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Automated Optimum Design of Machine Tool Structures for Static Rigidity, Natural Frequencies and Regenerative Chatter Stability

A computational capability for the automated optimum design of complex machine tool structures to satisfy static rigidity, natural frequency and regenerative chatter stability requirements is developed in the present work. More specifically, the mathematical programming techniques are applied to find the minimum-weight design of Warren-type lathe bed and horizontal knee-type milling machine structures using finite-element idealization. The Warren-type lathe bed is optimized to satisfy torsional rigidity and natural frequency requirements, whereas, the milling machine structure is optimized with constraints on static rigidity of the cutter centre, natural frequency and regenerative chatter stability.

Introduction

It is customary to base the structural design of any machine tool primarily upon the requirements of static rigidity and minimum natural frequency of vibration. The effects of different machining parameters like cutting speed, feed and depth of cut, as well as the size of the work piece, also have to be considered by a machine tool structural designer. For a tentative design, the machine tool is analyzed for natural frequencies, dynamic rigidity and chatter stability. Based on the results of this analysis, suitable modifications are made, by a process of trial and error, to satisfy the design requirements. This procedure of trial and error is adopted, mainly because of the complex nature of machine tool structures and also because of the lack of a suitable design procedure that can handle all the requirements simultaneously. A survey of the available literature indicates that the use of computers in the machine tool manufacturing industry has, up to the present time, been confined to the implementation of finite element techniques for the purpose of static and dynamic analysis only, and the potentialities of optimization techniques have not yet been exploited by the machine tool structural designers.

Taylor and Tobias [1][1] described the application of a finite-element program involving the use of slender beams to represent the structural parts of a radial-arm drill and a lathe. Cowley and Fawcett [2] analysed a plain milling machine for static deflections, natural frequencies and mode shapes. The authors have studied the effect of flexibilities between joints on natural frequencies and mode shapes. Badauri et al., [3] have studied, both analytically and experimentally, the effects of breadth-to-depth ratios and lacing angles of Warren-type lathe beds and investigated the possibility of obtaining optimum stiffness-to-weight ratios of Warren beams from their results. Thornley et al., [4] studied the static and dynamic behavior of Warren beams by experimental methods. A model milling machine was analysed for deflections, natural frequencies and mode shapes by a C.I.R.P. group in which professors from several European Universities and one Japanese University participated [5]. Andrews and Tobias [6] distinguished between forced vibration and chatter in horizontal milling and concluded that self-excited vibration was a significant limitation in horizontal milling, but forced vibration was relatively unimportant. Taylor [7] described a technique for predicting from design drawings the chatter stability of machine tool structures by computing responses to excitation from computed modes of vibration and assumed damping constants. Out of several available theories for relating chatter stability to response loci [8, 9, 10], the authors used regenerative chatter theory with penetration-rate effects neglected [8]. This process is illustrated with reference to a lathe model and three versions of a milling machine. The authors established the superiority of machines with box-type overarm over those with bar-type overarm, as characterized by higher values of minimum stability under all cutting conditions.

[1] Numbers in brackets designate References at end of paper.

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Tlusty and Polacek [11] analyzed the role of vibration of machine tool structures in the process of chatter with the idea of increasing their stabilities for all possible cutting conditions. With reference to horizontal milling machines, these authors recognized the "weak links" of the structure and recommended maximum rigidity for the overarm and its joint with the column. They also recommended that the natural frequency of vibration corresponding to the vertical mode should be slightly higher than the one corresponding to the horizontal mode. The satisfication of ths requirement insures a good stability in "down-milling," because of the advantageous interaction of both the modes. The effect of certain design features of horizontal milling machines was also investigated by Said [12]. Koenigsberger and Tlusty gave a state-of-the-art discussion of machine tool structural design in 1971 [13].

During the last few years several optimization investigations have been reported in the fields of civil engineering and aircraft structural design. Using the method of feasible directions, Fox and Kapoor [14] reported a capability for the minimum-weight optimum design of planar truss-frame structures with inequality constraints on the maximum dynamic displacements, stress and natural frequencies. In the fields of aircraft structural design, Rao [15] developed a method for the optimum design of aircraft wings to satisfy static, dynamic and aeroelastic requirements.

Statement of the Problem

When a means for predicting the behavior of any design is available, when limitations on the performance and other external constraints on the design can be stated, and when some acceptance criteria can be established, a design modification problem can be cast as a mathematical programming problem. A general mathematical programming problem can be stated as follows.

Minimize a multivariable function $f(\vec{X})$ subject to the constraints

$$g_j(\vec{X}) \leq 0, j = 1, m \tag{1}$$

where $\vec{X}$ is an n-dimensional vector consisting of the variables X_1, $X_2, \ldots, X_n$. The function $f(\vec{X})$ in equation (1) is called the objective or criterion function. The minimization of the weight of Warren-type lathe bed and horizontal knee-type milling machine structures is taken as the objective function in the present work.

Design Constraints

The design requirements to be satisfied in the case of Warren-type lathe beds are: *i*) the torsional rigidity must be greater than a specified quantity and *ii*) the natural frequencies of the structure are to be excluded from certain bands. In the case of milling machines, the design constraints are: *i*) the maximum deflection of the centre of the cutter in any direction should not exceed a certain prescribed value, *ii*) the natural frequencies of the structure are to be excluded from certain bands, and *iii*) the machine should not chatter under the stated cutting conditions. These constraints are stipulated so as to achieve a high-quality surface finish, to avoid mild harmonic forcing that might cause resonance and to increase the metal-removal rate which may be affected by the onset of chatter. In the design of metalcutting machine tools, the static- and dynamic-stiffness requirements are often more important than the load-carrying-capacity requirements, since the induced stresses corresponding to the permissible deformations are generally far less in value than those permissible for the various materials. Hence, strength was not considered as a design constraint in the present work.

Optimization Problem

The complete optimization problem can now be stated as follows:

i) For a lathe bed:
Minimize

$$f(\vec{X}) = \sum_{j=1}^{N_P} V_j \rho_j + i \sum_{j=1}^{N_F} V_{j+N_P} \rho_{j+N_P} \tag{2}$$

subject to

$$d_L^{(u)} - d_L \geqslant 0 \tag{3}$$

$$\omega_1 - \omega_1^{(l)} \geqslant 0 \tag{4}$$

$$\omega_2 - (\omega_1 + 100) \geqslant 0 \tag{5}$$

and

$$X_j^{(l)} \leq X_j \leq X_j^{(u)}, j = 1, 2, \ldots, n \tag{6}$$

where, X_1 = thickness of the main members, X_2 = thickness of the lacing diagonals, X_3 = width of the stiffener on the main and lacing diagonals, X_4 = depth of the stiffener on the main and lacing diagonals, and X_5 = width of the lathe bed as shown in Fig. 1.

ii) For a milling machine structure:
Minimize

$$f(\vec{X}) = \sum_{j=1}^{N_P} V_j \rho_j + \sum_{j=1}^{N_F} V_{j+N_P} \rho_{j+N_P} \tag{7}$$

Nomenclature

A_{avg} = average chip cross-sectional area
B = width of engagement
B_{LIM} = limiting chip width
d = depth of cut
d_c = maximum deflection of cutter centre
d_L = angle of twist of lathe bed
D = diameter of milling cutter
E = Young's modulus
f = objective function
F_T = tangential force of milling cutter
F_R = radial force on milling cutter
F_H = horizontal force on milling cutter
F_V = vertical force on milling cutter
P_A = axial force on milling cutter
g_i = ith inequality constraint
G = in-phase cross receptance
G_{MIN} = minimum negative in-phase cross receptance of cutter centre relative to table
$|G_{MIN}^{(u)}|$ = upper bound on absolute value of G_{MIN}
l = superscript used to denote lower bound
m = number of constraints
n = number of design variables
N = number of degrees of freedom
N_P = number of plate elements
N_F = number of frame elements
$P_T|_{avg}$ = average tangential force in plain milling
r = Coupling constant
r_{LIM} = Limiting value of r
r_k = kth penalty parameter
R = Resultant force on cutter centre
S_t = Feed per tooth in plain milling
t_{avg} = Average uncut chip thickness in plain milling
u = Superscript used to denote upper bound
v = Feed rate
V_j = Volume of jth element
V = Cutting velocity
X_j = jth design variable
$\vec{X}$ = Design vector
Z = Number of teeth on milling cutter
Z_i = Number of teeth in engagement
ρ_j = Density of jth element
ω_i = ith natural frequency of vibration
θ = Helix angle of milling cutter
χ = Angle of engagement of cutter with workpiece
χ_i = Angle of engagement of ith tooth
χ_{yi} = Angle between ith tooth and normal to cut surface χ = Friction angle
γ = Normal rake angle
ϕ = Shear angle
Φ = Penalty function
τ_s = Dynamic shear stress
ν = Poisson's ratio

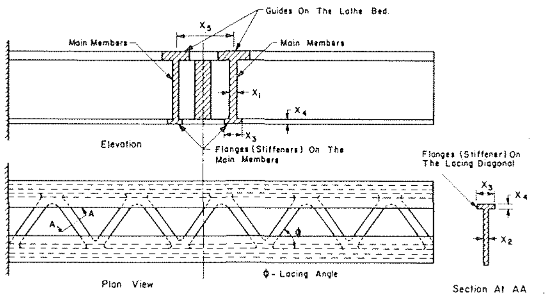

Fig. 1 Warren-Type Lathe Bed Details

subject to

$$d_c^{(u)} - d_c \geqslant 0 \tag{8}$$

$$|G_{\mathrm{MIN}}^{(u)}| - |G_{\mathrm{MIN}}| \geqslant 0 \tag{9}$$

$$\omega_1 - \omega_1^{(l)} \geqslant 0 \tag{10}$$

$$\omega_2 - (\omega_1 + 100) \geqslant 0 \tag{11}$$

and

$$X_j^{(l)} \le X_j \le X_j^{(u)}, j = 1, 2, \ldots, n \tag{12}$$

where, X_1 = breadth at the column at the base, X_2 = thickness of the overarm, X_3 = width of the machine, X_4 = thickness of the column and table, X_5 = breadth of the column at the top, and X_6 = square cross-sectional dimensions of the ribs on the overarm and its joint with the column as shown in Fig. 2.

Structural Analysis

Idealization. In the present work, the finite-element displacement method is used to model the machine tool structures. The idealization using triangular plate elements with a 3-term in-plane and a 9-term transverse displacement model [16, 17] and frame elements have been found to be efficient. The triangular plate elements are used to idealize the main members and lacing diagonals of lathe beds, and the column, overarm and table of milling machine structures. The frame elements are used to idealize the carriage guides, stiffeners on the main members and on the lacing diagonals of lathe beds. In the case of milling machines, the frame elements are used to model the ribs on the overarm, the overarm joint with the column, the arbor and the arbor support. In the present work, identically oriented finite elements of the same size and shape (having the same transformation matrix between local and global coordinate systems) are grouped together in generating element-stiffness and mass matrices. This resulted in savings of about 20 to 80 percent of the computer time in generating the global-stiffness and mass matrices.

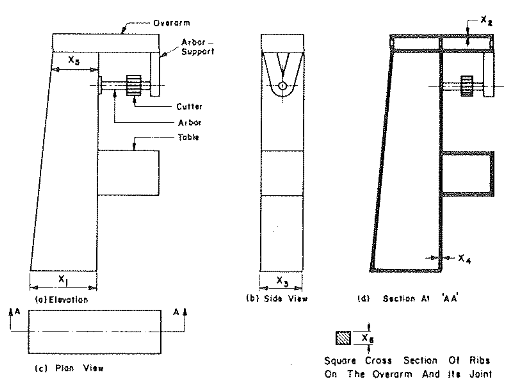

Fig. 2 Horizontal Knee-Type Milling Machine Structural Details

Static and Dynamic Analysis

For the dynamic optimization of large structures using finite-element methods, a designer is generally confronted with two problems, namely, the computer storage and the computer time. The eigenvalue problem has been solved by using one of the most efficient solution techniques developed by Bathe and Wilson for large structural systems [18]. In this technique, the Rayleigh-Ritz subspace iteration algorithm, which solves the eigenvalue problem directly without a transformation to the standard form, has been used. In this work, the Cholesky decomposition of symmetric banded matrices, storing only the upper triangular matrix, is used for solving the equilibrium equations. By using a judicious discretization and node-numbering scheme, it has been possible to reduce the band width of the stiffness matrix of the structure. A smaller band width, apart from reducing the computer storage, considerably reduces the computer time for static and eigen-solutions.

The frequency response of the structure has been obtained by using modal coordinates and by taking the damping matrix proportional to a linear combination of the stiffness and mass matrices. Since the present day knowledge is not sufficient to estimate the modal damping factors from the blueprints of a given structure, the values of damping factors have to be obtained from experimental results on similar structures. In the present work, equal modal damping factors of 0.06 have been used for the first few modes.

Regenerative Chatter Analysis

In machine tools, chatter occurs due to the interaction of the cutting forces and the machine tool structural dynamics. In this work, a simple dynamic cutting force relationship, assuming a direct proportionality between the force and the undeformed chip thickness, is taken. No definite criterion has been established so far for taking the critical proportionality constants between the force and the undeformed chip thickness in plain milling operation. A study of proportionality constants in plain milling has been made to choose a critical value for incorporation into the chatter stability constraint. A method of including G_{MIN} in the design is also developed. The change in the cutting force, P, due to the variation in uncut chip thickness, Y, is given by

$$\Delta P = -B \cdot r \cdot \Delta Y \tag{13}$$

The change in the cutting force will again result in a variation giving an undulating surface on the workpiece, and the regeneration of the undulation proceeds in subsequent cuts. To derive the limits of stability for the machine tool and the cutting system, it is necessary to assume a force relationship for the cutting process and relate this to the machine dynamics. The governing equations of motion cannot be solved directly, but the analysis can be simplified very much by assuming the condition at the limit of stability. In the present work, the basic theory of stability analysis as given by Tlusty and Polacek [19] is used in applying equation (13) for the design of milling machine structures. According to this analysis, at the limit of stability the following equation is obtained

$$\frac{1}{2Br} = -G \tag{14}$$

If the limiting or the maximum value of the coupling coefficient, r_{LIM}, can be estimated from all the commonly used cutting conditions, the structure can be designed for regenerative chatter stability by putting a constraint on G_{MIN} as

$$|G_{\mathrm{MIN}}^{(u)}| - |G_{\mathrm{MIN}}| \geqslant 0 \tag{15}$$

where

$$G_{\mathrm{MIN}}^{(u)} = -\frac{1}{2B_{\mathrm{LIM}} r_{\mathrm{LIM}}} \tag{16}$$

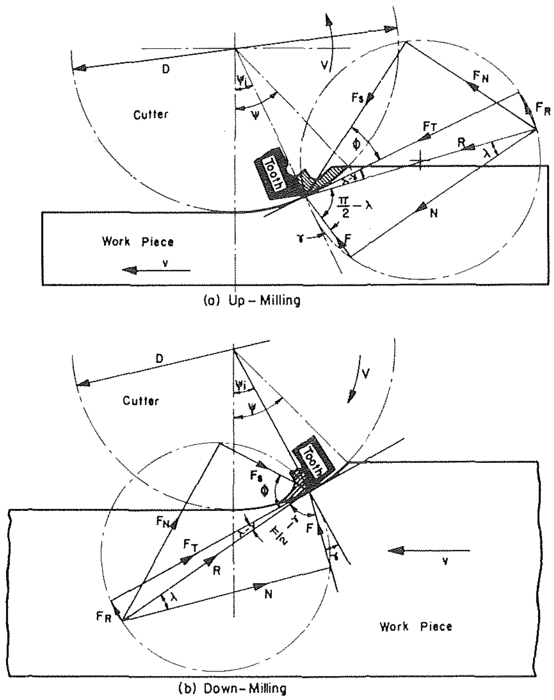

Fig. 3 Merchant's Circle Diagram for Cutting Forces in Plain Milling Simulated by a Rotating Single-Point Cutting Tool

Therefore it becomes essential to estimate r_{LIM} so that the value of G_{LIM} can be estimated.

Determination of r_{LIM}

i) Basic theory of metal cutting:

Referring to Fig. 3(*a*), the horizontal force, F_H, can be written as [20]

$$F_H = \frac{B \cdot \tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} (S_t \cdot \text{Sin}\chi_i) \times \text{Cos}(\lambda - \gamma - \chi_i) \ldots \quad (17)$$

and the vertical force, F_V, as

$$F_V = \frac{B \cdot \tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} (S_t \cdot \text{Sin}\chi_i) \, \text{Sin}(\chi - \gamma - \chi_i) \quad (18)$$

where $S_t \cdot \text{Sin}\ \chi_i$ indicates the uncut chip thickness at *i*th tooth. For a displacement, ΔY, of the cutter center relative to the workpiece along the normal to the cut surface, the uncut chip thickness at the *i*th tooth is given by $\Delta Y \cdot \text{Cos}\ \chi_{yi}$, where χ_{yi} is the angle between *i*th tooth and the normal direction to the cut surface as shown in Fig. 4. For this displacement, ΔY, the new horizontal force, F_H^*, and the new vertical force, F_V^*, become

$$F_H^* = \frac{B \cdot \tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} (S_t \, \text{Sin}\chi_i - \Delta Y \cdot \text{Cos}\chi_{yi}) \cdot \text{Cos}(\lambda - \gamma - \chi_i) \ldots \quad (19)$$

and

$$F_V^* = \frac{B \cdot \tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} (S_t \, \text{Sin}\chi_i - \Delta Y \cdot \text{Cos}\chi_{yi}) \cdot \text{Sin}(\lambda - \gamma - \chi_i) \ldots \quad (20)$$

For equations (13), (17), (18), (19) and (20), the value of r is obtained from

$$r_H = \frac{\tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} \text{Cos}\chi_{yi} \, \text{Cos}(\lambda - \gamma - \chi_i) \quad (21)$$

$$r_V = \frac{\tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} \text{Cos}\chi_{yi} \, \text{Sin}(\lambda - \gamma - \chi_i) \quad (22)$$

Therefore

$$r = (r_H{}^2 + r_V{}^2)^{1/2} \quad (23)$$

Similarly for down-milling,

$$r_H = \frac{\tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} \text{Cos}\chi_{yi} \, \text{Cos}(\lambda - \gamma + \chi_i) \quad (24)$$

$$r_V = \frac{\tau_s}{\text{Sin}\phi \, \text{Cos}(\phi + \lambda - \gamma)} \sum_1^{Z_i} \text{Cos}\chi_{yi} \, \text{Sin}(\lambda - \gamma + \chi_i) \quad (25)$$

For $\lambda = 18$ deg $\gamma = 10$ deg, $\phi = 37$ deg and $\tau_s = 353.05\ \text{N/m}^2$ [20], the values of r calculated for various values of χ are tabulated in Table 1.

ii) Coupling coefficients from Vulf's formula [21]

Based on the following equation for the average tangential force in horizontal milling,

$$P_T|_{\text{avg}} = \frac{c}{(t_{\text{avg}})^k} \cdot A_{\text{avg}} \quad (26)$$

the value of resultant force on the cutter is taken approximately as $P_T|_{\text{avg}}$ and the value of r is computed as [20]

$$r = \frac{c \cdot Z \cdot k}{\pi (\text{Cos}\phi)^{k+1}} \frac{d/D^{\frac{1-k}{2}}}{s_t{}^k} \quad (27)$$

where c and k are constants.

The static forces on the cutter, F_H, F_V and F_A, computed by using the parameters
$D = 100$ mm, $B = 90$ mm, $Z = 12$ teeth, $S_t = 0.1$ mm/tooth, $\chi = 30$ deg $\theta = 25$ deg, $c = 140$ (for mild steel) and $k = 0.28$ (for mild steel), are given by

$$F_H \simeq P_T|_{\text{avg}} = 8{,}237.88\ N$$

$$F_V \simeq 0.2 P_T|_{\text{avg}} = 1{,}647.58\ N$$

$$F_A \simeq 0.2 P_T|_{\text{avg}} = 1{,}647.58\ N$$

$$R = (F_H{}^2 + F_V{}^2 + F_A{}^2)^{1/2} = 8{,}581.13\ N$$

Substituting for the various parameters, equation (27) can be written as

$$r = (331) \left(\frac{d}{D}\right)^{0.36} \quad (28)$$

The values of r for various values of d/D are given in Table 2, along with the corresponding values of χ.

It can be observed from Tables 1 and 2 that the values of r obtained by using equation (28) agree very well with those obtained by using the basic theory of metalcutting. In this work, the value of r_{LIM} is taken as $1618 \times 10^6\ \text{N/m}^2$ and B_{LIM} as 0.09 m.

In the chatter stability constraint, the upper limit on $|G_{\text{MIN}}|$ is taken as

$$|G_{\text{MIN}}{}^{(u)}| = \frac{1}{2 B_{\text{LIM}} r_{\text{LIM}}} = 3.059 \times 10^{-9}\ m/N$$

Table 1 Values of Coupling Constants from Basic Theory of Metalcutting

	Coupling constants (N/m²)	
ψ	up-milling	down-milling
35°	1756×10^6	1550×10^6
40°	1559×10^6	1539×10^6
45°	1559×10^6	1530×10^6

Table 2 Values of Coupling Constants from Empirical Formula

ψ	(d/D)	(N/m²)
35°	.09	1363×10^6
40°	.116	1490×10^6
45°	.146	1618×10^6

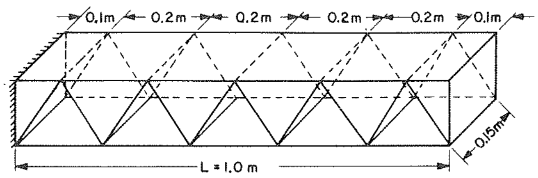

Fig. 5 Warren-Type Lathe Bed

Solution Procedure

The constrained optimum design problem is cast as a nonlinear mathematical programming problem. The interior penalty-function method, with a variable metric unconstrained minimization technique, is used to solve the minimum-weight design problem. In this method the objective function is augmented with a penalty term consisting of the constraints as shown below:

$$\Phi(\vec{X}, r_k) = f(\vec{X}) - r_k \sum_j \frac{1}{g_j(\vec{X})} \tag{29}$$

The minimizing step lengths in the unconstrained minimization are determined by the cubic interpolation method. The computation of the gradient and the slope of Φ-function has been carried out by a finite-difference method which incorporates the rates of changes of the response quantities with respect to the design varibles.

Illustrative Examples

The optimization problems formulated in the previous section have been solved to demonstrate the feasibility and effectiveness of designing complex structures with multiple behavior constraints. The computer program developed is quite general and the elements used for the idealization are sufficiently general to idealize several types of machine tool structures. The static and dynamic analysis programs as well as the optimization program are also written as general subroutines. Hence the optimum design of any other type of machine tool structure can be accomplished by using the present computer program by making little modifications.

Example 1 Lathe-Bed Design

The first example considered is a Warren-type lathe bed shown in Fig. 5. The overall dimensions of this bed are taken from the bed of lathe machine model No. MB02, year 1967, manufactured by Mysore Kirloskar Ltd., Harihar, India. This bed is optimized with a lower limit of 630 rad/s on the fundamental frequency and an upper limit of 0.00174 rads on the torsional deflection when a torque of 1274.91 N-m

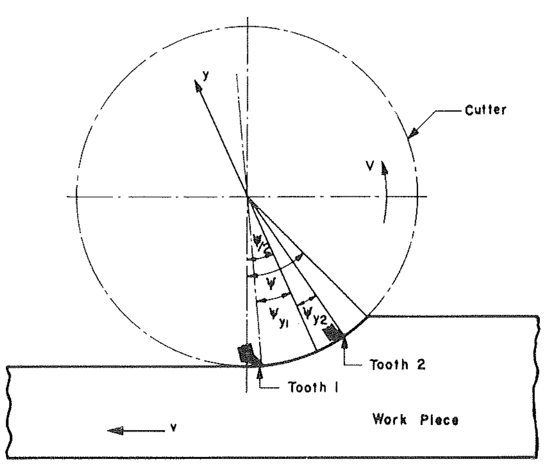

Fig. 4 Orientation of Cutting Teeth in Action with Respect to the Normal to the Cut Surface

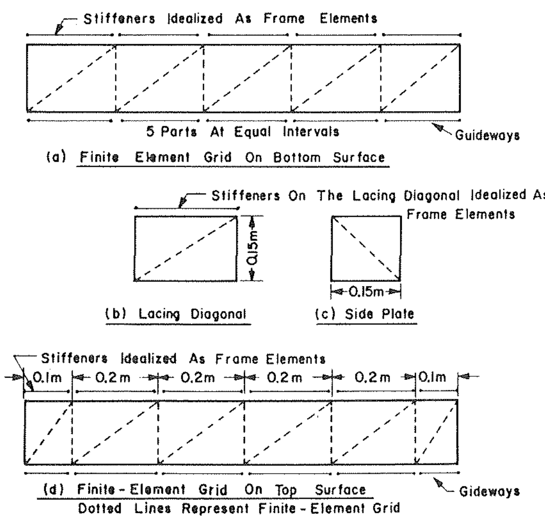

Fig. 6 Details of Idealization of Warren-Type Lathe Bed

is applied at one end of the structure by keeping the other end fixed. This torque corresponds to a power of 2.984 kW of the turning is done at the lowest spindle speed of 22 rpm. In this design, 4 design variables and 10 design constraints are considered. The finite-element modelling for this structure is shown in Fig. 6. The idealization of the structure consists of 26 node points, 46 triangular plate-bending elements and 32 frame elements. The number of degrees of freedom considered in the static and eigenvalue analysis is 132. The band width of the stiffness matrix is 36. The guides on the main members are taken to be rectangular in cross-section (0.065 m × 0.02 m).

The material of the structure is taken as grey cast iron with $\rho = 7.2 \times 10^3$ kg/m³, $E = 10.98 \times 10^{10}$ N/m², and $\nu = 0.25$. The lower limits on X_1, X_2, X_3 and X_4 are taken as 0.01 m, 0.01 m, 0.0175 m, and 0.01 m and the upper limits as 0.02 m, 0.02 m, 0.032 m, and 0.022 m, respectively. The behavior constraints include an upper bound of 0.00174 rad on the maximum torsional deflection and a lower bound of 630 rad/s on the fundamental natural frequency of the bed. The optimization results are shown in Table 3. The progress of the optimization path, showing the cumulative number of one-dimensional minimizations versus the weight of the structure, is given in Fig. 7. The least-weight design has a weight of 470.0 N with a reduction of 45.5 percent compared to the starting design.

At the optimum point, the side constraints, X_1, and X_2, are at their lower bounds, and none of the behavior constraints are active. The natural frequency, ω_1, gradually increased from 718 rad/s in the initial design to 735 rad/s for the optimum design, whereas, ω_2 gradually increased from 978 rad/s in the initial design to 949 rad/s for the optimum design. The number of one-dimensional minimizations is 10 and the total computer time taken is 35 minutes on an IBM 370/155 computer.

Example 2 Lathe-Bed Design

In this example, the same lathe bed shown in Fig. 5 is optimized by

Table 3 Optimization Results of Warren-Type Lathe Bed (Example 1)

Design Variable	Initial Design	Bounds Lower	Bounds Upper	Optimum Design
X_1	0.018 m	0.01 m	0.02 m	0.01029 m*
X_2	0.018 m	0.01 m	0.02 m	0.01029 m*
X_3	0.029 m	0.0175 m	0.0320 m	0.01970 m
X_4	0.019 m	0.01 m	0.0220 m	0.01110 m
Torsional Deflection (10^{-3} radians)	0.44	—	1.74	0.826
First National Frequency (radians/second)	718	630	—	735
Objective Function (weight of lathe bed)	865.4 N	—	—	470.0 N

Reduction in Weight Obtained by Optimization = 395.4 N (45.5%)
Number of One-Dimensional Minimizations = 10
Computer Time Required = 35 minutes on an IBM 370/155 system

* Active Constraint.

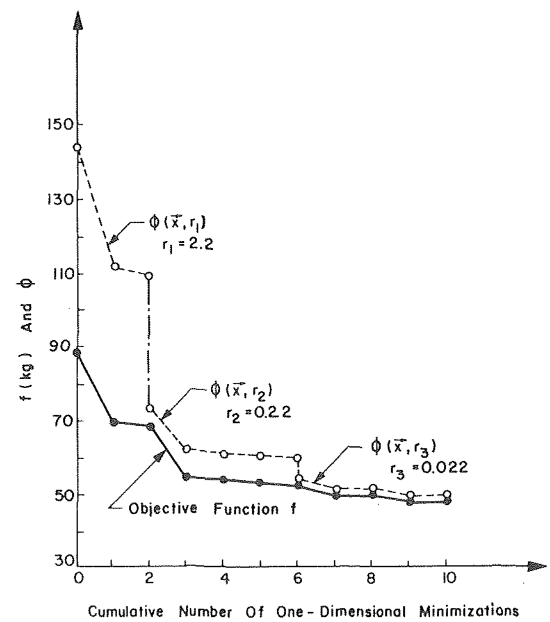

Fig. 7 Progress of Optimization Path for Example 1

taking 5 design variables and 14 design constraints. The design variables X_1, X_2, X_3 and X_4 are same as before. The width of the lathe bed is included as the fifth design variable, X_5, in this example. A limitation is also placed on the second natural frequency in this example. The initial design vector is chosen such that the behavior constraints are nearly satisfied at $\vec{X}_0$. This was done to see whether any weight could be removed from this starting design vector. The starting design corresponds to a weight of 857.8 N.

In addition to the side constraints taken in example 1, the following restrictions are also included in the present case:

$$X_2 - X_3 \leq 0$$

$$0.10\,m - X_5 \leq 0$$

$$X_5 - 0.18\,m \leq 0$$

The behavior constraints are taken as

$$d_L - 0.0005 \text{ rad} \leq 0$$

$$700 \text{ rad/s} - \omega_1 \leq 0$$

$$\omega_1 + 100 - \omega_2 \leq 0$$

The optimization results are shown in Table 4. The torsional deflection constraint is active in this example. The width of lathe bed, X_5, at optimum design is 0.135 m and it corresponds to a lacing angle

Table 4 Optimization Results of Warren-Type Lathe Bed (Example 2)

Design Variable	Initial Design	Bounds Lower	Bounds Upper	Optimum Design
X_1	0.0175 m	0.01 m	0.02 m	0.01605 m
X_2	0.0175 m	0.01 m	0.02 m	0.01553 m
X_3	0.0300 m	0.0175 m	0.0320 m	0.02764 m
X_4	0.0200 m	0.01 m	0.0220 m	0.01676 m
X_5	0.1325 m	0.10 m	0.18 m	0.13504 m
Torsional Deflection (10^{-3} radians)	0.453	0.50	0.5	0.499*
First Natural Frequency (radians/second)	718	700	—	734*
Second Natural Frequency (radians/second)	1272	$\omega_1 + 100$	—	979
Objective Function (weight of lathe bed)	857.8 N	—	—	759.1 N

Reduction in Weight Obtained by Optimization = 98.7 N (11.45%)
Number of One-Dimensional Minimizations = 14
Computer Time Required = 55 minutes on an IBM 370/155 system

* Active Constraint.

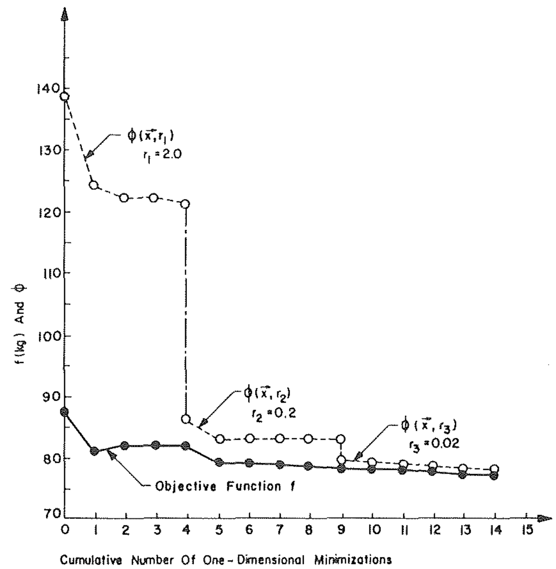

Fig. 8 Progress of Optimization Path for Example 2

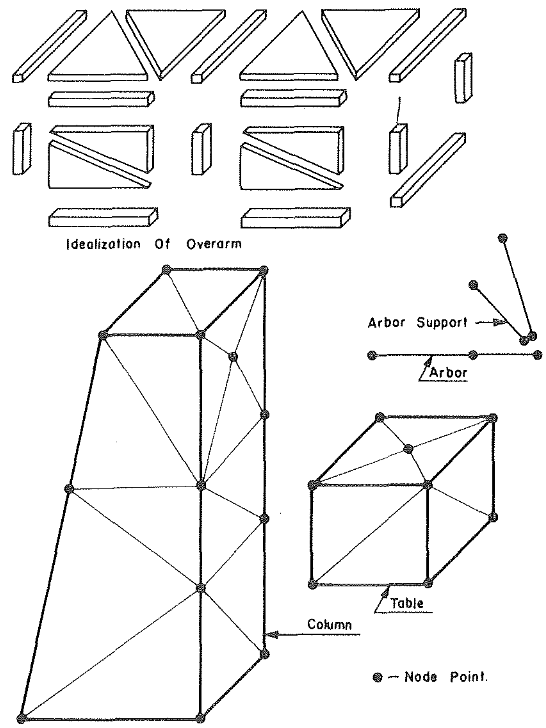

Fig. 10 Finite-Element Modelling of Horizontal Milling Machine (Examples 3(a) and 3(b))

of 53.5 deg. The proposed least weight design has a weight of 759.1 N with a reduction of 11.45 percent compared to the starting design. The progress of the optimization path is shown in Fig. 8. The number of one-dimensional minimizations is 14 and the computer time taken is 55 minutes.

Example 3(a) Horizontal Knee-Type Milling Machine Structural Design

The horizontal knee-type milling machine structure shown in Fig. 9 is considered for optimization in this example. The finite-element modelling for this structure is shown in Fig. 10. In this example, 6 design variables and 16 design constraints are considered. The idealization of the structure consists of 30 node points, 50 triangular plate-bending elements and 18 frame elements. The number of elastic degrees of freedom is 152 and the band width of the stiffness matrix is 42. In this problem, about 20 percent of the computer time was saved in generating the global-stiffness and mass matrices by grouping finite elements of the same size and shape having the same transformation matrix from local to global coordinate system.

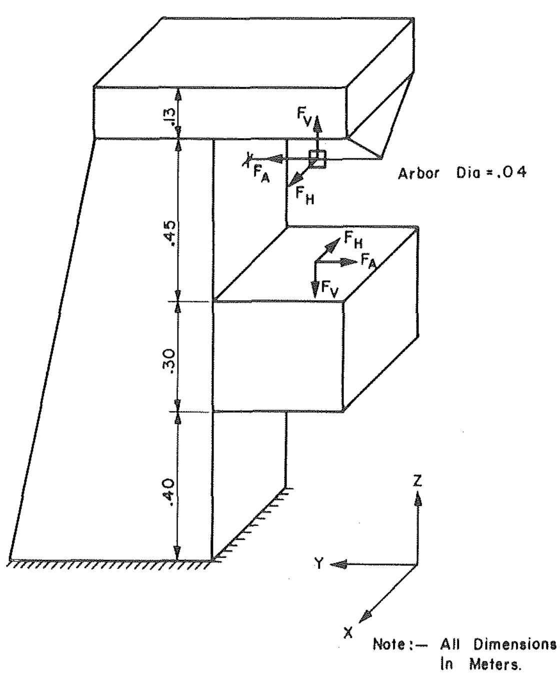

Fig. 9 Horizontal Milling Machine Structure (Examples 3(a) and 3(b)

The arbor diameter is taken as 0.04 m. The arbor support is idealized as two frame elements of rectangular cross section (0.10 m × 0.07 m). The thickness of the dovetail on front face of the column is taken as 0.035 m, and it is added to the thickness of the column, X_4, on the front face. The thickness of the dovetail on the bottom side of the overarm is taken as 0.025 m. This thickness is added to the thickness of the overarm, X_2, on the bottom side.

The geometrical constraints considered in this example are:

$$0.35 \text{ m} \leq X_1 \leq 0.54 \text{ m}$$

$$0.008 \text{ m} \leq X_2 \leq 0.040 \text{ m}$$

$$0.24 \text{ m} \leq X_3 \leq 0.35 \text{ m}$$

$$0.008 \text{ m} \leq X_4 \leq 0.040 \text{ m}$$

$$0.30 \text{ m} \leq X_5 \leq X_1$$

$$0.0 \text{ m} \leq X_6 \leq 0.06 \text{ m}$$

The behavior constraints are taken as:

$$d_c - 0.00009 \text{ m} \leq 0$$

$$|G_{\text{MIN}}| - 3.059 \times 10^{-9} \text{ m/N} \leq 0$$

$$850.0 \text{ rad/s} - \omega_1 \leq 0$$

$$\omega_1 + 100 - \omega_2 \leq 0$$

The material of the column, table, overarm and arbor support is taken as grey cast iron with the same material properties given in examples

Table 5 **Optimization Results of Horizontal Milling Machine (Example 3a)**

Design Variable	Initial Design	Bounds		Optimum Design
		Lower	Upper	
X_1	0.500 m	0.350 m	0.540	0.50120 m*
X_2	0.028 m	0.008 m	0.040 m	0.00987 m*
X_3	0.320 m	0.240 m	0.350 m	0.28826 m
X_4	0.028 m	0.008 m	0.040 m	0.00906 m*
X_5	0.420 m	0.400 m	X_1	0.41494 m
X_6	0.030 m	0.000 m	0.060 m	0.00896 m
Maximum Deflection of Cutter Centre In Any Direction (10^{-7} m)	661	—	900	893*
First Natural Frequency (ω_1) (radians/second)	1003	850	—	913*
Second Natural Frequency (ω_2) (radians/second)	1150	$\omega_1 + 100$	—	1024
Minimum Negative In-Phase Cross Receptance of Cutter Centre Relative to Table (10^{-11} m/N)	66.28	305.90	—	203.90
Objective Function (wt. of the milling machine including column, table and overarm)	7,256 N	—	—	2,949 N

Reduction of Weight Obtained by Optimization = 4,307 N (59.4%)
Computer Time Required = 105 minutes on an IBM 370/155 system
Cumulative Number of One-Dimensional Minimizations = 13

* Active constraint.

1 and 2. The material for the arbor is assumed as wrought steel with $\rho = 7.8$ kg/m^3, $E = 20.59 \times 10^{10}$ N/m^2 and $\nu = 0.3$.

The optimization results are tabulated in Table 5. The starting design corresponds to a weight of 7,256 N. The proposed optimum design corresponds to a weight of 2,949 N with a reduction of 59.4 percent in weight compared to the initial design. The active behavior constraints are static deflection, $d_c = 0.8932 \times 10^{-4}$ m, and the difference of ω_1 and ω_2, $\omega_1 - \omega_2 = 113$ rads/second. The constraints on ω_1 and G_{MIN} approached criticality as the optimization process progressed. Among the geometrical constraints, $X_2 = 0.00987$ m and $X_4 = 0.00905$ m are near to their respective lower bounds. The average computer time required for one one-dimensional minimization is eight minutes on an IBM 370/155 system. The progress of the optimization path as a plot of the f- and Φ-functions versus the cumulative number of one-dimensional minimizations is shown in Fig. 11. The total computer time taken for the optimization is about 105 minutes.

In order to see whether the optimum design obtained in example 3(b) corresponds to a local minimum or the absolute minimum in the design space, the space example has been considered with a different starting design vector. The plot of the f- and Φ-functions as the optimization progressed is shown in Fig. 12. It can be observed that the plot is similar to the one shown in Fig. 11. The optimization results for the example are shown in Table 6. The optimum design variables in the two cases agree well with each other except for small differences that might have occurred due to some roundoff errors and numerical instabilities in the optimization process. Although, merely on the basis of two trial starting designs, it is hard to say that the minimum obtained is the absolute minimum over the design space, finding the

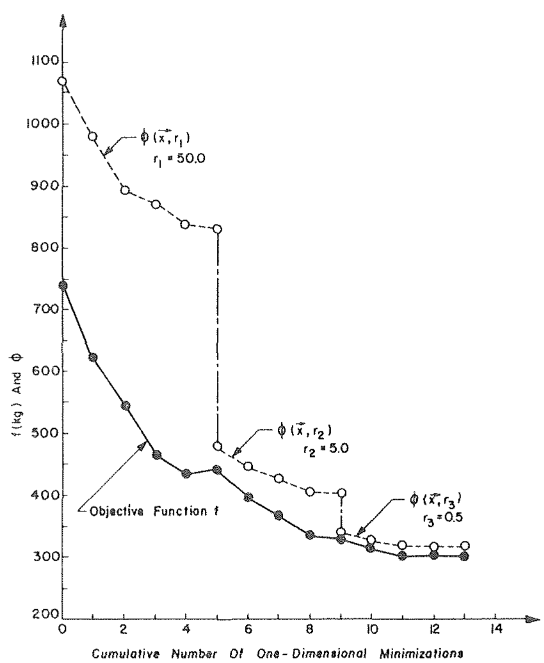

Fig. 11 Progress of Optimization Path for Example 3(a)

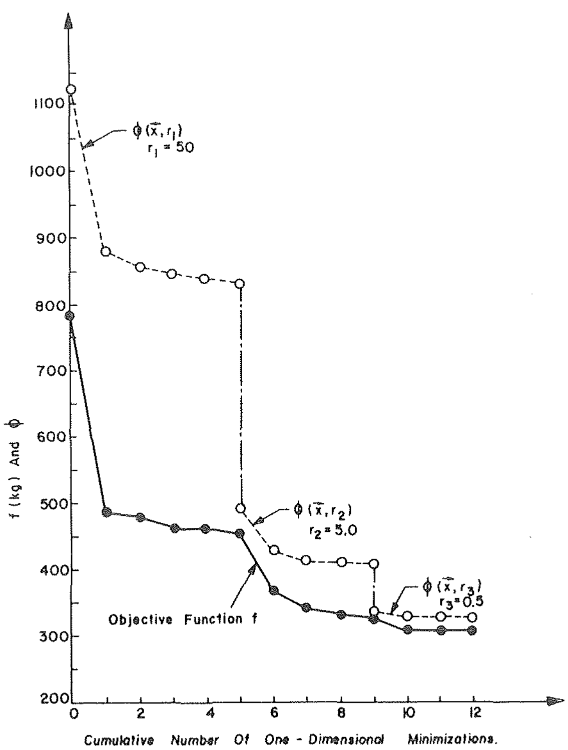

Fig. 12 Progress of Optimization Path for Example 3(b)

Table 6 Optimization Results of Horizontal Milling Machine (Example 3b)

Design Variable	Initial Design	Bounds		Optimum Design
		Lower	Upper	
X_1	0.520 m	0.350 m	0.540 m	0.50314 m
X_2	0.332 m	0.008 m	0.040 m	0.01069 m*
X_3	0.300 m	0.240 m	0.350 m	0.28978 m
X_4	0.030 m	0.008 m	0.040 m	0.09102 m*
X_5	0.450 m	0.400 m	X_1	0.42434 m
X_6	0.028 m	0.00 m	0.060 m	0.00866 m
Maximum Deflection of Cutter Centre In Any Direction (10^{-7} m)	662	—	900	884*
First Natural Frequency (ω_1) (radians/second)	961	850	—	914*
Second Natural Frequency (ω_2) (radians/second)	1187	$\omega_1 + 100$	—	1016*
Minimum Negative In-Phase Cross Receptance of Cutter Centre Relative to Table (10^{-11} m/N)	69.34	305.90	—	210.10
Objective Function	7,654 N	—	—	3,028 N

Reduction of Weight Obtained by Optimization = 4,626 N (60.4%)
Computer Time Required = 101 minutes on an IBM 370/155 system
Cumulative Number of One-Dimensional Minimizations = 12

* Active Constraint

similar least-weight design by starting from two different initial designs is at least a pointer in that direction.

Conclusions

The results of the example problems demonstrate the feasibility of automated optimum design of machine tool structures with static, dynamic and regenerative chatter stability constraints. In designing these complex machine tool structures with multiple behavior constraints, several analysis programs have been developed and incorporated into the optimization routine, hence it is difficult to summarize all the findings.

(1) The computational experience shows that the approximate methods used in the present work for evaluation the gradient and the slope of the Φ-function have been quite efficient and reliable without involving any significant errors. The progress of optimization has been quite smooth without any undue number of optimization steps.

(2) From the optimum results of the lathe bed (example 1), it has been found that the thickness of both the main members and the lacing diagonals decreased as the optimization progressed. At the optimum, the thicknesses are found to be the same and are at their lower bounds. This indicates that thin structures are preferable for this type of lathe bed. The widths and thicknesses of flanges (stiffeners) on the main members and the lacing diagonals also decreased as the optimization progressed. The results of example 2 show that the width of the lathe bed increased with a corresponding value of the lacing angle, 53.5 deg. This value of the lacing angle agrees very well with the results obtained by Badauri, Moshin and Thornley who analysed and tested various Warren beams under static torsional loads. It may also be mentioned that the thicknesses of both the main members and the lacing diagonals decreased approximately at the same rate, and at the optimum their values were same. Therefore, it can be concluded that it is preferable to have equal values of thickness for the main members and the lacing diagonals.

(3) The optimization results of the milling machine structure (examples 3(*a*) and 3(*b*)) show that the thicknesses of the overarm, the column and the table decreased as the optimization progressed. Even in horizontal milling machine structures, it can be concluded that thin structures are preferable. The cross-sectional areas of the square ribs on the overarm and its joint with the column also decreased as the optimization progressed.

(4) The results of the present study clearly indicate that the optimum design of machine tool structures with multiple behaviour constraints is feasible and the method proposed in this work can be used as a unified design procedure by machine tool structural designers. Although weight minimization is considered as the objective in the present work, other criteria like maximization of static rigidity, fundamental natural frequency and regenerative chatter stability can also be considered in the same manner.

References

1 Taylor, S., and Tobias, S. A., "Lumped Constants Methods for the Prediction of Vibration Characteristics of Machine Tool Structures," *Proc. 5th Int. Machine Tool Design and Research Conference*, Sept. 1964, pp. 37–52.

2 Cowley, A., and Fawcett, M., "Analysis of Machine Tool Structures by Computing Techniques," *Proc. 8th International Machine Tool Design and Research Conference*, 1967, pp. 119–138.

3 Badauri, M. W., Moshin, M. E., and Thornley, R. H., "The Static Analysis of Warren Beams Under Torsional and Bending Loads," *International Journal of Machine Tool Design and Research*, Vol. 3, 1963, pp. 177–191.

4 Thornley, R. H., and Howes, P. S., "Static and Dynamic Behaviour of Warren Type Machine Tool Structural Elements," JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 93 May 1971, pp. 467–476.

5 Peters, J., Vanherck, P., Vanden, L., Wolf, A. C. H. V., Opitz, H., Schlempter, K., Hoshi, T., Tobias, S. A., Taylor, S., Koenigsberger, F., and Cowley, A., "Cooperative Work in Computer Aided Design in the CIRP."

6 Andrew, C., and Tobias, S. A., "Vibration in Horizontal Milling," *International Journal of Machine Tool Design and Research*, Vol. 2, 1962, pp. 369–378.

7 Taylor, S., "The Design of Machine Tool Structures Using a Digital Computer," *Proc. 7th International Machine Tool Design and Research Conference*, 1966, pp. 369–383.

8 Gurney, J. P., and Tobias, S. A., "A Graphical Analysis of Regenerative Machine Tool Instability," JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. A.S.M.E., Vol. 84, Feb. 1962, pp. 103–112.

9 Gurney, J. P., "A General Analysis of Two-Degree of Freedom Instability in Machine Tools," *Journal of Mechanical Engineering Science*, Vol. 4, 1962.

10 Sweeney, G., and Tobias, S. A., "An Algebraic Method for the Determination of Dynamic Instability of Machine Tools," *International Research in Production Engineering*, 1963.

11 Tlusty, J. A., and Polacek, M., "Experience with Analysing Stability of Machine Tool Against Chatter," *Proc. 9th International Machine Tool Design and Research Conference*, 1968, pp. 521–570.

12 Said, S. M., "The Stability of Horizontal Milling Machines," *International Journal of Machine Tool Design and Research*, Vol. 14, Oct. 1974, pp. 245–265.

13 Koenigsberger, F., and Tlusty, J., *Machine Tool Structures*, Vol. 1, Pergamon Press, 1970.

14 Fox, R. L., and Kapoor, M. P., "Structural Optimization in the Dynamic Response Regime: A Computational Approach," *AIAA Journal*, Vol. 8, Oct. 1970, pp. 1788–1804.

15 Rao, S. S., Optimization of Complex Structures to Satisfy Static, Dynamic and Aeroelastic Requirements," International Journal for Numerical Methods in Engineering, Vol. 8, 1974, pp. 249–269.

16 Clough, R. W., and Tocher, J. L., "Finite Element Stiffness Matrices

for Analysis of Plate Bending," *Proceedings of Conference on Matrix Methods in Structural Mechanics*, WPAFB, Ohio, October 1965.

17 Przemieniecki, J. S., Theory of Matrix Structural Analysis, McGraw-Hill Book Co., New York, 1968.

18 Bathe, K. J. and Wilson, E. L., "Large Eigen Value Problems in Dynamic Analysis," *Journal of Engineering Mechanics Division*, Vol. 98, Dec. 1972, pp. 1471–1485.

19 Tlusty, J., and Polacek, M., "The Stability of Machine Tools Against Self-excited Vibrations in Machining," *Production Engineering Conference*, ASME, Pittsburgh, 1963.

20 Reddy, C. P., "Automated Optimum Design and Reliability Analyses of Machine Tool Structures," Ph.D. Thesis, December 1975, I.I.T. Kanpur, India.

21 Vulf, A. M., *Theory of Metal Cutting*, (in Russian), Moscow, 1963.