steels and concretes. Such diagrams would be useful in designing beam sections for a desired ductility in curvature.

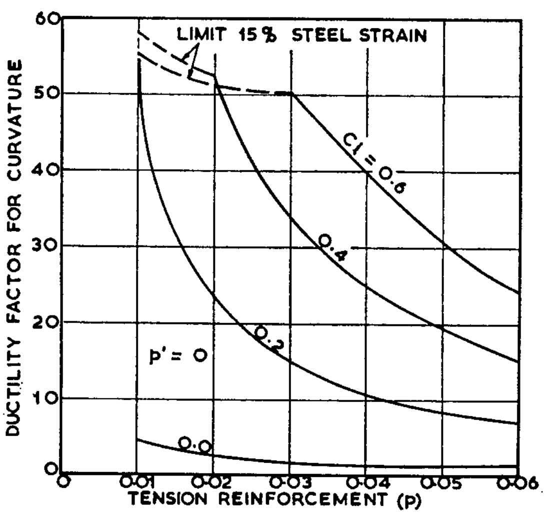

Figure 8. Variation of ductility factor for curvature of confined reinforced concrete sections in flexure ($p' = 0$)

CONCLUSIONS

1. Confinement improves the strength and ductility of concrete in compression. The improvement in ductility is an aspect which would be beneficial for seismic design of concrete structures. The strength factor and the

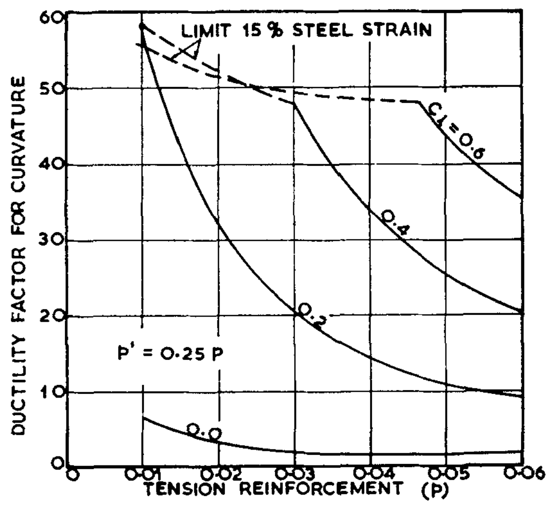

Figure 9. Variation of ductility factor for curvature of confined reinforced concrete sections in flexure ($p' = 0{\cdot}25p$)

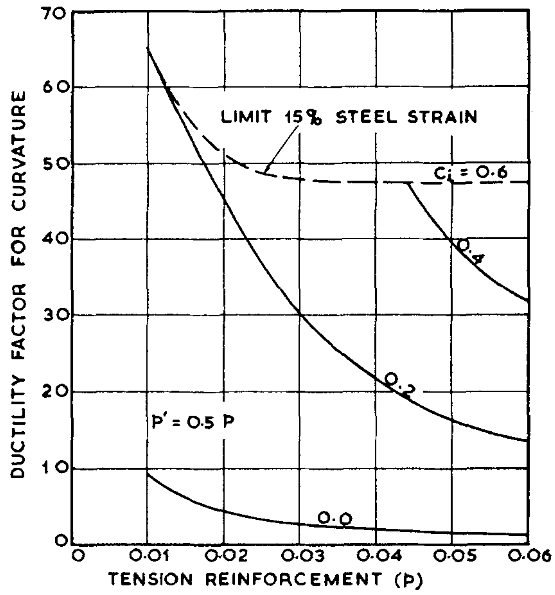

Figure 10. Variation of ductility factor for curvature of confined reinforced concrete sections in flexure ($p' = 0{\cdot}5p$)

ductility factor for strain are found to increase from 1 to 2·6 and 2·5 to 19·75 respectively, as shown in Figure 3, for a confinement index increasing from 0 to 0·7.

2. The assumed elastic–plastic M–χ diagrams represent satisfactorily the experimental plots. Figures 8, 9 and 10 illustrate the typical variation of ductility factors for curvature of reinforced concrete sections with confined compression concrete for different values of p, p' and C_i. Such plots would be useful in the seismic design of concrete structures where a beam section is required to possess a minimum prescribed ductility along with strength.

APPENDIX

Notation

b breadth of the rectangular beam

C_i confinement index

D_{fc} ductility factor for curvature
D_{fs} ductility factor for strain
d effective depth of the beam
E_c modulus elasticity of concrete
f_c stress in concrete
f'_c ($\bar{f}_c$) ultimate compressive strength of unconfined (confined) concrete
f_y yield strength of reinforcement or binder wire
I_{cr} moment of inertia of cracked transformed section
I_g moment of inertia of gross section
M bending moment
M_u ($\bar{M}_u$) ultimate moment of unconfined (confined) reinforced concrete section
p tension steel ratio
p' compression steel ratio
p_b volumetric ratio of binder, i.e. ratio of the volume of binder to the volume of confined concrete
$\bar{p}_b$ volumetric ratio when the pitch of binder is equal to the least lateral dimension of the specimen
S_f strength factor
W total load on the beam
ε_c compressive strain
ε_e ($\bar{\varepsilon}_e$) elastic strain of unconfined (confined) concrete specimen under compression
ε_u ($\bar{\varepsilon}_u$) ultimate strain of unconfined (confined) concrete specimen under compression
χ curvature
χ_e ($\bar{\chi}_e$) elastic curvature of unconfined (confined) reinforced concrete section
χ_u ($\bar{\chi}_u$) ultimate curvature of unconfined (confined) reinforced concrete section

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