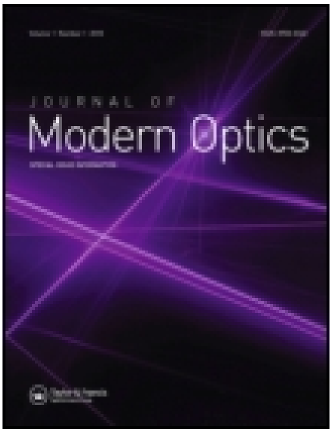

Optica Acta: International Journal of Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmop19

Correlation of Speckle Patterns Generated by a Diffuser Illuminated by Partially Coherent Light

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Published online: 03 Dec 2010.

OPTICA ACTA, 1983, VOL. 30, NO. 9, 1213–1216

Correlation of speckle patterns generated by a diffuser illuminated by partially coherent light

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(*Received 23 February 1983; revision received 14 July 1983*)

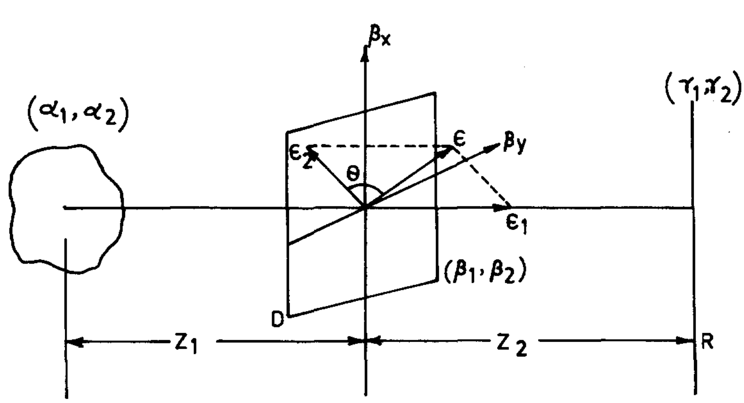
Abstract. In this paper it is shown that, under some experimental conditions, the speckle pattern generated by a diffuse object transilluminated by a partially space coherent light source suffers no decorrelation when the diffuse object is translated in three-dimensional space.

1. Introduction

Imaging or diffraction of diffuse objects with coherent or partially coherent light is always accompanied by speckle effects. The correlation of speckle patterns has been a subject of interest. In an earlier communication [1] Reddy *et al.* have shown that the speckle due to transillumination of a diffuser under partially space coherent light in two neighbouring planes remains correlated. In the present communication we shall show that, when the diffuser transilluminated by partially space coherent light is translated in three-dimensional space in an arbitrary direction, there exists a particular observation plane in which the corresponding speckle pattern remains correlated.

2. Theory

For mathematical simplicity, we consider only a one-dimensional case; extension to two dimensions is straightforward. In the geometry as shown in the figure, the



The diffuser, D, is illuminated by partially space coherent light and the resulting speckle is observed in the plane R before and after translation of D.

diffuser, of complex transmittance $D(\beta)$, is transilluminated by a partially space coherent light source whose mutual intensity function is $\Gamma(\alpha_1, \alpha_2)$.

Following Hopkins [2], we write the mutual intensity function at the plane of diffuser as

$$\Gamma(\beta_1, \beta_2) = \int_{\alpha_1}\int_{\alpha_2} \Gamma(\alpha_1, \alpha_2) \exp\left[\frac{ik}{2z_1}\{(\alpha_1-\beta_1)^2-(\alpha_2-\beta_2)^2\}\right] d\alpha_1\, d\alpha_2 \qquad (1)$$

where $k=2\pi/\lambda$ is the wavenumber associated with the wavelength λ used for illumination, and z_1 is the distance between the source and the diffuser.

Following the same propagation law, the mutual intensity function at the plane of observation is given by

$$\Gamma(\gamma_1, \gamma_2) = \int_{\alpha_1}\int_{\alpha_2}\int_{\beta_1}\int_{\beta_2} \Gamma(\alpha_1, \alpha_2) D(\beta_1) D^*(\beta_2) \times \exp\left[\frac{ik}{2z_1}\{(\alpha_1-\beta_1)^2-(\alpha_2-\beta_2)^2\}\right] \times \exp\left[-\frac{ik}{2z_2}\{(\beta_1-\gamma_1)^2-(\beta_2-\gamma_2)^2\}\right] d\alpha_1\, d\alpha_2\, d\beta_1\, d\beta_2 \qquad (2)$$

where z_2 is the distance between the diffuser and observation plane R.

The only observable parameter at this plane is intensity which can be obtained from $\Gamma(\gamma_1, \gamma_2)$ by letting $\gamma_1=\gamma_2=\gamma$. Therefore the intensity $I_1(\gamma)$ is given by

$$I_1(\gamma) = \int_{\beta_1}\int_{\beta_2} F_1(\beta_1, \beta_2) D(\beta_1) D^*(\beta_2) \exp\left[\frac{ik}{2}\left(\frac{z_2-z_1}{z_1 z_2}\right)(\beta_1^2-\beta_2^2)\right] \times \exp\left[-\frac{ik}{z_2}(\beta_2-\beta_1)\gamma\right] d\beta_1\, d\beta_2 \qquad (3)$$

where

$$F_1(\beta_1, \beta_2) = \int_{\alpha_1}\int_{\alpha_2} \Gamma(\alpha_1, \alpha_2) \exp\left[\frac{ik}{2z_1}(\alpha_1^2-\alpha_2^2)\right] \times \exp\left[\frac{ik}{z_1}(\alpha_2\beta_2-\alpha_1\beta_1)\right] d\alpha_1\, d\alpha_2. \qquad (4)$$

If the diffuser is translated along a certain direction in three-dimensional space as shown in the figure, by a distance ε, then ε_2 indicates the lateral-displacement vector $(\varepsilon_2\beta_x, \varepsilon_2\beta_y)$ and ε_1 is the axial component of the displacement along the z axis [3]. In the figure, θ stands for the angle between the directions of ε and ε_1. If ε is small enough to be written as

$$(z_1+\varepsilon_1)^{-1} \simeq \frac{1}{z_1}\left(1-\frac{\varepsilon_1}{z_1}\right),$$

and

$$(z_2-\varepsilon_1)^{-1} \simeq \frac{1}{z_2}\left(1+\frac{\varepsilon_1}{z_2}\right),$$

then the expressions (3) and (4) can be respectively given by

$$I_2(\gamma)=\int_{\beta_1}\int_{\beta_2} F_2(\beta_1,\beta_2)D(\beta_1)D^*(\beta_2)\exp\left[\frac{ik}{2}\left(\frac{z_2-z_1}{z_1z_2}\right)(\beta_1^2-\beta_2^2)\right]$$
$$\times\exp\left[-\frac{ik}{2}\left\{\frac{z_1^2+z_2^2}{(z_1z_2)^2}\right\}\varepsilon_1(\beta_1^2-\beta_2^2)\right]\exp\left[-\frac{ik}{z_2}(\beta_2-\beta_1)\right.$$
$$\left.\times\left\{\left(1+\frac{\varepsilon_1}{z_2}\right)\gamma+z_2\left(\frac{z_2-z_1}{z_1z_2}\right)\varepsilon_2-z_2\varepsilon_1\varepsilon_2\left(\frac{z_1^2+z_2^2}{z_1^2z_2^2}\right)\right\}\right]d\beta_1\,d\beta_2, \quad (5)$$

$$F_2(\beta_1,\beta_2)=\int_{\alpha_1}\int_{\alpha_2}\Gamma(\alpha_1,\alpha_2)\exp\left[\frac{ik}{2z_1}(\alpha_1^2-\alpha_2^2)\right]\exp\left[-\frac{ik\varepsilon_1}{2z_1^2}(\alpha_1^2-\alpha_2^2)\right]$$
$$\times\exp\left[\frac{ik}{z_1}\left(1-\frac{\varepsilon_1}{z_1}\right)\left\{\alpha_2(\beta_2-\varepsilon_2)-\alpha_1(\beta_1-\varepsilon_2)\right\}\right]d\alpha_1\,d\alpha_2. \quad (6)$$

It can be seen that besides the decorrelation term

$$\exp\left[\frac{-ik\varepsilon_1}{2z_1^2}(\alpha_1^2-\alpha_2^2)\right],$$

the terms of $F_1(\beta_1,\beta_2)$ and $F_2(\beta_1,\beta_2)$ are related homothetically in the ratio $(1-\varepsilon_1/z_1)$ in spite of the translation of the latter expression with respect to the former expression by a distance $-\varepsilon_2(1-\varepsilon_1/z_1)$. This decorrelation term is similar to that of the decorrelation term in [1, 4]. We can neglect this term provided it obeys the inequality

$$b^2\varepsilon_1/z_1^2 \ll \lambda$$

where $2b$ is the diameter of the source of illumination. However, under this condition $F_1(\beta_1,\beta_2)$ undergoes a radial magnification proportional to (ε_1/z_1) [1, 4] and $F_1(\beta_1,\beta_2)$ and $F_2(\beta_1,\beta_2)$ remain correlated. In expression (5) the term

$$\exp\left[-\frac{ik}{2}\left(\frac{z_1^2+z_2^2}{z_1^2z_2^2}\right)\varepsilon_1(\beta_1^2-\beta_2^2)\right]$$

is a function of the angular size of the diffuse object and it can be replaced by unity if

$$\frac{d^2}{z_2^2} \ll \frac{z_1^2\lambda}{\varepsilon_1(z_1^2+z_2^2)}$$

where d is the radius of the diffuser [5]. Thus, under these conditions, $I_1(\gamma)$ and $I_2(\gamma)$ are homothetic distributions. In the special case when $z_1=z_2=z$, the expressions (3), (4), (5) and (6) become, respectively,

$$I_1(\gamma)=\int_{\beta_1}\int_{\beta_2} F_1(\beta_1,\beta_2)D(\beta_1)D^*(\beta_2)\exp\left[\frac{-ik}{z}(\beta_2-\beta_1)\gamma\right]d\beta_1\,d\beta_2, \quad (7)$$

$$F_1(\beta_1,\beta_2)=\int_{\alpha_1}\int_{\alpha_2}\Gamma(\alpha_1,\alpha_2)\exp\left[\frac{ik}{2z}(\alpha_1^2-\alpha_2^2)\right]$$
$$\times\exp\left[\frac{ik}{z}(\alpha_2\beta_2-\alpha_1\beta_1)\right]d\alpha_1\,d\alpha_2, \quad (8)$$

$$I_2(\gamma)=\int_{\beta_1}\int_{\beta_2} F_2(\beta_1,\beta_2)D(\beta_1)D^*(\beta_2) \times \exp\left[\frac{-ik}{z}\left\{\left(1+\frac{\varepsilon_1}{z}\right)\gamma-\frac{2\varepsilon_1\varepsilon_2}{z}\right\}(\beta_2-\beta_1)\right]d\beta_1\,d\beta_2, \quad (9)$$

$$F_2(\beta_1,\beta_2)=\int_{\alpha_1}\int_{\alpha_2}\Gamma(\alpha_1,\alpha_2)\exp\left[\frac{ik}{2z}(\alpha_1^2-\alpha_2^2)\right] \times \exp\left[\frac{ik}{z}\left(1-\frac{\varepsilon_1}{z}\right)\left\{\alpha_2(\beta_2-\varepsilon_2)-\alpha_1(\beta_1-\varepsilon_2)\right\}\right]d\alpha_1\,d\alpha_2. \quad (10)$$

From expression (7) and (9) it is seen that the only cause for decorrelation between them is the presence of $F_2(\beta_1,\beta_2)D(\beta_1)D^*(\beta_2)$ and there will be complete decorrelation if the radial shift of $F_1(\beta_1,\beta_2)$ is larger than the size of the corresponding diffracting element of the diffuser D. Thus the correlation of speckle patterns depends on the source and on the structure of the diffuser D and is independent of its size. A photographic plate which records successively $I_1(\gamma)$ and $I_2(\gamma)$ gives rise, after processing, to a ring system [1, 4, 5], the contrast of which characterizes the degree of correlation. This property can be used to determine the roughness of the diffuser.

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