

OPTIMUM LOAD SHEDDING TAKING INTO ACCOUNT OF VOLTAGE  
AND FREQUENCY CHARACTERISTICS OF LOADS

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**Abstract** - This paper presents a method for optimal load curtailment in a power system taking into account of generator control effects and voltage and frequency characteristics of loads. The optimization problem is solved by a second-order gradient technique. Rigorous formulation and inclusion of all security related constraints assures that the solution always represents a secure system. It is shown that the solution speed can be increased by approximating the second-order matrix using the complementary Davidon-Fletcher-Powell update. The method is illustrated on a 10-bus and IEEE 30-bus systems. This method appears to be attractive for security studies and for operational planning.

**INTRODUCTION**

One of the important functions of a dispatching center is to control the power system during emergency conditions by dropping load or generation as necessary to prevent further system deterioration. Hajdu et al. [1] proposed a computational procedure for minimizing the curtailment of loads which is based on the Newton-Raphson technique for solving the power flow equations and the Kuhn-Tucker theorem for optimization. However the system reactive power or losses were not considered in their model. Subramanian [2] derived a model for sensitivity in power systems and used it in conjunction with linear programming for the solution of load shedding problems. But the method completely overlooks all equipment and operational requirements. Abdullah Khan and Kuppuswamy [3] proposed a method for precomputing optimum load-shedding strategy under emergency conditions using a linearized power flow model obtained from sensitivity relations. The successive linear programming was employed to obtain the solution of the problem. The method assumes that the voltage profile remains constant. This is not true especially under emergency conditions. Chan and Schwepp [4] used linear programming algorithm to solve the problem of rescheduling generation and shedding loads in an emergency state. The algorithm

minimizes the control actions necessary to remove all the constraint violations and to serve a maximum of load. In a linear programming problem, the objective function and constraints are linear. But the power system problems are in general non-linear and are therefore to be approximated as linearised problems for the application of linear programming. The authors [5] presented a quadratic formulation for minimizing load curtailment which provides reduced transmission losses, uniform power distribution and uniform voltage profile. Rashed et al. [6] proposed a method for steady state optimal load shedding and solved it using an optimal load-flow solution technique, taking a load bus as the reference. The Lagrangian multipliers are used to incorporate the equality constraints into the objective function. The use of Lagrangian multipliers may cause the solution to oscillate near the optimal point due to duality gap. Medicherla et al. [7,8] proposed a method for generation rescheduling and load shedding to alleviate line overloads.

In all the above methods the effect of frequency variation due to generation-demand imbalance is not considered. Okamura et al. [9] presented a new power flow model for analysing the steady state behaviour of power systems under normal as well as abnormal operating conditions. Frequency was treated as one of the variables whereas in the conventional methods of power flow calculations, the frequency is assumed to be constant. This method is useful only to obtain the load-flow solution. But it does not give optimal load curtailment to correct an emergency situation. Saadat [10] presented an alternate formulation for load shedding including the effect of control devices. The problem was solved by a power perturbation technique. It has been found that the perturbation technique takes more C.P.U. time than the method presented in this paper. This paper presents a method for optimal load curtailment taking into account of generator control effects and voltage and frequency characteristics of loads. The optimization problem is solved by a second-order gradient technique. The flat-frequency control, flat tie-line control and/or flat tie-line frequency bias control, employed to limit frequency fluctuations, can be incorporated in the method. The method is illustrated on a 10-bus system and IEEE 30-bus system.

**Notations**

$P_G, Q_G$	- Real and reactive power generations
$P, Q$	- Real and reactive power injected at a bus
$\bar{V}_1$	- Nominal value of voltage

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$P_D, Q_D$	- Non-curtailed real and reactive power demand
$P_D, Q_D$	- Actually supplied real and reactive power demand
$G_{ij} + j B_{ij}$	- $ij^{\text{th}}$ element of bus admittance matrix
$N$	- Number of buses in a system
$NG$	- Number of generator buses
$\epsilon$	- Specified tolerance
$\psi_{ij}$	- Maximum voltage phase angle difference between buses $i$ and $j$
$\alpha, \beta, \gamma$	- Weightage factors
$\delta$	- Voltage angle
$\Delta f$	- System frequency deviation, per unit Hz
$R$	- Speed regulation in per unit
$P_R$	- Rated output of generator
$P_G$	- Scheduled output of generator
$P_z, q_z$	- Portion of total load proportional to constant impedance load
$P_c, q_c$	- Portion of total load proportional to $N^{\text{th}}$ power of voltage
$P_p, q_p$	- Portion of total load proportional to constant power load
$K_p, K_q$	- Frequency characteristics of load

Superscripts 'min' and 'max' denote the lower and upper limits respectively.

### PROBLEM FORMULATION

The objective of the frequency dependent load shedding problem is to minimize the load curtailment and frequency deviation from the nominal value. It may be expressed mathematically as

Minimize

$$F = \sum_{i=1}^N [\alpha_i (P_{Di} - \bar{P}_{Di})^2 + \beta_i (Q_{Di} - \bar{Q}_{Di})^2 + \gamma (f - \bar{f})^2] \quad (1)$$

The minimization of (1) is subject to the following constraints.

### Generation Constraints

Under normal conditions, system frequency is maintained constant and generators are operated at a scheduled voltage and output. When the system is disturbed by loss of generation or tie-line support, the discrepancy between the generation and demand causes a change in frequency. The speed control mechanism will then act to compensate the power deficit. Generator real power output is adjusted by the static response of the governor. This may be expressed as :

$$P_{Gi} = \bar{P}_{Gi} - \frac{P_{R1}}{R_1} \Delta f \quad (2)$$

and

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad i = 1, 2, \dots, NG \quad (3)$$

Combining (2) and (3), the real power generation constraint may be expressed as follows:

$$P_{Gi}^{\min} \leq \bar{P}_{Gi} - \frac{P_{R1}}{R_1} \Delta f \leq P_{Gi}^{\max} \quad (4)$$

The reactive power generation is also constrained to lie within its maximum and minimum limits

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad (5)$$

### Load Constraints

It is well established that the system lead characteristics affect the overall performance of a power system under disturbed conditions. The system leads may be decomposed in several ways. By examining suitably chosen individual lead components separately, and then as a composite group, the resultant lead characteristics can be evaluated. Berg has shown that loads in a power system can be represented for simulation studies by assuming voltage and frequency dependence of leads to be of an exponential nature [11].

$$P_{Di} = P_{Dio} \left( \frac{V_i}{V_{10}} \right)^{P_{Vi}} \left( \frac{w_i}{w_{10}} \right)^{P_{Wi}} \quad (6)$$

$$Q_{Di} = Q_{Dio} \left( \frac{V_i}{V_{10}} \right)^{Q_{Vi}} \left( \frac{w_i}{w_{10}} \right)^{Q_{Wi}} \quad (7)$$

$$i = 1, 2, \dots, N$$

where the subscript o refers nominal values.  $P_v$ ,  $P_w$  and  $Q_v$ ,  $Q_w$  are the characteristic voltage and frequency parameters for the active and reactive load components, respectively. Okamura et al. [9] employed another load model described as follows :

$$P_{Di} = \bar{P}_{Di} (1 + K_p \Delta f) [P_p + P_c \left( \frac{V_1}{V_1} \right)^{N_1} + P_z \left( \frac{V_1}{V_1} \right)^2] \quad (8)$$

$$Q_{Di} = \bar{Q}_{Di} (1 + K_q \Delta f) [Q_p + Q_c \left( \frac{V_1}{V_1} \right)^{N_2} + Q_z \left( \frac{V_1}{V_1} \right)^2] \quad (9)$$

Both the real and reactive components of loads are assumed to be composed of three parts. One part is independent of the system voltage, the second changes as the  $N^{\text{th}}$  power of voltage and the third changes as the square of the system voltage. In this model, the frequency deviations are considered to affect the three load components in exactly the same manner.

### System Equations

The following equations must be satisfied in order to obtain the balance of real and reactive power at each node  $i$

$$P_{Gi}(f) - P_{Di}(V, f) - P_i(V, \delta) = 0 \quad (10)$$

$$Q_{Gi}(f) - Q_{Di}(V, f) - Q_i(V, \delta) = 0 \quad (11)$$

$$i = 1, 2, \dots, N$$

where

$$P_i(V, \delta) = V_i \sum_{j=1}^N V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] \quad (12)$$

and

$$Q_1(V, \delta) = V_i \sum_{j=1}^N V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] \quad (13)$$

Voltage Constraints

In order to satisfy statutory legal requirements and design limitations the voltage magnitudes are restricted to lie between the specified upper and lower limits

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (14)$$

$i = 1, 2, \dots, N$

Transmission Constraints

The transfer of power through transmission lines is limited by the thermal and stability limits of the lines. This can be achieved by limiting the angular displacement between the busbars concerned.

$$|\delta_i - \delta_j| \leq \psi_{ij} \quad (15)$$

$i = 1, 2, \dots, N-1$   
 $j = i + 1, \dots, N$

The optimal load curtailment problem may, therefore, be described by the equations (1), (4), (5), (6) and (7) or (8) and (9), (10), (11), (14) and (15).

The equality constraints (10) and (11) can be absorbed into the objective function by substituting for  $P_{Di}$  and  $Q_{Di}$ . The modified objective function will be: Minimize

$$F = \sum_{i=1}^N [\alpha_i (P_{Gi} - P_i - \bar{P}_{Di})^2 + \beta_i (Q_{Gi} - Q_i - \bar{Q}_{Di})^2] + \gamma (f - \bar{f})^2 \quad (16)$$

Loss of a generator or a transmission line may cause violation of one or more of the constraints (4), (5), (14) and (15). The constrained minimization problem may be converted into an unconstrained minimization problem by incorporating the violating constraints into the objective function using penalty functions of the form

$$w_j = \begin{cases} K_1 (P_{Gj}^{\max} - P_{Gj})^2 & \text{if } P_{Gj} > P_{Gj}^{\max} \\ K_1 (P_{Gj}^{\min} - P_{Gj})^2 & \text{if } P_{Gj} < P_{Gj}^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

$$u_j = \begin{cases} K_2 (Q_{Gj}^{\max} - Q_{Gj})^2 & \text{if } Q_{Gj} > Q_{Gj}^{\max} \\ K_2 (Q_{Gj}^{\min} - Q_{Gj})^2 & \text{if } Q_{Gj} < Q_{Gj}^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

$$y_j = \begin{cases} K_3 [\psi_{ij}^2 - (\delta_i - \delta_j)^2]^2 & \text{if } |\delta_i - \delta_j| > \psi_{ij} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

$i = 1, 2, \dots, N; i \neq j$

$$v_j = \begin{cases} K_4 (V_j^{\max} - V_j)^2 & \text{if } V_j > V_j^{\max} \\ K_4 (V_j^{\min} - V_j)^2 & \text{if } V_j < V_j^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

where  $K_1, K_2, K_3$  and  $K_4$  are penalty factors.

DECOMPOSITION OF THE PROBLEM

The memory requirement is reduced and the solution speed is increased by splitting the problem into two subproblems. The subproblem I is a function of real power and angles and subproblem II is a function of reactive power and voltages. In a power system change of frequency indicates real power mismatch and change of voltage indicates reactive power mismatch. So the weightage term for frequency deviation in the objective may be included in the subproblem which minimises the real power mismatch. The two sub-problems may therefore, be stated as follows:

Subproblem I

$$F(\delta, f) = \sum_{i=1}^N \alpha_i (P_{Gi} - P_i - \bar{P}_{Di})^2 + \sum_j w_j + \sum_j y_j + \gamma (f - \bar{f})^2 \quad (21)$$

Subproblem II

$$G(V) = \sum_{i=1}^N \alpha_i (Q_{Gi} - Q_i - \bar{Q}_{Di})^2 + \sum_j u_j + \sum_j v_j \quad (22)$$

The two subproblems are solved iteratively until  $\max [F(\delta, f), G(V)] \leq \epsilon$ . A second-order gradient technique developed for load-flow studies is employed for the solution. The technique has been described in detail in reference 12. In this technique, the corrections in voltage angles  $[\Delta\delta]$ , magnitudes  $[\Delta V]$  and frequency deviation  $\Delta f$  are obtained by solving the following equations

$$\begin{bmatrix} R \\ \vdots \\ R \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 F}{\partial \delta_i \partial \Delta \delta} & \cdots & \frac{\partial^2 F}{\partial \delta_{N-1} \partial \Delta \delta} & \frac{\partial^2 F}{\partial \Delta \delta^2} \\ \hline \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 F}{\partial \Delta f \partial \Delta \delta} & \cdots & \frac{\partial^2 F}{\partial \Delta f \partial \Delta \delta} & \frac{\partial^2 F}{\partial \Delta f^2} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \vdots \\ \Delta \delta \end{bmatrix} \quad (23)$$

$$[S] [\Delta V] = -[\Delta G] \quad (24)$$

where  $[R]$  and  $[S]$  are the first and second derivative matrices of  $F(\delta, f)$ , with respect to  $\delta_i$ ,  $i = 1, 2, \dots, N-1$  and  $[G]$  and  $[S]$  are the first and second derivative matrices of  $G(V)$  with respect to  $V_i$ ,  $i = 1, 2, \dots, N$ . The  $N$ th bus is taken as the reference. The solution speed is increased if  $[R]$  and  $[S]$  are approximated

using the complementary Davidon-Fletcher-Powell update, described in reference 13.

### SYSTEM STUDIES

Normal Load-Flow : The proposed method can be used to obtain load-flow solution and optimal load-flow solution under normal as well as abnormal operating conditions. The method absorbs and distributes the supply-demand imbalance among all the generators on the basis of the governor characteristics whereas in the conventional methods of load flow calculation, the imbalance is supplied by a slack generator. The latter results in the major distortions in load-flow distribution.

The proposed method was applied to a 10-bus system [9]. The data for the system are given in Appendix. The load model, described by the equations (8) and (9), was used in the study. The constants of the load model were assumed to be the same as those given in reference 9. The weightage factors  $\alpha_j, \beta_j$  and  $\gamma$  were taken as unity. The load-flow solution of the 10-bus system obtained with frequency as variable is given in Table 1. The frequency at the end of solution was 50 Hz. The total generation required was 3883.1 MW.

TABLE 1  
LOAD-FLOW SOLUTION OF THE 10-BUS SYSTEM  
WITH FREQUENCY AS VARIABLE

Bus No.	Voltage p.u.	Angle deg.	Load MW	Generation MW	Load Mvar	Generation Mvar
1	1.0211	- 8.49	0.0	0.0	0.0	0.0
12	1.0211	- 8.49	0.0	0.0	0.0	0.0
2	1.0147	- 9.93	0.0	0.0	494.2	0.0
3	1.0000	-29.77	500.0	0.0	200.0	424.9
4	0.9653	-30.47	621.8	0.0	0.0	0.0
42	0.9653	-30.47	621.8	0.0	0.0	0.0
5	0.9494	-31.02	1500.0	0.0	0.0	0.0
6	1.0000	-22.14	450.0	150.0	1605.9	371.8
9	1.0000	0.00	0.0	0.0	791.5	210.1
92	1.0000	0.00	0.0	0.0	791.5	210.1

Table 2 shows the load-flow solution for the same system with constant frequency and without considering the voltage characteristic of loads. The generation at slack bus was 820 MW and the total generation was 3945 MW. Further it may be observed that voltage magnitude of load buses is improved if load characteristics are considered. The total real power generation decreased due to the reduction of load at bus 2 and also due to re-distribution of the supply-demand unbalance among all the generators. The total reactive power generation also decreased due to the increase of voltage magnitude at load buses.

### Flat-Frequency Control

In flat-frequency control, the power outputs of generators within a prescribed area are automatically regulated to maintain scheduled system frequency. Power output of the regulating generators is given by

$$P_{Gi} = \bar{P}_{Gi} + \eta_i P_{Rq} \quad (25)$$

where  $i$  represents the node of the regulating generator,  $P_{Rq}$  is the supply insufficiency in a given area,  $\eta_i$  the load distribution fac-

tor among the regulating generators and  $\sum \eta_i$  is 1.0. Since system frequency remains constant,  $\Delta f$  in (23) is replaced by the new variable  $P_{Rq}$ . Table 3 gives the load-flow solution obtained with flat-frequency control. The supply insufficiency was assumed to be distributed equally between the generators at buses 9 and 92. The supply insufficiency was only 14.2 MW whereas it was 20 MW in the conventional method of load-flow solution.

In a similar manner flat tie-line control and/or flat tie-line frequency bias control on interconnected systems can be incorporated in the method.

TABLE 2  
LOAD-FLOW SOLUTION OF THE 10-BUS SYSTEM WITH  
FREQUENCY ASSUMED TO BE CONSTANT

Bus No.	Voltage	Angle	Load		Generation	
			deg.	MW	Mvar	MW
1	1.0198	- 8.81	0.0	0.0	0.0	0.0
12	1.0198	- 8.81	0.0	0.0	0.0	0.0
2	1.0131	-10.40	0.0	0.0	500.0	0.0
3	1.0000	-30.68	500.0	0.0	200.0	448.2
4	0.9634	-31.41	650.0	0.0	0.0	0.0
42	0.9634	-31.41	650.0	0.0	0.0	0.0
5	0.9482	-31.70	1500.0	0.0	0.0	0.0
6	1.0000	-22.68	450.0	150.0	1625.0	383.6
9	1.0000	- 0.22	0.0	0.0	800.0	218.5
92	1.0000	0.00	0.0	0.0	820.0	221.5

TABLE 3  
LOAD-FLOW SOLUTION OF 10-BUS SYSTEM WITH FLAT  
FREQUENCY CONTROL

Bus No.	Voltage	Angle	Load		Generation	
			deg.	MW	Mvar	MW
1	1.0200	- 8.67	0.0	0.0	0.0	0.0
12	1.0200	- 8.67	0.0	0.0	0.0	0.0
2	1.0134	-10.25	0.0	0.0	500.0	0.0
3	1.0000	-30.41	500.0	0.0	200.0	443.5
4	0.9636	-31.17	650.0	0.0	0.0	0.0
42	0.9636	-31.17	650.0	0.0	0.0	0.0
5	0.9484	-31.46	1500.0	0.0	0.0	0.0
6	1.0000	-22.44	450.0	150.0	1625.0	381.3
9	1.0000	0.00	0.0	0.0	807.1	218.4
92	1.0000	0.00	0.0	0.0	807.1	218.4

### Optimal Load-Flow Under Emergency Conditions

If a system is transiently and dynamically stable after a line or generator outage, a new steady state can be obtained by solving post-transient load-flow. As frequency changes the real and reactive generations are reallocated according to the steady-state characteristics of control devices. The loads also change based on the voltage and frequency characteristics. The proposed method takes account of the above features in determining rescheduling and/or load curtailment. To illustrate the application of the method to obtain optimal load-flow under emergency conditions, it was assumed that there was 50 percent generation loss at bus 9 of the 10-bus system. This results in decrease of frequency to 49.7931 Hz. In reference 9, it was assumed arbitrarily that 60 percent of the load at bus 4 would be shed if the frequency dropped below 49.8 Hz

With the pre-emergency values as the starting values, the optimal solution obtained by the proposed method is shown in Table 4. The operating conditions of the system are taken the same as those given in Table 3 of reference 9. The optimal solution indicates that the emergency condition can be met by rescheduling the generation and without any load curtailment. With this rescheduling, the frequency increased to 49.81 Hz. The penalty factor for generation violation was taken as two. The solution was obtained in only two iterations and the C.P.U. time taken by DEC 2050 computer was 0.75 second.

TABLE 4  
OPTIMAL LOAD-FLOW SOLUTION OF 10-BUS SYSTEM  
FOR 50 PERCENT GENERATION LOSS AT BUS 9

Bus No.	Voltage p.u.	Angle deg.	Load		Generation	
			MW	Mvar	MW	Mvar
1	1.0247	- 9.39	0.0	0.0	0.0	0.0
12	1.0247	- 9.39	0.0	0.0	0.0	0.0
2	1.0196	- 8.63	0.0	0.0	558.5	0.0
3	1.0000	-28.49	500.0	0.0	227.3	394.8
4	0.9668	-29.39	645.7	0.0	0.0	0.0
42	0.9668	-29.39	645.7	0.0	0.0	0.0
5	0.9475	-28.60	1500.0	0.0	0.0	0.0
6	1.0000	-19.03	450.0	150.0	1824.0	331.6
9	1.0000	- 4.69	0.0	0.0	437.5	151.1
92	1.0000	0.00	0.0	0.0	875.0	205.1

#### Application to Large Systems

The method was then tried on the IEEE 30-bus system [14]. The buses 1, 2 and 11 are taken as generator buses. The minimum and maximum limits of real and reactive power generations and the regulation are given in Table 5.

TABLE 5  
THE OPERATING LIMITS OF THE GENERATORS OF IEEE 30-BUS SYSTEM

Bus No.	$P_G^{\min}$ MW	$P_G^{\max}$ MW	$Q_G^{\min}$ Mvar	$Q_G^{\max}$ Mvar	Reg. percent
1	0.0	175.0	-20.0	30.0	5.0
2	0.0	70.0	-30.0	30.0	5.0
11	0.0	75.0	-30.0	30.0	6.0

The pre-emergency condition, given in Table-6 was taken as the starting condition. The voltage and frequency constants of the loads at all buses were assumed as follows :

$$p_p=0.2, \quad p_c=0.3, \quad p_z=0.5, \quad K_p=0.10, \quad N_1=1.0$$

$$q_p=0.2, \quad q_c=0.3, \quad q_z=0.5, \quad K_q=0.05, \quad N_2=1.0$$

The optimum post-transient load-flow solution obtained is shown in Table 7. The solution was obtained in 4 iterations and the CPU time taken by DEC 2050 computer was 6.22 seconds. The same solution was also obtained by appro-

TABLE 6  
PRE-EMERGENCY LOAD FLOW-SOLUTION FOR IEEE 30-BUS SYSTEM

Bus No.	Voltage p.u.	Angle deg.	Load		Generation		+CAP/-REA Mvar
			MW	Mvar	MW	Mvar	
1	1.0600	0.00	0.0	0.0	170.0	57.1	0.0
2	1.0182	- 3.02	21.7	12.7	65.0	5.0	0.0
3	1.0041	- 5.21	2.4	1.2	0.0	0.0	0.0
4	0.9913	- 6.26	7.6	1.6	0.0	0.0	0.0
5	0.9484	-11.12	94.2	19.0	0.0	5.0	0.0
6	0.9811	- 7.30	0.0	0.0	0.0	0.0	0.0
7	0.9594	- 9.44	22.8	10.9	0.0	0.0	0.0
8	0.9689	- 7.88	30.0	30.0	0.0	5.0	0.0
9	1.0321	- 6.32	0.0	0.0	0.0	0.0	0.0
10	1.0296	- 9.39	5.8	2.0	0.0	0.0	19.0
11	1.0754	0.14	0.0	0.0	60.0	25.8	0.0
12	1.0496	-10.64	11.2	7.5	0.0	0.0	0.0
13	1.0827	-10.64	0.0	0.0	0.0	25.6	0.0
14	1.0328	-11.34	6.2	1.6	0.0	0.0	0.0
15	1.0276	-11.22	8.2	2.5	0.0	0.0	0.0
16	1.0343	-10.39	3.5	1.8	0.0	0.0	0.0
17	1.0258	- 9.90	9.0	5.8	0.0	0.0	0.0
18	1.0161	-11.29	3.2	0.9	0.0	0.0	0.0
19	1.0124	-11.13	9.5	3.4	0.0	0.0	0.0
20	1.0159	-10.75	2.2	0.7	0.0	0.0	0.0
21	1.0168	-10.00	17.5	11.2	0.0	0.0	0.0
22	1.0172	-10.03	0.0	0.0	0.0	0.0	0.0
23	1.0134	-11.30	3.2	1.6	0.0	0.0	0.0
24	1.0029	-11.06	8.7	6.7	0.0	0.0	4.3
25	0.9908	-11.25	0.0	0.0	0.0	0.0	0.0
26	0.9727	-11.68	3.5	2.3	0.0	0.0	0.0
27	0.9927	-11.12	0.0	0.0	0.0	0.0	0.0
28	0.9755	- 7.82	0.0	0.0	0.0	0.0	0.0
29	0.9716	-12.42	2.4	0.9	0.0	0.0	0.0
30	0.9597	-13.37	10.6	1.9	0.0	0.0	0.0

TABLE 7  
POST-TRANSIENT LOAD-FLOW SOLUTION OF THE IEEE 30-BUS SYSTEM

Frequency = 49.99 Hz

Bus No.	Voltage p.u.	Angle deg.	Load		Generation		+CAP/-REA Mvar
			MW	Mvar	MW	Mvar	
1	1.0600	0.00	0.0	0.0	170.0	25.13	0.0
2	1.0267	- 3.59	20.66	10.75	0.0	0.0	0.0
3	1.0305	- 4.24	0.41	0.24	0.0	0.0	0.0
4	1.0226	- 5.11	5.82	0.44	0.0	0.0	0.0
5	0.9729	-10.49	94.20	12.66	0.0	5.0	0.0
6	1.0163	- 5.87	0.00	0.00	0.0	0.0	0.0
7	0.9914	- 8.27	21.64	9.84	0.0	0.0	0.0
8	1.0055	- 6.36	29.38	29.89	0.0	5.0	0.0
9	1.0738	- 2.87	0.00	0.00	0.0	0.0	0.0
10	1.0797	- 5.60	4.09	1.00	0.0	0.0	19.0
11	1.1096	4.78	0.00	0.00	75.0	24.18	0.0
12	1.0990	- 7.28	9.79	6.90	0.0	0.0	0.0
13	1.1339	- 7.14	0.00	0.00	0.0	28.2	0.0
14	1.0877	- 7.68	4.50	0.63	0.0	0.0	0.0
15	1.0840	- 7.48	6.66	1.62	0.0	0.0	0.0
16	1.0866	- 6.72	1.63	0.90	0.0	0.0	0.0
17	1.0774	- 6.10	7.50	5.23	0.0	0.0	0.0
18	1.0759	- 7.26	1.32	0.00	0.0	0.0	0.0
19	1.0718	- 7.04	8.11	2.66	0.0	0.0	0.0
20	1.0740	- 6.69	0.00	0.00	0.0	0.0	0.0
21	1.0700	- 6.08	16.61	11.02	0.0	0.0	0.0
22	1.0711	- 6.10	0.00	0.00	0.0	0.0	0.0
23	1.0738	- 7.34	1.32	0.67	0.0	0.0	0.0
24	1.0636	- 7.02	7.27	1.54	0.0	0.0	4.3
25	1.0573	- 7.19	0.00	0.00	0.0	0.0	0.0
26	1.0482	- 7.35	1.73	1.36	0.0	0.0	0.0
27	1.0535	- 7.36	0.00	0.00	0.0	0.0	0.0
28	1.0157	- 6.13	0.00	0.00	0.0	0.0	0.0
29	1.0436	- 8.26	0.48	0.00	0.0	0.0	0.0
30	1.0329	- 9.11	9.49	0.82	0.0	0.0	0.0

ximating the Hessian matrices using the CDFFP update. The time saved in the latter method was found to be 16 percent. Since load shedding in any real power system takes place in discrete blocks, load in the block nearest to the actual value obtained may be shed. If P and Q load shedding are not independent, then the problem size is reduced and solution speed is increased.

#### CONCLUSIONS

A method has been presented to minimize the load curtailment in preserving a system following a sudden major supply outage or tripping of tie-line breakers. The method takes account of generator control effects and the voltage and frequency characteristics of loads. It has been observed that the distribution of the supply insufficiency among the generators instead of one slack generator reduces the total generation required to meet a given demand. It has been shown that the automatic load frequency control can be incorporated into the proposed model. It has been found that the use of the quasi-Newton method for large systems reduces the C.P.U. time.

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#### APPENDIX

The impedance and line charging data of the 10-bus system are given in Table A1 and the transformer data in Table A2.

TABLE A1

Line designation	Resistance p.u.*	Reactance p.u.*	Line charging p.u.*
1-2	0.10	0.65	0.013
1-9	0.0	0.2	0.0
1-92	0.0	0.2	0.0
2-3	0.104	0.65	0.013
3-4	0.03	0.1	0.0
4-5	0.03	0.2	0.0
5-6	0.03	0.1	0.0
6-1	0.104	0.65	0.013
1-4	0.104	0.65	0.013
12-42	0.104	0.65	0.013

\* Impedance and line charging susceptance in p.u. on 1000 MVA base. Line charging one-half of total charging of of line. Buses 1 and 12 and 4 and 42 are directly connected.

TABLE A2  
TRANSFORMER DATA

Transformer designation	Tap setting*
9-1	1.05
92-1	1.05

\* Off-nominal turns ratio.

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#### Discussion

**A.O. Ekwue** (Queen Mary College, University of London, U.K.): The authors have in their paper presented an adequate review of techniques used for optimal load shedding as reported in the literature and then proposed a nonlinear programming method taking into account generator control effects and voltage and frequency characteristics of loads.

Load shedding, in practice, is usually the last resort during emergency situations to restore system integrity, hence very fast solution procedures must be employed. In the light of the fact that nonlinear programming methods are slow, may have convergence problems and are computationally expensive, a comparison of the authors' approach and those of Medicherla et al in References 7 and 8 would be appreciated.

Frequency consideration have been as a result of many loads consisting of rotating machines. Would the authors' method be applied to a system where the load composition is unknown?

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**K.A. Palaniswamy, J. Sharma and K.B. Misra**, We thank the discusser for his interest in our paper. Reference 7 presents mathematical models based on linearized relationships between line currents and state variables and bus injected powers and state variables to alleviate line overloads in a power systems. In Reference 8 systems studies are presented. It may be noted that the solution is obtained based on the concept of the pseudoinverse of a matrix and that it is not unique. Each line overload alleviation takes one or two passes and each pass requires approximately  $\frac{1}{2}$  to  $\frac{3}{4}$ th of the computational time of the initial load flow. Alleviation of line overloads is only considered in the above References.

In our paper a method, which solves the generation rescheduling and load shedding problem as an optimization problem, is presented. The method can be used to obtain secure operation state for both generation outage and line outage conditions. This method considers not only line overloads but also the operating limits of the generators and load characteristics. The number of iterations required varies from 2 to 4 for generation outages. The computation time for each iteration is approximately 25 percent of the computational time of the initial load flow. The number of iterations required for alleviating line overloads using our method is only 2 or 3 and the computation time for each iteration is approximately 30 percent of the computation time of the initial load flow. Although the method presented in Reference 7 and our method are suggested for operational planning studies, our method gives unique solution. The load curtailment, if any, is minimum. There is no appreciable difference in the computation time between the two methods. However in our method the major corrections in the state variables take place in the first iteration itself and if an approximate solution as proposed in the Reference 7 is enough for restoring system integrity during emergency situations the solution at the end of the first iteration may be considered. In that case our method will be much faster than the other.

In response to the second question, the load composition can be suitably assumed if it is not known. Reference 11 gives typical values of the parameters for different types of loads often encountered in practice.

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