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Performance analysis of asynchronized synchronous machine

B. W. HOGG [a] & M. RAMA MURTHI [b]

[a] Department of Electrical Engineering and Electronics, University of Liverpool, Brownlow Hill, P.O. Box 147, Liverpool, L69 3BX, England

[b] Department of Electrical Engineering, Regional Engineering College, Warangal, A.P 4, India

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Performance analysis of asynchronized synchronous machine†

B. W. HOGG

Department of Electrical Engineering and Electronics,
University of Liverpool, Brownlow Hill, P.O. Box 147,
Liverpool L69 3BX, England

M. RAMA MURTHI

Department of Electrical Engineering, Regional Engineering College,
Warangal 4 (A.P.), India

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An asynchronized synchronous machine has two field windings and two special excitation control systems. The paper examines the performance of this type of machine during both synchronous and asynchronous operating conditions. The results illustrate the effects of subsidiary feedback signals in the control loops on equilibrium conditions and stability. They are supported by analysis, which establishes the conditions for stability and shows how these depend on the additional control signals.

Notation

δ = angle between quadrature axis and voltage vector of power system (called rotor angle).
ω_0 = system frequency, rad/sec.
ω = rotor angular velocity, rad/sec.
$p = d/dt$, differential operator.
H = inertia constant, kWs/kVA.
K_d = mechanical damping coefficient.
P = active power, p.u.
Q = reactive power, p.u.
R = resistance, p.u. (of armature if no subscript is used).
T_i = input torque from prime mover, p.u.
T_e = electrical output torque, p.u.
V = system voltage, p.u.
v = voltage, p.u.
i = current, p.u.
X = reactance, p.u.
μ = regulator gain.
θ = angle between stator e.m.f. and ordinate axis.
$E_f = (E_{fd}^2 + E_{fq}^2)^{1/2}$, resultant field excitation.
T' = transient time constant, sec.
T'' = sub-transient time constant, sec.
$X(p)$ = operational reactance.
$G(p)$ = operational function of p.

† Communicated by the Authors.

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Subscripts

0 = initial quantity.
d = direct axis.
q = quadrature axis.
fd = direct-axis field.
fq = quadrature-axis field.
ad = direct-axis mutual.
aq = quadrature-axis mutual.
b = busbar.
t = machine terminals (for currents, voltage and power).
t = transmission system (for impedances).
kd = direct-axis damper.
kq = quadrature-axis damper.
1δ = regulator, first derivative.
2δ = regulator, second derivative.
d0, q0 = field, open circuit.

1. Introduction

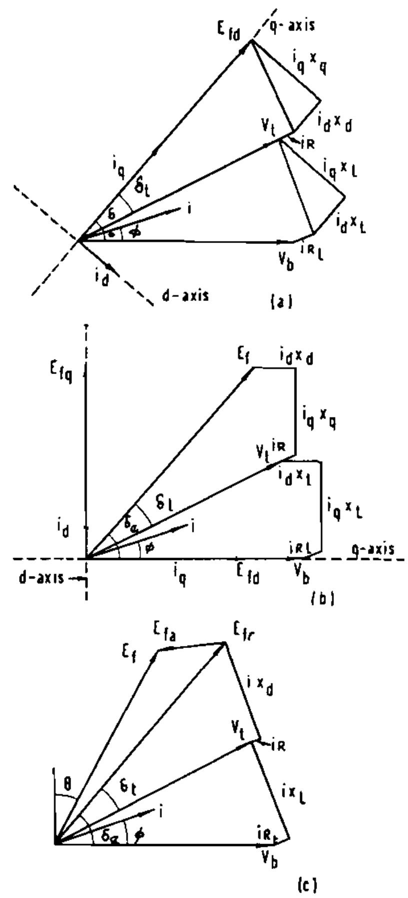
The phasor diagram for a conventional round-rotor synchronous generator supplying power to busbars at a leading power factor is shown in fig. 1 (*a*).

Any change in power can be brought about by changing the input to the prime mover, which alters the rotor angle. Increase of excitation with constant prime-mover input moves the phasor E_{fd} towards V_b, and since the excitation is rigidly fixed to the rotor, the rotor angle decreases. When automatic regulators are used the reference quantity can be adjusted to obtain the required change of excitation. A given amount of power at a specified power factor can be transmitted over a transmission line only when the sending voltage has a particular value. If the busbar voltage is constant, then for different reactive power conditions the terminal voltage has to be maintained at various levels. Since all the station auxiliaries are connected to the station transformer it is necessary to keep the terminal voltage constant under all conditions. Hence, in normal practice, any change in reactive power transmitted is brought about by changing the tap setting on the main step-up transformer. However, in a computer simulation it does not make any difference whether the regulator reference is changed or the tap setting is changed, provided saturation and the slight variations in transmission system parameters due to tap changes are neglected.

If a generator has two mutually perpendicular field windings, the resultant excitation is the vector sum of E_{fd} and E_{fq}. The provision of dual excitation permits a number of alternative control schemes to be considered (Rama Murthi and Hogg 1972).

To transfer a given power at a particular power factor over a transmission line, the magnitude of the resultant excitation and its electrical angular displacement with respect to the busbar voltage must be the same as in the conventional machine. However, the resultant excitation can be obtained by any suitable combination of voltages on the two field windings. The physical position of the rotor depends on this choice. Alternatively, a fixed position of the rotor can be maintained by suitably controlling, either manually or

Fig. 1



Phasor diagrams for equilibrium conditions. (*a*) Conventional synchronous generator. (*b*) Synchronous machine with two field-windings at 90°. (*c*) Asynchronized synchronous machine.

automatically, the excitation on one of the windings and adjusting the excitation on the other winding as is required to suit the system conditions. The phasor diagram shown in fig. 1 (*b*) corresponds to the case when the rotor angle δ is zero. Neglecting the small variations due to stator-circuit resistances, it can be seen from this diagram that when $\delta = 0$ (that is, when the quadrature axis is in alignment with busbar voltage) any change in d-axis excitation brings about a change in the component of excitation in phase with the busbar voltage, and alteration of the q-axis excitation changes the quadrature component. This means that a change of d-axis excitation changes the reactive power, and a change of input power makes it necessary to adjust the q-axis excitation to maintain δ at zero. This is usually achieved by an angle regulator on the q axis, and the accuracy of maintaining this angle depends on the loop gain (Rama Murthi *et al.* 1970). Obviously, the d-axis excitation system should be capable of producing both positive and negative excitation so that the machine can absorb or generate reactive power.

If the angle between the two field windings is other than 90°, the rotor angle must be held at a different value (Soper and Fagg 1969) to achieve the same flexibility of operation. The principle is that one of the windings should directly control the excitation in phase with the busbar or terminal voltage.

A further development of dual excitation is the asynchronized synchronous machine (Botvinnick 1964). This is referred to as an a.s. machine.

During steady operating conditions, to produce a constant torque, it is necessary for the stator e.m.f. phasor due to the resultant excitation flux, (E_t), to rotate at the same speed as the voltage vector of the power system, but displaced at a particular angle depending on the line and power conditions. Constant excitation on the field windings, therefore, produces constant torques when the rotor is running at steady synchronous speeds, but at rotor speeds other than synchronous this is no longer true. However, by employing a suitable excitation control, (Botvinnick 1964) it is possible to ensure that E_t and V rotate at the same speed even during steady asynchronous running of the rotor. This means that such a control scheme should make E_t move backwards on the rotor at a speed equal to the slip speed, if the rotor is running at supersynchronous speeds, or move forward if the rotor is running subsynchronously (Smith 1958). A phasor diagram similar to that for the conventional machine can still be drawn, fig. 1 (*c*), but no fixed positions of the d and q axes can be represented on the diagram during asynchronous running. Asynchronous operation also induces slip-frequency currents in the rotor windings, because of the relative movement of the rotor in the field of the armature flux. These rotor currents cause an asynchronous component of e.m.f. (E_{ta}) to be induced in the stator. Thus, the total induced e.m.f. (E_{tr}) in the stator, due to the rotor currents, is the resultant of the asynchronous component (E_{ta}) and the normal synchronous component E_t. There are methods (Botvinnick 1964), however, which compensate for this asynchronous component of e.m.f., and such compensation makes the operation of the a.s. machine very similar to that of a synchronous machine under steady-state asynchronous conditions.

A change in the magnitude of the excitation (E_t), that is, in the synchronous component of e.m.f., does not change the reactive power, since the angle θ is fixed by the control system. However, the synchronous component of torque changes, and if the total torque is not altered, the unbalance of torque on the shaft is nullified by a change in the asynchronous component of e.m.f., and hence the asynchronous torque, which is only possible with a change of slip speed. Therefore, if the reactive power is to be changed without altering the original slip conditions, both the magnitude of E_t and its phase position θ should be altered.

This paper presents the results of computer simulation and analysis of the performance of an a.s. machine.

2. Mathematical model

The system consists of a single generator supplying power to a large system through a transformer and two parallel transmission lines. The mathematical representation of the machine and transmission system is based on Park's model (1929, 1933). It is assumed that there are two identical field windings in the direct and quadrature axes, respectively, and the rotor iron is represented by one equivalent damper winding in each axis. These equations and the values of parameters are given in the Appendix.

A special form of excitation control is an essential part of the a.s. machine (Botvinnik 1964). This consists basically of sine and cosine functions of rotor angle, e.g.

$$E_{\mathrm{fd}} = E_{\mathrm{f}} \sin(\delta + \theta), \tag{1}$$

$$E_{\mathrm{fq}} = E_{\mathrm{f}} \cos(\delta + \theta). \tag{2}$$

Additional derivative signals are employed to improve the accuracy of regulation and the stability limits. The equations are

$$E_{\mathrm{fd}} = (E_{\mathrm{f}} + \mu_{1\delta} p\delta + \mu_{2\delta} p^2\delta) \sin(\delta + \theta), \tag{3}$$

$$E_{\mathrm{fq}} = (E_{\mathrm{f}} + \mu_{1\delta} p\delta + \mu_{2\delta} p^2\delta) \cos(\delta + \theta). \tag{4}$$

There is also a time delay in each regulator loop.

If the generator is to be run at steady asynchronous speeds, the excitation system may be required to handle very large amounts of power, which depend on the slip and the power developed by the generator. This is due to the unbalance that exists between the stator power and the input shaft power. For this reason the analysis is confined to small values of slip.

3. Equilibrium conditions

3.1. *Analysis*

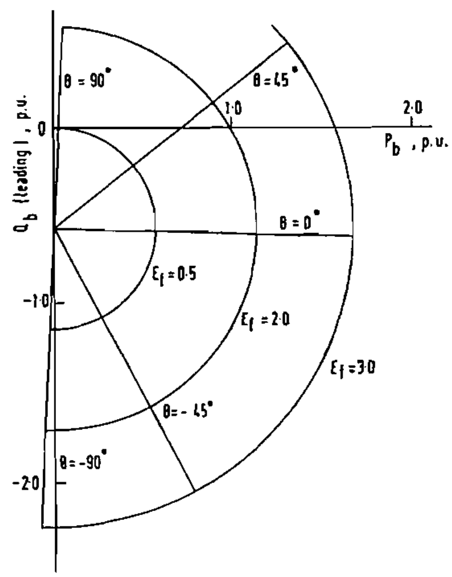
If the rotor is assumed to be running at a constant slip, s_0, and the excitation control is as given by eqns. (1) and (2), all the quantities (currents, voltages, flux, etc.) will vary sinusoidally at slip-frequency, but have different phase positions, as viewed from the rotor structure. Therefore, these quantities can be expressed in a more general form as $A_i \cos\delta + B_i \sin\delta$. For example, ψ_{fd} can be taken to be $A_{\mathrm{fd}} \cos\delta + B_{\mathrm{fq}} \sin\delta$, and so on. These substitutions are made in eqns. (7) to (18), and the sine and cosine terms on either side are equated. The resulting equations can be expressed in matrix form, and, for a known busbar voltage and an assumed set of E_{f}, θ, etc., all the A and B values can be found by a standard inversion procedure. Thus the various quantities are now known as functions of δ, in the form $A_i \cos\delta + B_i \sin\delta$.

The values of active and reactive power can now be calculated. These will be independent of δ because of the form of excitation control chosen. The terminal voltage will, however, be a function of δ, and can be obtained from eqns. (21)–(22) in the form $A_{\mathrm{vt}} \cos\delta + B_{\mathrm{vt}} \sin\delta$. When the excitation control contains derivative functions of δ, as in eqns. (3) and (4), the value of E_{f} in the above procedure is replaced by $(E_{\mathrm{f}} + \mu_{1\delta} s_0 w_0)$, as $p^2\delta$ is zero in the steady state.

3.2. *Results*

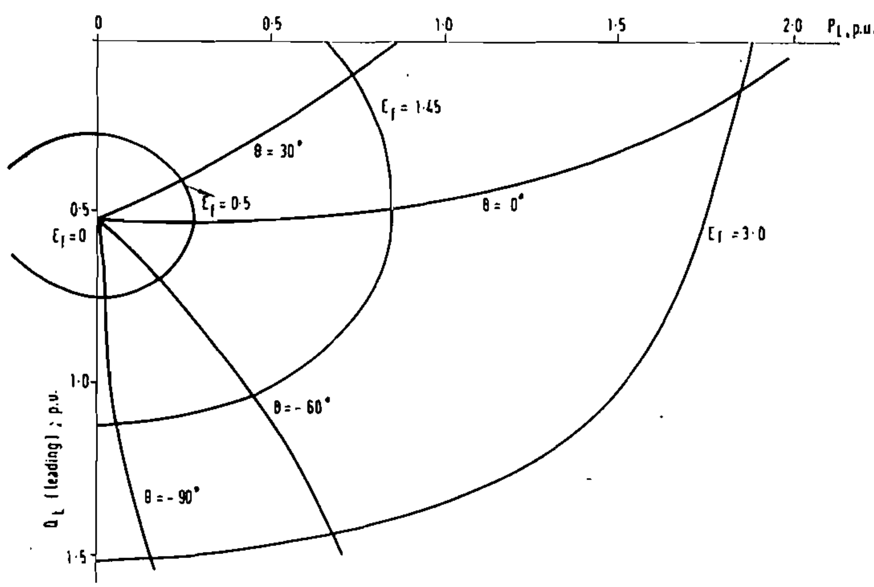
The steady-state characteristics can be illustrated by graphs showing the effects of E_{f} and θ on the real power (P) and reactive power (Q) for constant values of slip. Figures 2–6 show that adjustments of both E_{f} and θ are necessary to obtain a particular set of P and Q. In fig. 2 the slip is zero, and θ is varied from $-90°$ to $+90°$. A similar figure can be obtained for negative P_{b} if E_{f} is made negative, or alternatively if θ becomes $\theta + 180°$. Figure 3 shows the active and reactive power at the machine terminals.

Fig. 2



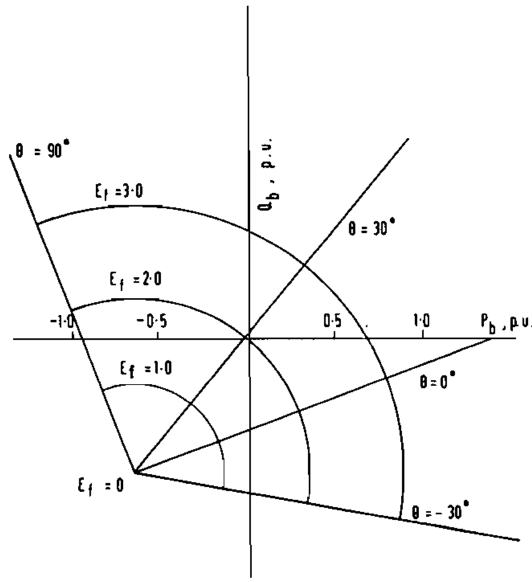
P_b/Q_b characteristics of a.s. machine, $s_0=0\%$.

Fig. 3



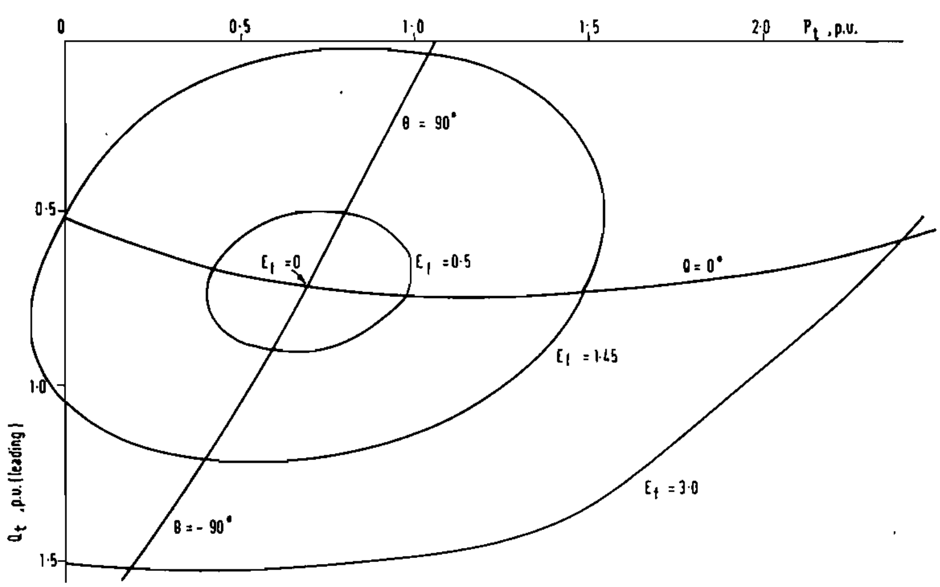
P_t/Q_t characteristics of a.s. machine, $s_0=0\%$.

Fig. 4



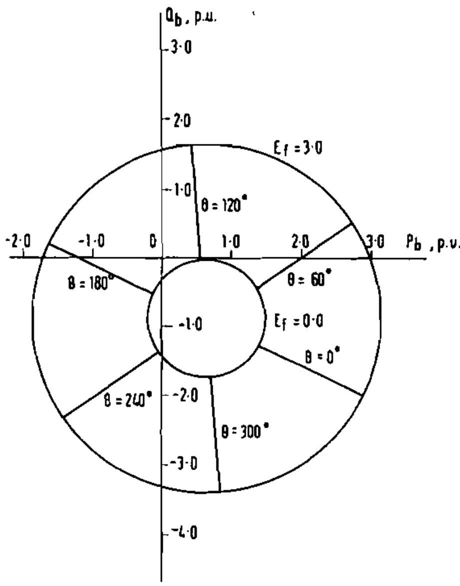
P_b/Q_b characteristics of a.s. machine, $s_0 = -0{\cdot}1\%$.

Fig. 5



P_t/Q_t characteristics of a.s. machine, $s_0 = +0{\cdot}1\%$.

Fig. 6



P_b/Q_b characteristics of a.s. machine, $s_0=0{\cdot}1\%$, $\mu_{1\delta}=5{\cdot}0$, $\mu_{2\delta}=0$.

The effect of operating at non-zero values of slip is shown in fig. 4 for $s=-0{\cdot}1\%$. The point corresponding to $E_f=0$ is now in the third quadrant, and the line $\theta=0$ has rotated. This figure also shows that operation at negative slip requires increased excitation to produce the desired output at the busbar. The graph moves into the fourth quadrant when the slip is positive, as shown in fig. 5.

In computing the above results, it has been assumed that $\mu_{1\delta}=\mu_{2\delta}=0$. If the excitation is defined by eqns. (3) and (4), and the machine is run at constant non-zero slip, then $p^2\delta=0$, but the term $\mu_{1\delta}p\delta$ will be constant. Figure 6 shows the P/Q curves when $\mu_{1\delta}=5$. For $E_f=0$, the graph is no longer defined by a single point, but becomes a circle whose radius depends on the value of $\mu_{1\delta}$. Negative values of E_f must be used to obtain operating points within this circle.

The curves in figs. 2–6 show the equilibrium conditions of the machine for both synchronous and asynchronous conditions, but do not give any indication of stability.

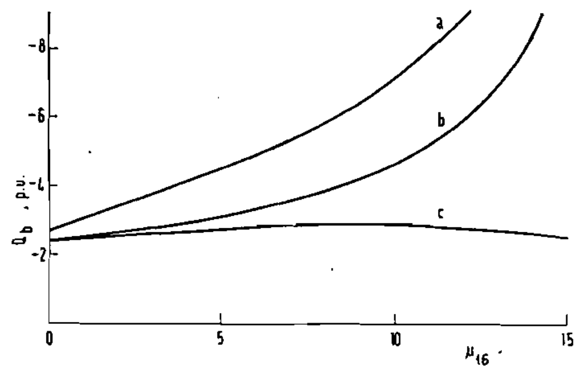
4. Stability

4.1. *Method of computation*

In an a.s. machine the concept of synchronous stability is no longer valid, in the sense of achieving a constant rotor angle, and is replaced by that of speed stability, i.e. obtaining a steady value of $p\delta$.

The steady-state speed stability is examined by applying small oscillations theory to linearize the system equations at an operating point, so that the closed-loop transfer function $\Delta T_i/\Delta\delta$ is determined (Rama Murthi *et al.* 1970). It is shown in the Appendix that, for an a.s. machine, the constant term of the

Fig. 7



Steady-state speed stability boundaries. (*a*) $P_b = 1 \cdot 0$ p.u., with $p^2\delta$ feedback. (*b*) $P_b = 0$, with $p^2\delta$ feedback. (*c*) $P_b = 0$, without $p^2\delta$ feedback.

characteristic equation is zero. The equation can therefore be divided by p, to give the characteristic equation of the transfer function $\Delta T_i / \Delta p\delta$. This is examined for stability by applying the Routh–Hurwitz criterion.

4.2. *Results and analysis*

The results are based on zero initial slip, and presented as boundaries in the $\mu_{1\delta}/Q_b$ plane. With no feedback signals from $p\delta$ and $p^2\delta$ ($\mu_{1\delta} = \mu_{2\delta} = 0$) the reactive power limit is independent of P_b and is $-V^2/X'_d$. To increase the speed stability, the excitation must depend on the slip, as defined in eqns. (3) and (4). Stability boundaries for $P_b = 0$ and $P_b = 1 \cdot 0$ p.u. are shown in fig. 7. These curves were obtained using a detailed model, and illustrate the results of the approximate analysis in the Appendix. When $\mu_{2\delta} = 0$, the reactive absorption limit increases to a maximum as $\mu_{1\delta}$ is increased, and then decreases for higher values of $\mu_{1\delta}$. Two conditions must be satisfied for stability.

1. The reactive absorption must be less than the limit given by

$$Q_{b0} = -\frac{V^2}{X_d} \cdot \frac{T_{d0}' + T_{d0}''}{T_d' + T_d''} - \mu_{1\delta} \frac{V \cos\theta}{X_d(T_d' + T_d'')}, \tag{5}$$

where $T_{d\sigma}'$, etc. are as defined by Adkins (1959).

2. The following inequality must be satisfied

$$\mu_{2\delta} - \mu_{1\delta} \frac{T_d' T_d''}{T_d' + T_d''} > \frac{V T_{d0}' T_{d0}''}{\cos\theta} - \frac{T_d' T_d''(T_{d0}' + T_{d0}'')}{T_d' + T_d''}. \tag{6}$$

As $\mu_{1\delta}$ increases, the reactive absorption limit, as set by condition 1, increases because $\cos\theta$ is positive when $P_b > 0$. But if $\mu_{1\delta}$ exceeds a certain limit (for any $\mu_{2\delta}$), then condition 2 is not satisfied and instability occurs.

The stability can be improved by compounding $p\delta$ with a positive $p^2\delta$ signal, which satisfies condition 2 so that the limit set by 1 can be achieved. An indefinite increase of $\mu_{2\delta}$ does not raise the boundary beyond a certain level, because Q_{b0} depends on $\mu_{1\delta}$ but not $\mu_{2\delta}$.

The limit can reach very high values when both derivative signals are used, and is generally greater with large values of P_b. This can be observed from condition 1, in which the term containing $\mu_{1\delta}$ depends on $\cos\theta$. If Q_b is constant, $\cos\theta$ increases with P_b when $Q_b < -V^2/X_d$, so the limit specified by condition 1 also increases, provided that $\mu_{1\delta}$ is positive.

5. Conclusions

The paper illustrates the characteristics of an asynchronized synchronous machine during synchronous and steady asynchronous operating conditions. A method for examinating speed stability is presented, and the results are interpreted by analysis (Appendix). The results underline the importance of $p\delta$ and $p^2\delta$ feedback signals in the control loops of an a.s. machine, and provide a basis for selecting suitable values of feedback gains ($\mu_{1\delta}$ and $\mu_{2\delta}$).

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Appendix

1. *System equations*

The mathematical representation of the synchronous machine employs Park's equations (1929, 1933), and the rotor iron is represented by one damper winding in each axis. The equations are :

$$v_d = \frac{1}{\omega_0} p\psi_d - \frac{\omega}{\omega_0}\psi_q - Ri_d, \tag{7}$$

$$v_q = \frac{1}{\omega_0} p\psi_q + \frac{\omega}{\omega_0}\psi_d - Ri_q, \tag{8}$$

$$v_{fd} = \frac{1}{\omega_0} p\psi_{fd} + R_{fd} i_{fd}, \tag{9}$$

$$v_{fq} = \frac{1}{\omega_0} p\psi_{fq} + R_{fq} i_{fq}, \tag{10}$$

$$v_{kd} = 0 = \frac{1}{\omega_0} p\psi_{kd} + R_{kd} i_{kd}, \tag{11}$$

$$v_{kq} = 0 = \frac{1}{\omega_0} p\psi_{kq} + R_{kq} i_{kq}, \tag{12}$$

$$\psi_d = -X_d i_d + X_{ad} i_{fd} + X_{ad} i_{kd}, \tag{13}$$

$$\psi_q = -X_q i_q + X_{aq} i_{fq} + X_{aq} i_{kq}, \tag{14}$$

$$\psi_{fd} = -X_{ad} i_d + X_{fd} i_{fd} + X_{ad} i_{kd}, \tag{15}$$

$$\psi_{fq} = -X_{aq} i_q + X_{fq} i_{fq} + X_{aq} i_{kq}, \tag{16}$$

$$\psi_{kd} = -X_{ad}i_d + X_{ad}i_{fd} + X_{kd}i_{kd}, \quad (17)$$

$$\psi_{kq} = -X_{aq}i_q + X_{aq}i_{fq} + X_{kq}i_{kq}, \quad (18)$$

$$T_e = \psi_d i_q - \psi_q i_d, \quad (19)$$

$$T_i = \frac{2H}{\omega_0} p^2\delta + K_d p\delta + T_e. \quad (20)$$

All quantities are expressed in per-unit values, except time (seconds) and rotor angle (radians).

The transmission system is represented by lumped series inductance and resistance, and the transformer magnetizing impedance is neglected. The equations are

$$v_{db} = v_d - R_t i_d + \frac{\omega}{\omega_0} X_t i_q - \frac{X_t}{\omega_0} p i_d, \quad (21)$$

$$v_{qb} = v_q - R_t i_q - \frac{\omega}{\omega_0} X_t i_d - \frac{X_t}{\omega_0} p i_q. \quad (22)$$

Parameters

The parameters are basically those of a 200 Mw 16·5 kv turbogenerator, which was assumed to have a quadrature-axis field-winding identical to the direct-axis field-winding.

Generator

$X_d = 1{\cdot}60$ p.u.,	$X_q = 1{\cdot}60$ p.u.,
$X_{fd} = 1{\cdot}60$ p.u.,	$X_{fq} = 1{\cdot}60$ p.u.,
$X_{ad} = 1{\cdot}45$ p.u.,	$X_{aq} = 1{\cdot}45$ p.u.,
$X_{kd} = 1{\cdot}51$ p.u.,	$X_{kq} = 1{\cdot}51$ p.u.,
$R = 0{\cdot}002$ p.u.,	$V = 1{\cdot}00$ p.u.,
$R_{fd} = 0{\cdot}001$ p.u.,	$R_{fq} = 0{\cdot}001$ p.u.,
$R_{kd} = 0{\cdot}009$ p.u.,	$R_{kq} = 0{\cdot}009$ p.u.,
$H = 3{\cdot}00$ kwsec/kVA,	$K_d = 0{\cdot}004$ p.u.

Transmission line

$R_t = 0{\cdot}10$ p.u./line, $X_t = 0{\cdot}30$ p.u./line.

Transformer

$X = 0{\cdot}10$ p.u.

2. *Analysis*

The symbols used in this analysis are standard for synchronous machines (Adkins 1959).

The stator voltage equations can be written in terms of operational quantities, as follows

$$V \cos \delta = G_d(p) \,.\, E_{fd} - X_d(p) \,.\, i_d - Ri_q, \quad (23)$$

$$V \sin \delta = G_q(p) \,.\, E_{fq} + X_q(p) \,.\, i_q - Ri_d. \quad (24)$$

Also, the torque eqns. (19) and (20), can be combined.

$$\therefore \quad T_{\rm i}=\left(\frac{H}{\pi f}p^2+K_{\rm d}p\right)\cdot\delta+\psi_{\rm d}i_{\rm q}-\psi_{\rm q}i_{\rm d}, \tag{25}$$

where

$$\psi_{\rm d}=V\cos\delta+Ri_{\rm q}$$

and

$$\psi_{\rm q}=-(V\sin\delta+Ri_{\rm d}).$$

Assuming small perturbations, and linearizing eqns. (23)–(25) by making first-order approximations in the Taylor series, and using the suffix 0 for initial quantities,

$$\therefore \quad \Delta T_{\rm i}=\left(\frac{H}{\pi f}p^2+K_{\rm d}p+Q_{\rm b0}\right)\Delta\delta+(V\cos\delta_0+2Ri_{\rm q0})\Delta i_{\rm q} + (V\sin\delta_0+2Ri_{\rm d0})\Delta i_{\rm d}, \tag{26}$$

where

$$Q_{\rm b0}=Vi_{\rm d}\cos\delta_0-Vi_{\rm q0}\sin\delta_0,$$

$$0=-V\sin\delta_0\,.\,\Delta\delta+X_{\rm d}(p)\,.\,\Delta i_{\rm d}+R\Delta i_{\rm q}-G_{\rm d}(p)\,.\,\Delta E_{\rm fd}, \tag{27}$$

$$0=V\cos\delta_0\,.\,\Delta\delta-X_{\rm q}(p)\,.\,\Delta i_{\rm q}+R\Delta i_{\rm d}+G_{\rm q}(p)\,.\,\Delta E_{\rm fq}. \tag{28}$$

The excitation of the a.s. machine is defined by eqns. (3) and (4). Linearizing, and assuming $s_0=0$, then

$$\Delta E_{\rm fd}=[E_{\rm f}\cos(\delta_0+\theta)+(\mu_{1\delta}p+\mu_{2\delta}p^2)\sin(\delta_0+\theta)]\Delta\delta \tag{29}$$

and

$$\Delta E_{\rm fq}=[-E_{\rm fq}\sin(\delta_0+\theta)+(\mu_{1\delta}p+\mu_{2\delta}p^2)\cos(\delta_0+\theta)]\Delta\delta. \tag{30}$$

Assuming perfect symmetry, and neglecting the armature circuit resistance and mechanical damping coefficient ($R=0$, $K_{\rm d}=0$),

$$\therefore \quad X_{\rm d}(p)=X_{\rm q}(p)=\frac{1+T_{\rm d}'p}{1+T_{\rm d0}'p}X_{\rm d},$$

$$G_{\rm d}(p)=G_{\rm q}(p)=\frac{1}{1+T_{\rm d0}'p}.$$

Substituting eqns. (29) and (30) in (27) and (28),

$$\Delta i_{\rm d}=\frac{\Delta\delta}{X_{\rm d}(p)}[V\sin\delta_0+G_{\rm d}(p)E_{\rm f}\cos(\delta_0+\theta)+(\mu_{1\delta}p+\mu_{2\delta}p^2)\sin(\delta_0+\theta)], \tag{31}$$

$$\Delta i_{\rm q}=\frac{\Delta\delta}{X_{\rm d}(p)}[V\cos\delta_0-G_{\rm d}(p)E_{\rm f}\sin(\delta_0+\theta)+(\mu_{1\delta}p+\mu_{2\delta}p^2)\cos(\delta_0+\theta)]. \tag{32}$$

Substituting in eqn. (26) and simplifying,

$$\frac{\Delta T_{\rm i}}{\Delta\delta}=\frac{H}{\pi f}p^2+Q_{\rm b0}+\frac{1}{X_{\rm d}(p)}[V^2+G_{\rm d}(p)\{-VE_{\rm f}\sin\theta + V(\mu_{1\delta}p+\mu_{2\delta}p^2)\cos\theta]. \tag{33}$$

Substituting for $X_d(p)$ and $G_d(p)$, the characteristic equation reduces to the form

$$a_4p^4+a_3p^3+a_2p^2+a_1p+a_0=0, \tag{34}$$

where

$$a_4=\frac{H}{\pi f}T_d'T_d'',$$

$$a_3=(T_d'+T_d'')\frac{H}{\pi f},$$

$$a_2=T_d'T_d''Q_{b0}+\frac{V^2}{X_d}T_{d0}'T_{d0}''+\frac{\mu_{2\delta}V\cos\theta}{X_d},$$

$$a_1=(T_d'+T_d'')Q_{b0}+\frac{V^2}{X_d}(T_{d0}'+T_{d0}'')+\frac{\mu_{1\delta}V\cos\theta}{X_d},$$

$$a_0=Q_{b0}+\frac{V^2}{X_d}-E_f\frac{V\sin\theta}{X_d}.$$

Under initial conditions ($s_0=0$),

$$E_{fd}=E_f\sin(\delta+\theta),$$

$$E_{fq}=E_f\cos(\delta+\theta),$$

$$V\sin\delta=X_qi_q-Ri_d-E_{fq},$$

$$V\cos\delta=-X_di_d+Ri_q+E_{fd}.$$

$$\therefore \qquad Q_{b0}=-\frac{V^2-E_fV\sin\theta}{X_d}.$$

Therefore $a_0=0$ for *all* conditions.

Dividing across by p in eqn. (34) gives the characteristic equation of the transfer function $\Delta T_i/\Delta p\delta$, which determines speed stability :

$$a_4p^3+a_3p^2+a_2p+a_1=0. \tag{35}$$

The first column of the Routh array is

$$a_4,\ a_3,\ a_2-\frac{a_1a_4}{a_3},\ a_1.$$

An examination of these terms leads to the following observations :

(i) When subsidiary signals from $p\delta$ and $p^2\delta$ are not used, $\mu_{1\delta}=\mu_{2\delta}=0$, so that for stability $a_1>0$ (since a_4 and a_3 are positive), i.e.

$$Q_{b0}>-\frac{V^2}{X_d}\left(\frac{T_{d0}'+T_{d0}''}{T_d'+T_d''}\right), \tag{36}$$

and when damper windings are not considered the limit is $Q_b=-V^2/X_d'$.

(ii) With an increase in $\mu_{1\delta}$, Q_{b0} can be made more negative while a_1 remains positive. This is evident from the expression for a_1, since $\cos\theta$ is positive because $-90° < \theta < 90°$ for all positive values of P_b. The limit of Q_{b0} is now

$$Q_{b0} = -\frac{V^2}{X_d}\left(\frac{T_{d0}' + T_{d0}''}{T_d' + T_d''}\right) - \frac{\mu_{1\delta} V \cos\theta}{X_d(T_d' + T_d'')}. \tag{37}$$

(iii) However, when Q_{b0} has a large negative value, a_2 is decreased so that another first term of the Routh array, namely $(a_2 - a_1 a_4/a_3)$, may become negative. Substituting the values of a_1, a_2, a_3 and a_4 it can be seen that for this term to be positive the following inequality should be satisfied :

$$\frac{V^2}{X_d}\left[T_{d0}'T_{d0}'' - T_d'T_d''\left(\frac{T_{d0}' + T_{d0}''}{T_d' + T_d''}\right)\right] + \frac{V\cos\theta}{X_d}\left[\mu_{2\delta} - \mu_{1\delta}\left(\frac{T_d'T_d''}{T_d' + T_d''}\right)\right] > 0. \tag{38}$$

This can be achieved by increasing the value of $\mu_{2\delta}$.

(iv) Equation (37) gives the limit of Q_{b0} for any fixed value of $\mu_{1\delta}$ and does not depend on $\mu_{2\delta}$. Thus, by changing $\mu_{1\delta}$ alone this limit can be changed. However, this is possible only if inequality (38) is satisfied, which requires a certain minimum value of $\mu_{2\delta}$.

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