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Diffracted field characteristics of Straubel class of apodisation filters

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Abstract. The diffracted field characteristics of Straubel class of apodisation filters have been investigated with reference to point spread function, relative central intensity, total transmission factor, Strehl ratio, second order moment, encircled energy and dispersion factors.

Keywords. Diffraction; apodisation.

1. Introduction

In optics and in telecommunication, the suppression of secondary side lobes in the diffraction pattern is highly desirable in certain cases. The process of achieving this is known as apodisation. By properly choosing the transmission function of the pupil of the system, the intensity in the outer parts of the diffraction pattern can be totally suppressed or at least considerably reduced without increasing the dimensions of the pupil. There is a consequent increase in the width of the central maximum, diminishing the resolving power of the system for objects of equal intensity. Many apodisation filters have been proposed in instrumental optics (Jacquinot and Roizen-Dossier 1964) for various purposes. It is, however, found that most of these filters cannot be implemented due to practical difficulties in their fabrication.

Now, Straubel class of apodisation filters has evoked considerable interest (Kavathekar and Singh 1970; Singh *et al* 1970). The optical transfer function of a system with these filters in incoherent light has been thoroughly investigated by Rao and Mondal (1975) to determine the limiting resolvable frequencies. The sine wave response of an optical system in partially coherent light with these filters has also been studied (Mondal and Rao 1975). However, nothing has been reported so far on the point image characteristics of such filters. Furthermore, it is well known that the process of apodisation will not simultaneously improve all the imaging properties of the system. It can achieve only an improvement of certain desirable image qualities at the expense of some others. It is necessary, therefore, to investigate the diffracted field characteristics of these filters and with this end in view, we have presented a few important parameters associated with the diffraction pattern of an aperture with Straubel class of apodisation filters. The computed results presented here will help in determining the relative performance of these filters with respect to an optical system with conventional airy type of objectives.

2. Theory

The amplitude of diffracted light due to a particular pupil function with a circular symmetry is given by (Boivin 1964)

$$G(y, Z) = 2\pi i \Omega a^2 \int_0^1 f(r^2)\, e^{iyr^2/2}\, J_0(Zr)\, r dr \tag{2.1}$$

$G(y, Z)$ is thus the finite Hankel transform of the pupil function multiplied by the factor $e^{iyr^2/2}$. With reference to figure 1 the other symbols have the following significance:

$$\Omega = \frac{B}{\lambda R R^*}\, e^{[(2\pi i/\lambda)(R-R^*)]} \tag{2.2}$$

where B = brightness of the source; λ = wavelength of light;

also $y/2 = (2\pi/\lambda)\ C_1 \cdot a^2$ (2.3)

where $C_1 = 1/2\ (1/R - 1/R^*)$ and a is the radius of the pupil.

Finally,

$$Z = (2\pi/\lambda) \cdot (\sigma/R) \cdot a. \tag{2.4}$$

The factor outside the integral of (2.1) is a constant geometric factor which depends only on the geometry of the arrangement. This factor has no contribution to the diffraction pattern which depends only on the form of the pupil function. We can thus omit this factor for the purpose of calculating the effect of a particular pupil function on the diffraction pattern and we can write (2.1) as

$$G(y, Z) = 2 \int_0^1 f(r^2)\, e^{iyr^2/2}\, J_0(Zr)\, r dr. \tag{2.5}$$

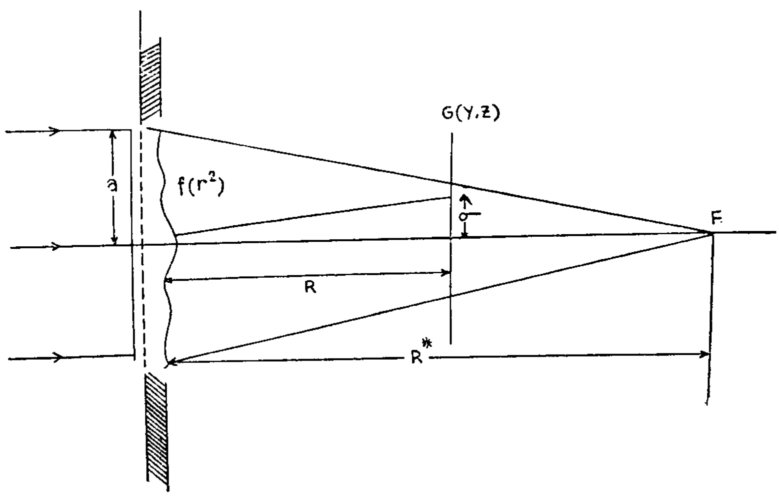

Figure 1. Diffraction by a circular aperture.

In the case of a free aperture, $f(r^2) = 1$ and we can write for the diffraction pattern due to a free aperture

$$0(y, Z) = 2 \int_0^1 e^{iyr^2/2} J_0(Zr)\, r dr. \tag{2.6}$$

This function was first evaluated by Lommel (1885). Now, in this paper, we propose to investigate the various characteristics in the diffracted field due to Straubel class of apodisation filters which are given mathematically by

$$f(r^2) = (1 - r^2)^P. \tag{2.7}$$

For a filter of this type, the diffracted amplitude is given, according to (2.5), by

$$G(y, Z) = 2 \int_0^1 (1 - r^2)^P e^{iyr^2/2} J_0(Zr)\, r dr. \tag{2.8}$$

At the Gaussian focal plane, $y = 0$, so that

$$G(0, Z) = 2 \int_0^1 (1 - r^2)^P J_0(Zr)\, r dr. \tag{2.9}$$

For a free aperture $P = 0$ and (2.9) takes the well-known form

$$0(0, Z) = 2 \int_0^1 J_0(Zr)\, r dr. \tag{2.10}$$

The intensity of the diffracted light can be easily obtained from the relation,

$$I = G \, . \, G^* \tag{2.11}$$

where G^* is the complex conjugate of G.

3. Results and discussions

(i) *Point spread function*

Figure 2 shows the distribution of the diffracted light amplitude at the Gaussian focal plane and figure 3 shows the corresponding point spread function for various values of P. The point spread function, which is the intensity distribution in the image plane due to a point object, plays an important role in determining the resolution of the optical system. According to the Rayleigh criterion of resolution, the resolving power increases as the first zero of the point spread function moves towards the centre of the pattern. For an airy type of objective corresponding to $P = 0$, the first zero of the point spread function occurs at $Z = 3 \cdot 83$. As the value of P is increased, the point of zero intensity moves away from the centre with a consequent loss of resolving power. From figure 3, we find that the zeroes for $P = 1, 2, 3$ and 4 occur respectively at $Z = 4 \cdot 0$, $5 \cdot 6$, $6 \cdot 0$ and $7 \cdot 0$. To reveal this, the point spread function has been drawn in an enlarged scale in the region $Z = 4 \cdot 0$ to $Z = 8 \cdot 0$ by the side of the original scale for non-zero values of P. It is obvious, of course, that by increasing P, the secondary maxima or the side-lobes are totally suppressed. If we restrict ourselves to the main aim of apodisation as to suppress the side lobes in the diffraction pattern of a point object so that the image resembles more closely to that of the object, we find that

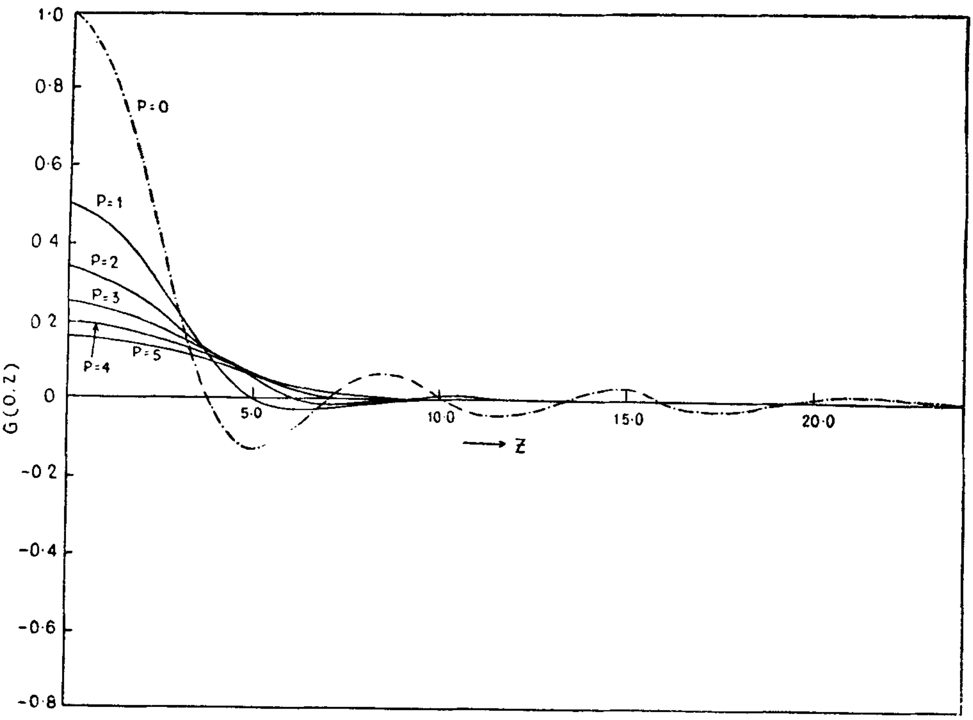

Figure 2. Distribution of diffracted light amplitude.

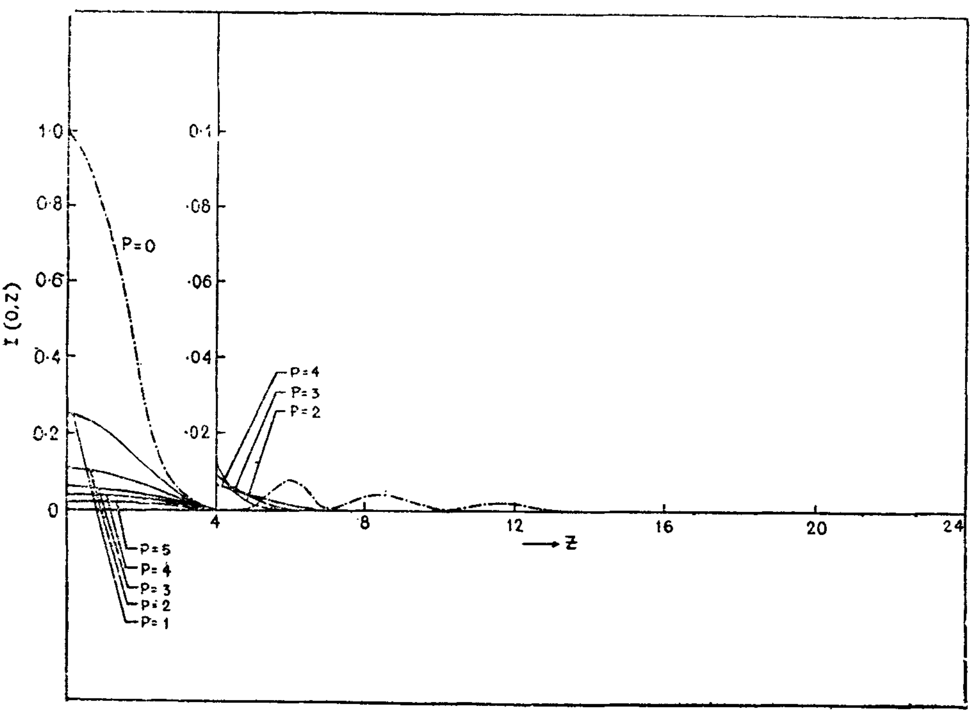

Figure 3. Point spread function for various values of *P*.

these filters are quite good 'apodisers', but their resolution aspect is significantly worse.

(ii) *Relative central irradiance and Strehl definition*

To determine the efficiency of non-airy type of pupils, two important parameters need to be investigated. The first is the relative value of the central irradiance in the diffraction pattern defined by

$$B(0) = \left| \frac{G(0,0)}{G_A(0,0)} \right|^2 \tag{3.1}$$

where $G(0,0)$ represents the central amplitude due to an airy type of pupil function and $G_A(0,0)$ that due to a non-airy type of objective. $B(0)$ is an important parameter because it gives an idea of relative strength of the central lobe in the diffraction pattern due to non-airy objectives. The reciprocal of this parameter is the Strehl definition ratio S (Kusakawa 1972).

(iii) *Total transmission factor*

Although the above parameter $B(0)$ is a good measure of the efficiency of a non-airy pupil, yet it does not give the total transmission factor relative to that due to the airy pupil function. The total amount of light flux received in the gaussian focal plane is very important from the consideration of the photometric point of view. The t tal transmission factor of the non-airy objectives is defined by

$$\tau = 2 \int_0^1 [f(r^2)]^2 \, r dr. \tag{3.2}$$

(iv) *Second order moment*

Another important parameter is the second order moment defined by

$$\triangle = \frac{\int_0^1 [f'(r^2)]^2 \, r dr}{\int_0^1 f(r^2) \, r dr} \tag{3.3}$$

which is normally strongly dependent on the distant feet of the diffracted field. The minimization of the second order moment maximizes the central maximum. The problem of minimization of the second order moment is known as Wiener apodisation problem and has been studied by Kusakawa and Okudaira (1972). The computed values of the above four parameters $B(0)$, S, τ and $\triangle$ for various values of P have been presented in table 1. These four parameters together determine the efficiency of the non-airy pupil functions.

(v) *Encircled flux and dispersion factors*

Now, the flux concentration in the neighbourhood of the centre of the diffraction pattern is measured by means of a parameter known as the encircled energy factor. The importance of the encircled energy within the circle of a specified radius as a point image assessment parameter was first pointed out by Rayleigh (1902). We

Table 1. Values of $B(0)$, τ, S and Δ

P	$B(0)$	τ	S	Δ
0	1·00	1·00	1·00	0·00
1	4·00	0·33	0·25	4·00
2	9·00	0·20	0·11	4·00
3	16·00	0·14	0·06	4·80
4	25·00	0·11	0·04	5·71
5	36·00	0·09	0·03	6·67
6	49·00	0·08	0·02	7·64
7	64·00	0·07	0·02	8·62
8	81·00	0·06	0·01	9·60
9	100·00	0·05	0·01	10·60
10	121·00	0·05	0·01	11·58

may define this encircled flux factor $E(\delta)$ as the ratio of the flux inside a circle of radius δ centered on the focus of the diffraction pattern to the total flux in the pattern. Thus

$$E(\delta) = \frac{\int_0^\delta |G(0, Z)|^2 Z dZ}{\int_0^\infty |G(0, Z)|^2 Z dZ} \tag{3.4}$$

which can again be written as

$$E(\delta) = 1 - D(\delta) \tag{3.5}$$

where

$$D(\delta) = \frac{\int_\delta^\infty |G(0, Z)|^2 Z dZ}{\int_0^\infty |G(0, Z)|^2 Z dZ} \tag{3.6}$$

is known as the dispersion factor. The minimization of the dispersion factor, under the constraint that the Strehl definition must have a certain prespecified value, is known as the modified Luneberg apodisation problem and the pupil function which minimizes the dispersion factor $D(\delta)$ under this restriction has been determined by Kusakawa (1972).

Figures 4 *a* and 4 *b* show the encircled flux factor and the dispersion factor curves respectively for various values of P. For airy type of pupils corresponding to $P = 0$, the first, second and third dark rings are at $\delta = 3{\cdot}83$, $7{\cdot}02$ and $11{\cdot}62$ respectively. It is found that the first dark ring already covers about 83% of the light flux and more than 90% of the light is contained within the circle bounded by the second dark ring. With non-zero values of P, the secondary side-lobes are completely suppressed and there are no secondary dark rings. The above

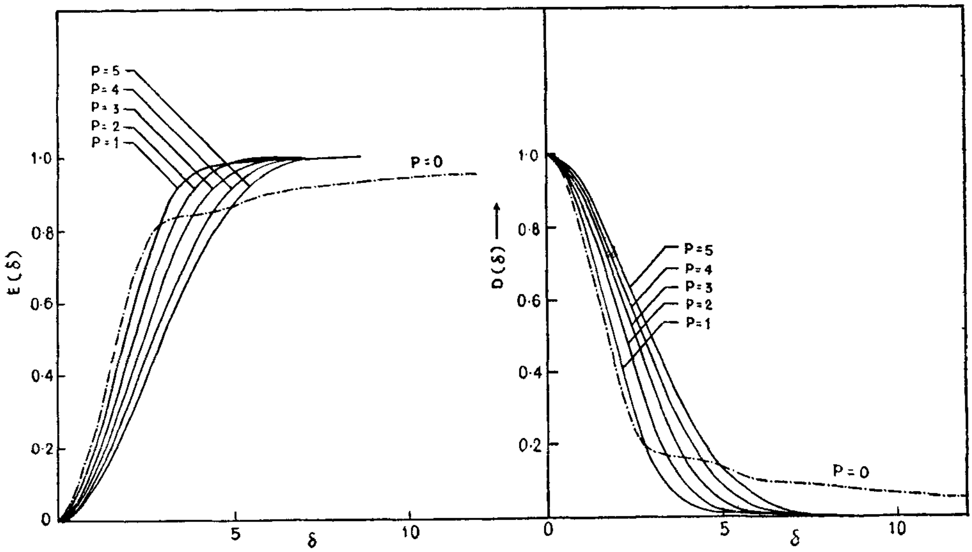

Figure 4 *a*. Curves showing total encircled energy $E(\delta)$ *versus* δ for various values of P. 4 *b*. Curves showing dispersion factor $D(\delta)$ *versus* δ for various values of P.

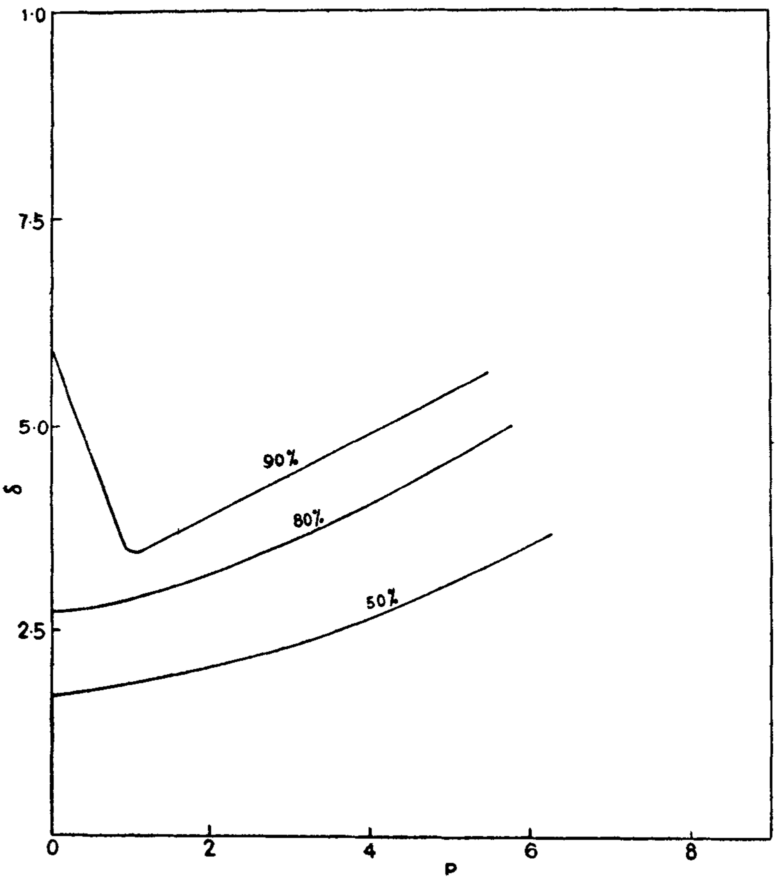

Figure 5. Curves showing the dependence cf the radius of a circle containing a fixed percentage of light energy on P.

criterion of assessing the performance of these filters, therefore, does not hold good. We can, however, show the dependence of the radius of the circle containing a given percentage of light energy on the value of P. This dependence has been shown graphically in figure 5 for 50%, 80% and 90% of the total energy.

Before concluding the discussions, it may be pointed out that the performance of an apodiser in partially coherent light may be quite unexpected and contrary to their desired performance. In fact, Biswas (1972) has studied the effects of coherence on the performance of some apodisers and concluded that it is very important to take into account the state of coherence of the aperture illumination. This aspect is, however, outside the scope of the present paper and we propose to publish in a separate communication the results of our investigations on two point resolution in partially coherent light with Straubel class of apodisation filters.

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