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CONSOLIDATION DUE TO SHEAR LOADS DISTRIBUTED OVER A RECTANGULAR AREA

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SUMMARY

The paper presents an analytical study of the consolidation of a semi-infinite clay layer subjected to a shear load distributed over a rectangular area. Biot's theory is made use of, along with the three displacement functions suggested by Verruijt. A complex Fourier and Laplace transformation technique enables the solution to be obtained in terms of non-dimensional parameters. Settlements and pore pressures under the loaded area are evaluated for two types of surface drainage conditions. Extension of the solutions to point loads is also suggested. The Mandel–Cryer effect is seen in the behaviour of the pore pressure.

INTRODUCTION

Present methods of estimating magnitudes and rates of settlements are based on the conventional one-dimensional Terzaghi[10] and pseudo three-dimensional Rendulic[8] theories of consolidation. Biot[3] proposed a comprehensive three-dimensional theory of primary consolidation based on a poro-elastic approach which takes care of simultaneous pore pressure diffusion and total stress distribution.

Most of the previous studies based on Biot's theory were confined to normal loads.[5,7] However, the plane strain shear load problem was investigated by the authors[2] and the problem of shear load distributed over a circular area by Schiffman and Fungaroli.[9] Rectangular and square footings are common in foundation engineering practice. Inclined loads on such footings can be split into vertical[5] and horizontal components. The present study concerns the consolidation behaviour of a semi-infinite clay layer subjected to uniform shear load acting over a rectangular area.

STATEMENT OF THE PROBLEM AND METHOD OF SOLUTION

The boundary-value problem under investigation is shown in Figure 1, which also shows the coordinate system chosen (with the positive z-axis taken into the medium). The initial and boundary conditions are:

$$\begin{aligned}
&\sigma_{zz}=0, \quad \sigma_{yz}=0, \quad \text{when } z=0, \quad |x|<\infty, \quad |y|<\infty, \quad t>0\\
&\sigma_{xy}=q \quad \text{when} \quad z=0, \quad |x|<b, \quad |y|<b\lambda, \quad t>0\\
&\quad\;\; =0 \quad \text{when} \quad z=0, \quad |x|>b, \quad |y|>b\lambda, \quad t>0\\
&\sigma=0 \quad \text{when} \quad z=0, \quad |x|<\infty, \quad |y|<\infty, \quad t>0
\end{aligned} \tag{1}$$

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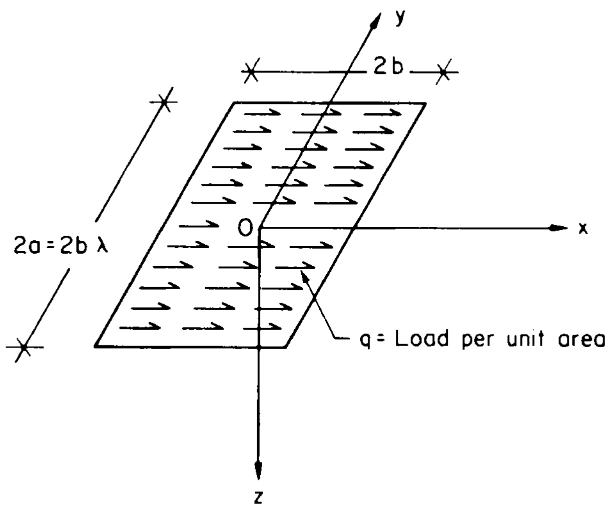

Figure 1. Uniformly distributed shear load over a rectangular area

for a pervious drainage boundary at the surface, and

$$\frac{\partial \sigma}{\partial z} = 0 \quad \text{when} \quad z = 0, \quad |x| < \infty, \quad |y| < \infty, \quad t > 0$$

for an impervous drainage boundary at the surface.
Also $\bar{e} = 0$ throughout the medium initially and

$$\sigma_{ij} = 0 \quad \text{when} \quad |x| \to \infty, \quad |y| \to \infty, \quad |z| \to \infty, \quad t > 0$$

where

σ_{ij} is the total stress tensor which is composed of the effective stress tensor σ'_{ij} (i.e., stress carried by the soil skeleton through solid contacts) and excess pore pressure σ (i.e., stress taken by water present in the voids of soil),

$\bar{e}$ is the volumetric strain,

q is the shear load intensity and

$\lambda = (a/b)$ is the length to breadth ratio of the rectangular area.

The governing equations of Biot's theory[3] for a saturated soil are:

$$\nabla^2 u_i - (2n-1)\frac{\partial \bar{e}}{\partial x_i} - \frac{1}{G}\frac{\partial \sigma}{\partial x_i} = 0$$
$$c\nabla^2 \bar{e} = \frac{\partial \bar{e}}{\partial t} \tag{2}$$

where

u_i is the displacement vector of the soil skeleton,

G is the shear modulus,

n is an auxiliary effective elastic parameter defined as $n = (1-\nu)/(1-2\nu)$,

ν is the effective Poisson's ratio,

$c = (2Gnk/\gamma_w)$ is the coefficient of consolidation,

k is Darcy's coefficient of permeability,

γ is the unit weight of water and

∇^2 is the three-dimensional Laplacian operator.

The governing equations (2) are satisfied if the displacement functions E, S and Q[11] are the solutions of

$$c\nabla^4 E = \nabla^2\left(\frac{\partial E}{\partial t}\right), \qquad \nabla^2 S = 0, \qquad \nabla^2 Q = 0 \tag{3}$$

σ_{zz}, σ_{xz}, σ_{yz}, σ, $\bar{e}$ and w are related to E, S and Q by

$$\begin{aligned}
\frac{\sigma_{zz}}{2G} &= \left(\frac{\partial^2}{\partial_z^2} - \nabla^2\right)E - z\frac{\partial^2 S}{\partial_z^2} + \frac{\partial S}{\partial z} \\
\frac{\sigma_{xz}}{2G} &= \frac{\partial^2 E}{\partial x \partial z} - z\frac{\partial^2 S}{\partial x \partial z} - \frac{\partial^2 Q}{\partial y \partial z} \\
\frac{\sigma_{yz}}{2G} &= \frac{\partial^2 E}{\partial y \partial z} - z\frac{\partial^2 S}{\partial y \partial z} + \frac{\partial^2 Q}{\partial x \partial z} \\
\frac{\sigma}{2G} &= \frac{\partial S}{\partial z} - n\nabla^2 E, \qquad \bar{e} = +\nabla^2 E, \qquad w = -\frac{\partial E}{\partial z} - S + z\frac{\partial S}{\partial z}
\end{aligned} \tag{4}$$

where w is the displacement in the z direction.

Following McNamee and Gibson,[7] normalization is done by dividing all stresses by q, dividing all lengths by b and time by (b^2/c). Henceforth, only normalized variables are used (unless otherwise stated) and no special notation is employed for this purpose. Integral transforms are used in the analysis.

Thus the (normalized) variable E is transformed into $\bar{E}$ as follows:

$$\bar{E}(\alpha, \beta, z, p) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} E(x, y, z, t) \exp\left(i(\alpha x + \beta y) - pt\right) \mathrm{d}t \, \mathrm{d}x \, \mathrm{d}y \tag{5}$$

and similarly E is obtained from $\bar{E}$ by inversion, i.e.,

$$E(x, y, z, t) = \frac{1}{8\pi^3 i}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{h-i\infty}^{h+i\infty} \bar{E}(\alpha, \beta, z, p) \exp\left(-i(\alpha x + \beta y) + pt\right) \mathrm{d}p \, \mathrm{d}\alpha \, \mathrm{d}\beta \tag{6}$$

Transforming equations (3) and using the last two conditions of (1), $\bar{E}$, $\bar{S}$ and $\bar{Q}$ are given by

$$\begin{aligned}
\bar{E} &= A_1 \exp(-rz) + A_2 \exp\left(-rz(1+s)^{1/2}\right) \\
\bar{S} &= B \exp(-rz), \; \bar{Q} = C \exp(-rz)
\end{aligned} \tag{7}$$

where $r = \sqrt{(\alpha^2 + \beta^2)}$ and $p = r^2 s$. The constants of integration in equation (7), on using the first of the four conditions of equation (1) are:

$$\begin{aligned}
A_1 &= -\frac{i\alpha D(1+ns)}{Hr^3}, \qquad A_2 = i\alpha D/Hr^3 \\
B &= -\frac{i\alpha nsD}{Hr^2}, \qquad C = \frac{i\beta D}{r^3}
\end{aligned} \tag{8}$$

where

$$H = 1 + ns - \sqrt{(1+ns)}, \; D = \frac{2 \sin\alpha \sin\lambda\beta}{G\alpha\beta s r^2}$$

in the case of a pervious boundary, whereas

$$A_1 = -\frac{i\alpha DJ}{Ir^3}, \qquad A_2 = \frac{i\alpha D}{Ir^3}$$

$$B = \frac{i\alpha nsD\sqrt{(1+s)}}{Ir^2}, \qquad C = \frac{i\beta D}{r^3}$$

where $J = ns\sqrt{(1+s)}+1$ and $I = 1+(ns-1)\sqrt{(1+s)}$ in the case of an impervious boundary. Substitution of equation (8) in equation (7) and effecting Laplace inversion, one obtains the following displacement functions:

$$2GE = -\frac{i}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp(-i(\alpha x+\beta y))}{\beta r^3}\Big[\frac{2n+rz}{2n-1}\exp(-rz) - \exp(-r^2 t)(I_4+\exp(-rz)I_5)\Big]\, d\alpha\, d\beta$$

$$2GS = -\frac{in}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp - i(\alpha x+\beta y)-\alpha z)}{\beta r^2}\Big[\frac{2n}{2n-1} - \exp(-r^2 t)I_6\Big]\, d\alpha\, d\beta \qquad (9)$$

$$2GQ = \frac{i}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp(-i(\alpha x+\beta y)-rz)}{\alpha r^3}\, d\alpha\, d\beta$$

in the case of a pervious boundary, and

$$2GE = -\frac{i}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp(-i(\alpha x+\beta y))}{\beta r^3}\Big[\frac{2n+rz}{2n-1}\exp(-rz) - \exp(-r^2 t) \times (I_1 - \exp(-rz)I_2) - \frac{2\exp(-(1-\delta)r^2 t)(\exp(-rz\sqrt{\delta}) - \sqrt{\delta}\exp(-rz))}{(1-\delta)((n+1)^2-(2-n\delta)^2)}\Big]\, d\alpha\, d\beta$$

$$2GS = -\frac{in}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp(-i(\alpha x+\beta y)-rz)}{\beta r^2} \times\Big[\frac{2}{2n-1} - \exp(-r^2 t)I_3 - \frac{2\sqrt{\delta}\exp(-(1-\delta)r^2 t)}{(n+1)^2-(2-n\delta)^2}\Big]\, d\alpha\, d\beta$$

$$2GQ = \frac{iq}{\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp(-i(\alpha x+\beta y)-rz)}{\alpha r^3}\, d\alpha\, d\beta$$

in the case of an impervious boundary, where $I_1, I_2, \ldots, I_6$ and δ in equations (9) and (10) are identical to expressions obtained by McNamee and Gibson[7] except that r should be substituted in the place of α. Once the displacement functions are known, all the other variables of interest can be obtained using the relations given in equations (4).

RESULTS AND DISCUSSION

Results for pervious drainage boundary

The settlement as obtained from equations (9) and (4) is

$$2Gw_{z=0} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\alpha \sin\beta\lambda \exp(-i(\alpha x+\beta y))}{\beta r^2}\Big(\frac{1}{2n-1} - n\exp(-r^2 t)I_6\Big)\, d\alpha\, d\beta \qquad (11)$$

The immediate settlement, w_0, i.e. w at $t=0$, is zero and so is the settlement for $n \to \infty$ (or $\nu \to 0{\cdot}5$). Thus, the time-dependent (consolidation) settlement is the same as the total settlement for the problem. The ultimate settlement at points lying on the x-axis is obtained by letting $y=0$ and $t \to \infty$ in equation (11), i.e.,

$$2Gw(x,0,0,\infty)=\frac{4}{(2n-1)\pi^2}\int_0^\infty\int_0^\infty \frac{\sin\alpha\,\sin\beta\lambda\,\sin\alpha x}{\beta r^2}\,d\alpha\,d\beta$$
$$=\frac{1}{2n-1}\left[\begin{matrix}x\ (\text{for } x<1)\\ 1\ (\text{for } x>1)\end{matrix}-\frac{1}{\pi}\left\{\tan^{-1}\frac{2x\lambda}{\lambda^2+1-x^2}\right.\right.$$
$$\left.\left.+x\tan^{-1}\frac{2\lambda}{\lambda^2+x^2-1}+\frac{\lambda}{2}\ln\frac{\lambda^2+(1-x)^2}{\lambda^2+(1+x)^2}\right\}\right] \tag{12}$$

Equation (12) suggests that settlement is positive for $x>0$ and is negative for $x<0$, which means that half the footing in the direction of the load settles and the other half correspondingly heaves up with the result that there will not be any settlement at the centre of the footing (i.e., at $x=0$). For $n=1$ ($\nu=0$) and $x=1$ (i.e., midpoint of an edge of the footing), equation (12) reduces to:

$$2Gw\,(1,0,0,\infty)=1-\frac{2}{\pi}\left[\tan^{-1}\left(\frac{2}{\lambda}\right)+\frac{\lambda}{4}\ln\left(\frac{\lambda^2}{\lambda^2+4}\right)\right]$$

This settlement increases with the λ-value, and in the limit as $\lambda \to \infty$, $2Gw\ (1,0,0,\infty)$, which was obtained by the authors[2] for the plane strain uniform shear load problem. It is also seen that the higher the n-value (or ν-value), the smaller will be the settlements.

The time–settlement relations obtained by numerical evaluation of the double integral in equation (11) using Filon's rule are shown in Figures 2 and 3.

The pore pressure is obtained as

$$\sigma=\frac{in}{\pi^2}\int_{-\infty}^\infty\int_{-\infty}^\infty \frac{\sin\alpha\,\sin\beta\lambda\,\exp\left(-i(\alpha x+\beta y)-r^2t\right)}{\beta r}\left[I_4-\frac{1}{r^2}\frac{d^2I_4}{dz^2}-\exp(-rz)I_6\right]d\alpha\,d\beta \tag{13}$$

which for $n=x=1$, $y=0$, reduces to

$$\sigma=\frac{4}{\pi^2}\int_0^\infty\int_0^\infty \frac{\sin^2\alpha\,\sin\beta\lambda\,\exp(-rz)}{\beta r}\left[\operatorname{erfc}\left(r\sqrt{t}-\frac{z}{2\sqrt{t}}\right)-\operatorname{erfc}\left(r\sqrt{t}\right)\right]d\alpha\,d\beta$$

and for $n \to \infty$, $x=1$, $y=0$ is given by

$$\sigma=\frac{2}{\pi^2}\int_0^\infty\int_0^\infty \frac{\sin^2\alpha\,\sin\beta\lambda}{\beta r}\left[\exp(-rz)\operatorname{erfc}\left(r\sqrt{t}-\frac{z}{2\sqrt{t}}\right)-\exp(rz)\operatorname{erfc}\left(r\sqrt{t}+\frac{z}{2\sqrt{t}}\right)\right]d\alpha\,d\beta \tag{14}$$

where erfc (x) refers to the complementary error function. The initial pore pressure (at $y=0$) is given by

$$\sigma_0=\frac{4}{\pi^2}\int_0^\infty\int_0^\infty \frac{\sin\alpha\,\sin\beta\lambda\,\sin\alpha x\,\exp(-rz)}{\beta r}\,d\alpha\,d\beta \tag{15}$$

It is seen from equations (13), (14) and (15) that pore pressures are absent at $x=0$, positive for $x>0$ and negative for $x<0$. This pattern of pore pressure is consistent with the settlement pattern discussed earlier.

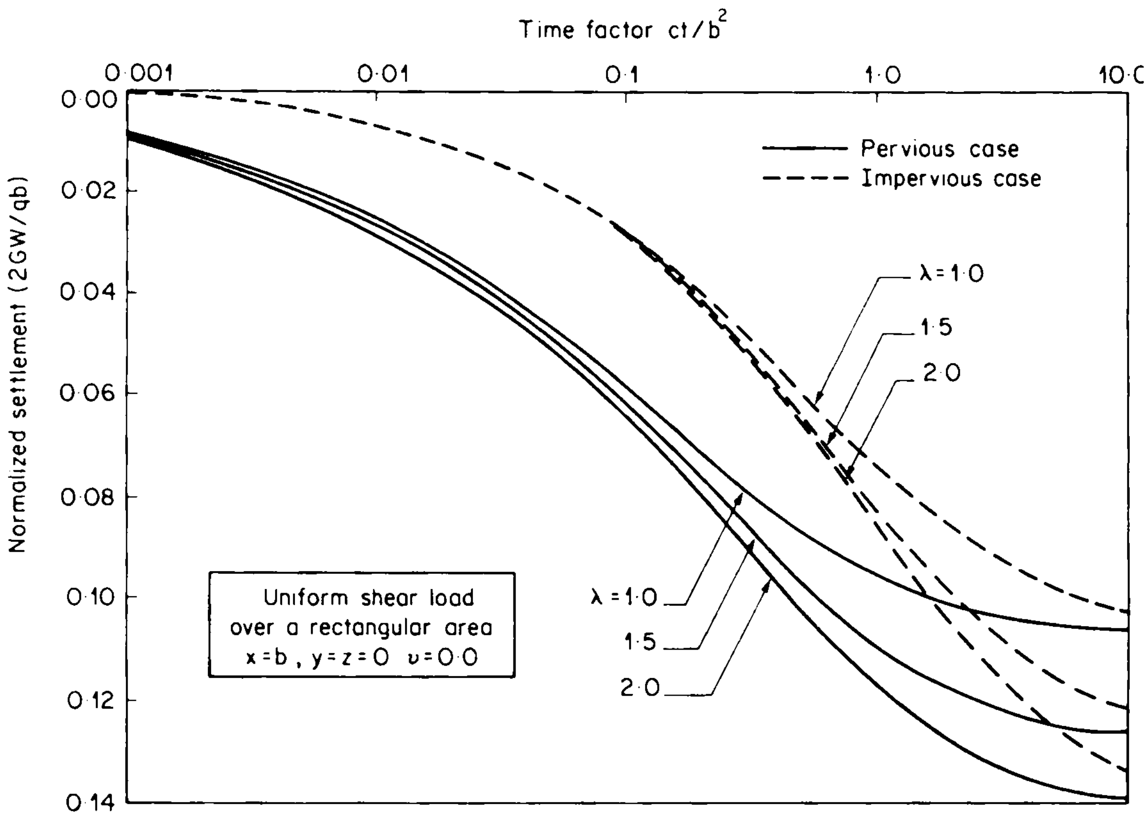

Figure 2. Time–settlement relation for $\nu = 0{\cdot}0$

Time–pore pressure relations under the edge of square and rectangular footing ($\lambda = 1{\cdot}5$) are shown in Figures 4 and 5. The most striking result of these figures is that while pore pressure for the case $n \to \infty$ decreases with time monotonically from its initial maximum value, for the case of $n = 1$ it increases with time beyond its initial value before decreasing steadily. This peculiar pore pressure phenomenon is known as the Mandel–Cryer effect.[4,6] Pore pressures predicted from Terzaghi's one-dimensional consolidation theory and from pseudo three-dimensional theory do not exhibit this effect. The possible reasons for the manifestation of this effect are shown in Figures 6 and 7. Figure 6 shows that during the initial stages of consolidation, the total volumetric stress

$$\sigma_v = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

increases while the effective volumetric stress

$$\sigma'_v = \frac{\sigma'_{xx} + \sigma'_{yy} + \sigma'_{zz}}{3}$$

remains constant. This causes a temporary build up of pore pressure ($\sigma = \sigma_v - \sigma'_v$). In ordinary diffusion theories like that of Terzaghi, it is assumed that σ_v does not change during

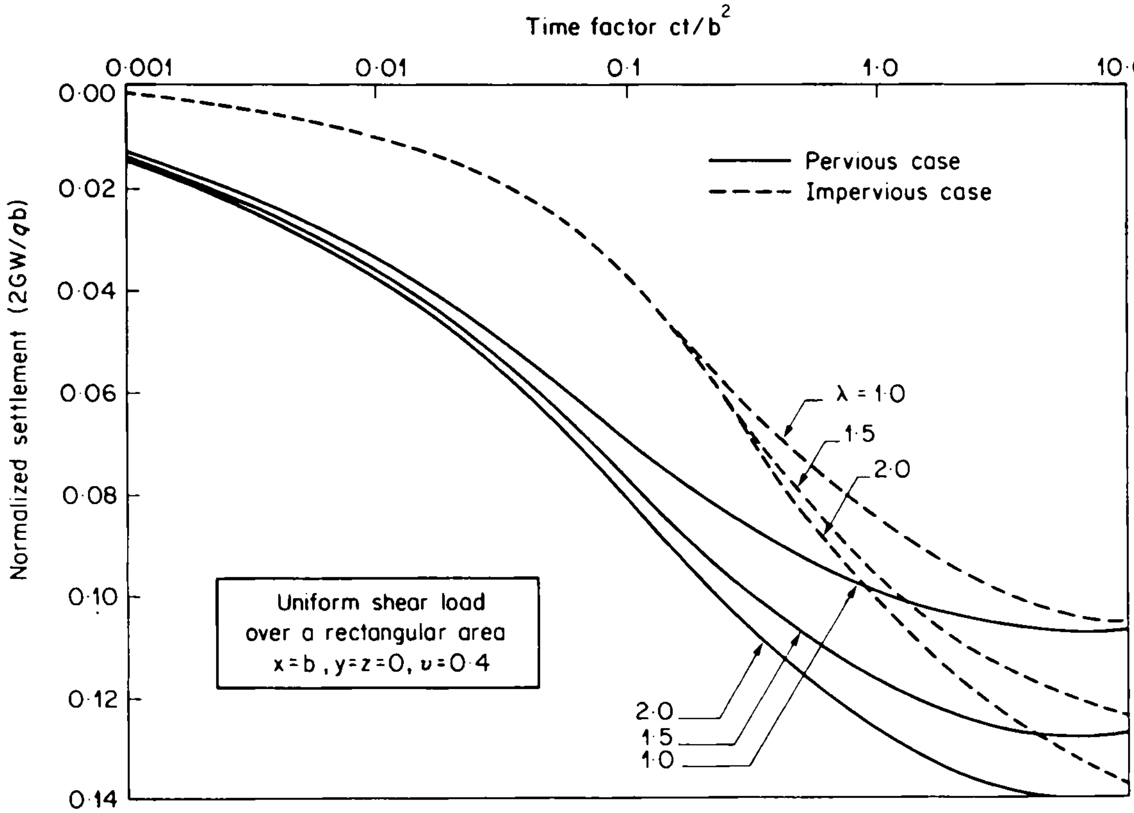

Figure 3. Time–settlement relation for $\nu = 0{\cdot}4$

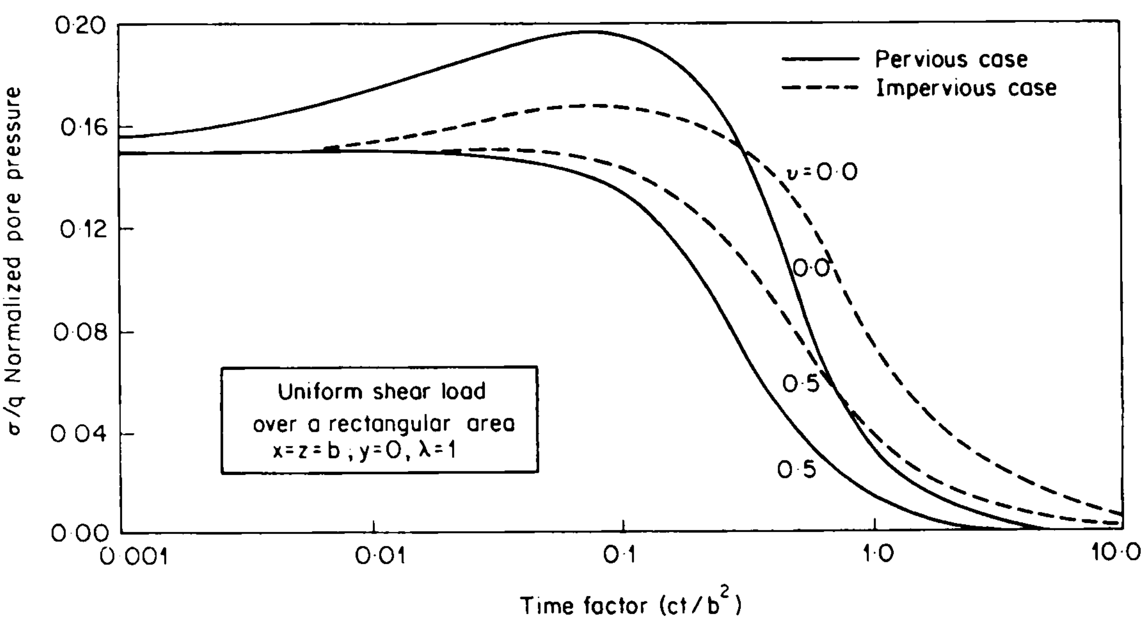

Figure 4. Time–pore pressure relation for $\lambda = 1{\cdot}0$

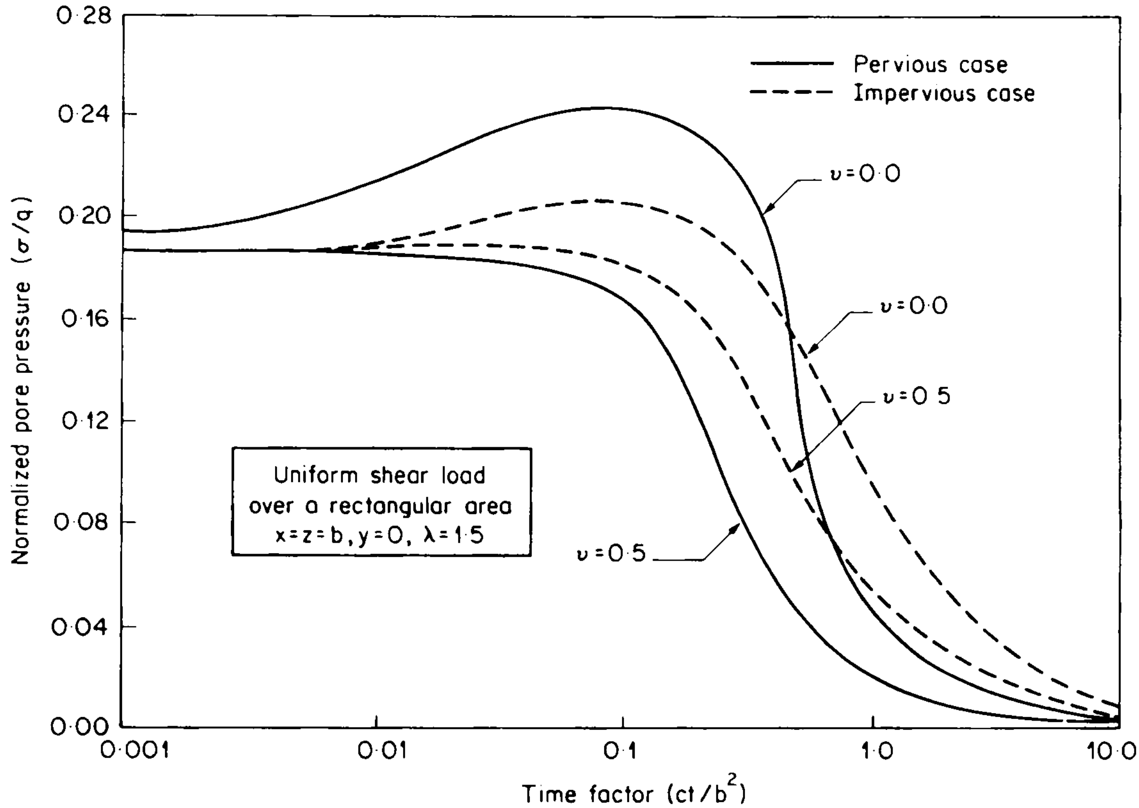

Figure 5. Time–pore pressure relation for $\lambda = 1{\cdot}5$

consolidation. The increase in σ_v as predicted by Biot's theory is due to the fact that some internal redistribution of stresses take place.

Another way of looking at the Mandel–Cryer effect is that, during the consolidation process, the elastic parameters of soil (comprising soil skeleton and pore water) change from initial underdrained values to final drained (effective) parameters. In Biot's theory, it is assumed that the effective stress parameters G and ν (or n) are constant. For example, σ_v' can be related to $\bar{e}$ through the relation

$$\sigma_v' = \frac{2G\,(1+\nu)}{3\,(1-2\nu)}\bar{e}$$

A similar relation for σ_v in terms of the total stress parameters G_t and ν_t could be written as

$$\sigma_v = \frac{2G_t\,(1+\nu_t)}{3\,(1-2\nu_t)}\bar{e}$$

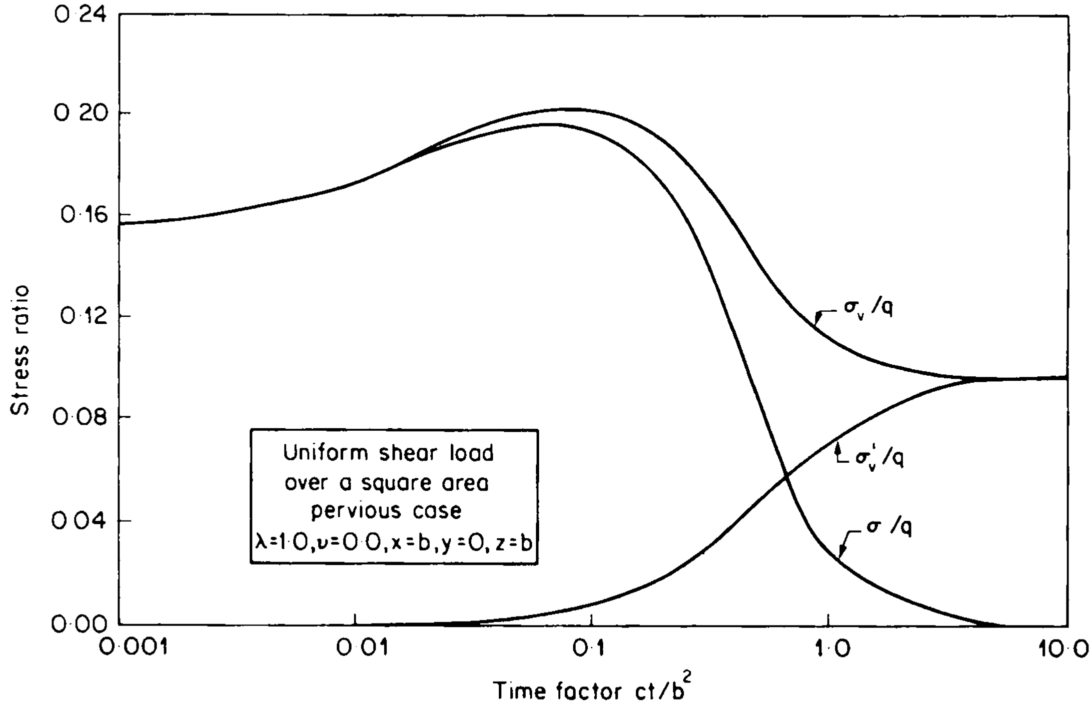

Figure 6. Time–total volumetric stress relation for $\lambda = 1{\cdot}0$

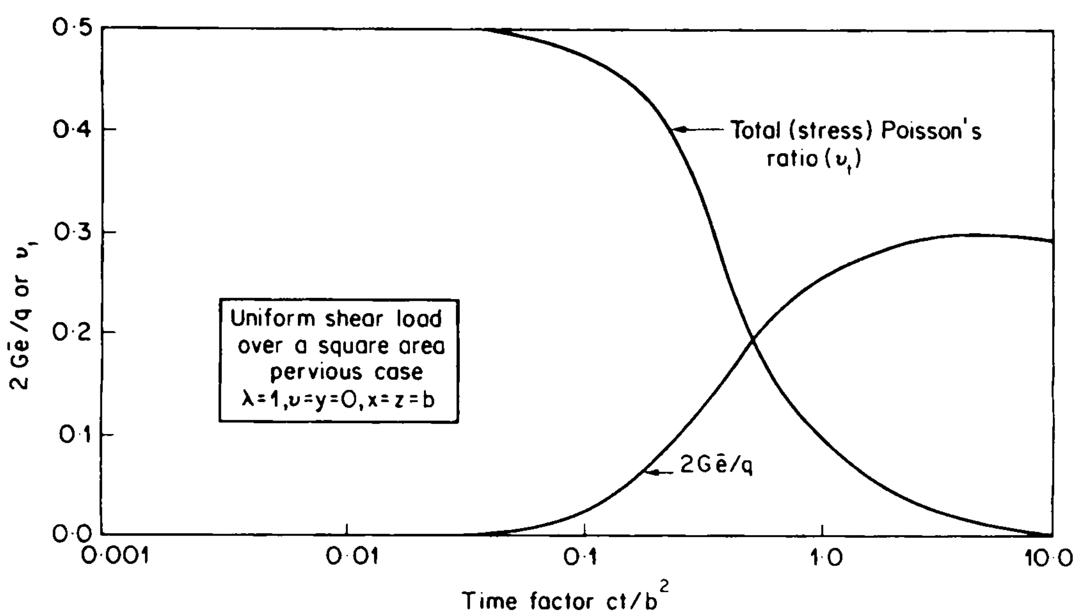

Figure 7. Variation of total stress, Poisson's ratio and volumetric strain with time

As pore water is incapable of taking any shear, the values of G and G_t must be equal. From the common value of $\bar{e}$ in the preceding two expressions, one obtains the following relation:

$$\nu_t = \frac{\sigma_v(1+\nu)-(\sigma_v-\sigma)(1-2\nu)}{(\sigma_v-\sigma)(1-2\nu)+2\sigma_v(1+\nu)}$$

$$= \frac{\sigma}{3\sigma_v-\sigma} \quad \text{for} \quad \nu = 0$$

Figure 7 shows the variation of ν_t and $\bar{e}$ with time. It is interesting to note that the ν_t value is 0·5 initially (signifying the initial condition of incompressibility) and gradually decreases with time to its final (effective) value ν.

Results for impervious drainage boundary

The settlements and pore pressure for the impervious case are obtained similarly and are shown in Figures 2 to 5 along with the results for the pervious case for comparison. It is obvious that initial and final values of settlements and pore pressures are the same for both the drainage cases. Initial values of these parameters are also independent of n (or ν). Rates of settlement and pore pressure dissipation are slower in the impervious case than in the pervious case. The Mandel–Cryer effect is seen for $n = 1$ even in the impervious case, the intensity of the effect, however, being smaller than that in the pervious case.

Incidentally, it may be observed[1] that the excess pore pressures predicted from pseudo three-dimensional theory[8] coincide with those obtained by Biot's theory with $n \to \infty$ or $\nu \to 0{\cdot}5$.

SOLUTIONS FOR POINT CONCENTRATED SHEAR LOAD

Solutions for the point shear load problem are obtained as particular cases of the corresponding solutions of the rectangular shear load problem discussed earlier, making use of the following condition in various dimensionalized variables:

$$P = \lim_{\substack{b\to\infty \\ q\to\infty}} 4b^2 q\lambda \tag{16}$$

where P is the concentrated shear load at the origin of coordinate axes in the x-direction.

Only the final dimensional results are quoted in the following for both the drainage conditions.

Results for pervious drainage boundary

$$2Gw_{z=0} = \frac{Pi}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\alpha \exp(-i(\alpha x+\beta y))}{r^2}\left[\frac{1}{2n-1} - n \exp(-r^2 ct) I_6\right] d\alpha \, d\beta$$

$$= \frac{P}{2\pi}\int_0^{\infty} J_1(\alpha R)\left[\frac{1}{2n-1} - n \exp(-\alpha^2 ct) I_6\right] d\alpha \tag{17}$$

where $R^2 = x^2 + y^2$ and I_4, I_5, I_6, etc. are identical to similar integral expressions referred to earlier with the exception that 'ct' is substituted for 't'.

For $n = 1$, equation (18) reduces to

$$2Gw_{\substack{z=0\\n=1}} = \frac{P}{2\pi R}\left[\exp\left(\frac{-R^2}{4ct}\right)M\left(\frac{1}{2}; 1; \frac{R^2}{4ct}\right)\right]$$

where $M(\alpha; \beta; x)$ is the confluent hypergeometric function. Figure (8) shows the result of the above equation. For $t \to \infty$, equation (17) reduces to

$$2Gw_{\substack{z=0\\t\to\infty}} = \frac{P}{2\pi R(2n-1)} \tag{18}$$

Pore pressure is obtained as

$$\sigma = \frac{Pin}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\alpha \exp(-i(\alpha x + \beta y) - r^2 ct)}{r}\left[I_4 - \frac{1}{r^2}\frac{d^2 I_4}{dz^2} - \exp(-rz)I_6\right] d\alpha\, d\beta$$

$$\sigma_{t=0} = \frac{P}{2\pi}\int_0^{\infty}\alpha J_1(\alpha R)\exp(-\alpha z)\, d\alpha = \frac{PR}{2\pi}\frac{1}{(R^2+z^2)^{3/2}}$$

$$\sigma_{n=1} = \frac{P}{2\pi}\int_0^{\infty}\alpha J_1(\alpha R)\left[\exp(-\alpha z)\left\{\operatorname{erfc}\left(\alpha\sqrt{ct} - \frac{z}{2\sqrt{ct}}\right) - \operatorname{erfc}(\alpha\sqrt{ct})\right\}\right] d\alpha$$

$$\sigma_{n\to\infty} = \frac{P}{4\pi}\int_0^{\infty}\alpha J_1(\alpha R)\left[\exp(-\alpha z)\operatorname{erfc}\left(\alpha\sqrt{ct} - \frac{z}{2\sqrt{ct}}\right) - \operatorname{erfc}\left(\alpha\sqrt{ct} + \frac{z}{2\sqrt{ct}}\right)\exp(\alpha z)\right] d\alpha$$

Pore pressure results are shown in Figure 9.

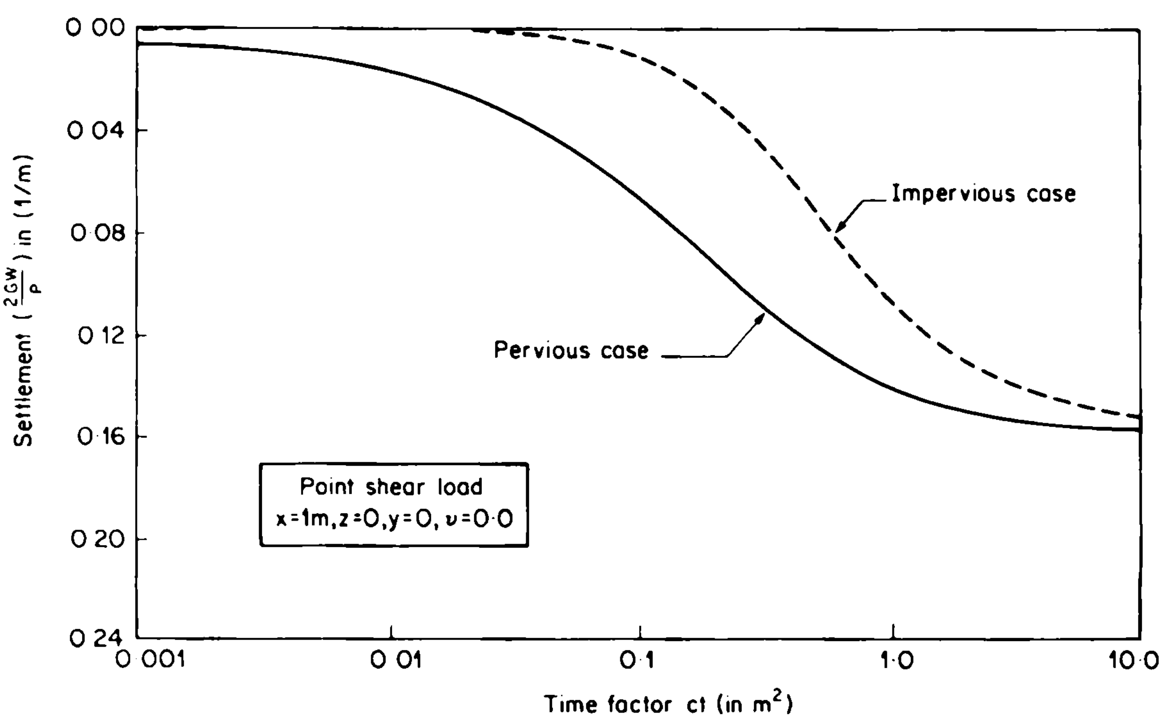

Figure 8. Time–settlement relation for point shear load

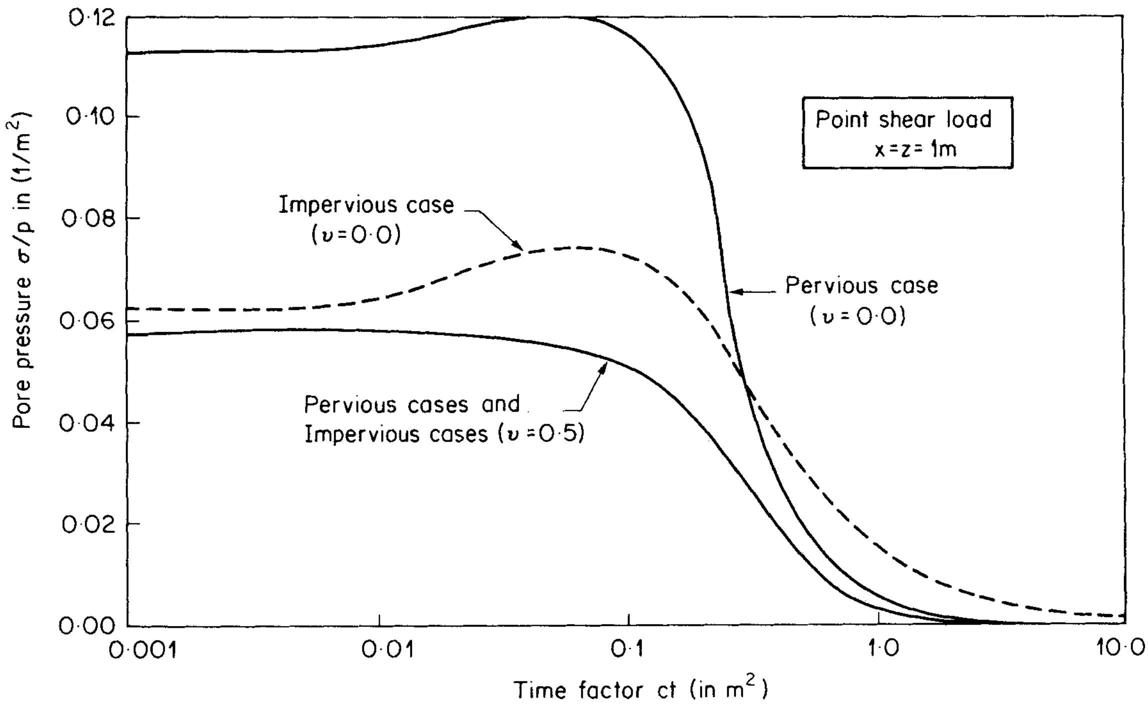

Figure 9. Time–pore pressure relation for point shear load

Results for impervious drainage boundary

The results for settlements and pore pressures obtained for this case are shown in Figures 8 and 9.

APPENDIX

Notation

The following symbols are used in this paper:

A_1, A_2 = constants of integration used in equation (7)
a = half the length of the rectangular loaded area
B = constant of integration in equation (7)
b = half the width of the rectangular loaded area
C = constant of integration in equation (7)
c = coefficient of consolidation
$D = 2 \sin \alpha \sin \beta\lambda / G\alpha\beta s r^2$
E = displacement function
$\bar{E}$ = transformed value of E after normalization
exp = exponential base
erf (x) = error function
$\bar{e}$ = volumetric strain
G = shear modulus

$H = 1 + ns\sqrt{(1+s)}$
$I = 1 + (ns - 1)\sqrt{(1+s)}$
I_1 to I_6 = integrals obtained in References 5 and 7
i = imaginary unit
$J_1(x)$ = Bessel function
k = coefficient of permeability
lim = limit of
$M(\alpha; \beta; x)$ = confluent hypergeometric function
$n = (1-\nu)/1-2\nu)$
P = point shear load
Q = displacement function
$\bar{Q}$ = transformed displacement function
p = parameter of Laplace transformation
q = intensity of shear load per unit area
$r = \sqrt{(\alpha^2+\beta^2)}$
$R = \sqrt{(x^2+y^2)}$
S = displacement function
$\bar{S}$ = transformed value of S
$s = p/r^2$
t = time
u = displacement in the x direction
u_i = displacement in the x_i direction
v = displacement in the y direction
w = displacement in the z direction
w_0 = immediate surface settlement (i.e., w at $z = 0$, $t = 0$)
x = horizontal coordinate in the direction of shear load
x_i = ith component of the spatial vector $\vec{x}$
y = horizontal coordinate perpendicular to the direction of shear load
z = vertical coordinate
α, β = parameters of Fourier transforms
γ_w = unit weight of water
$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$
$\delta = (n+2-\sqrt{(n^2+4n)})/2n$
ν = effective (stress) Poisson's ratio
ν_t = total (stress) Poisson's ratio
σ = excess pore water pressure
σ_{ij} = total stress tensor
σ_v = total volumetric stress
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ = total normal stresses
$\sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ = total shear stresses
$\lambda = (a/b)$

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