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# TRANSIENT ANALYSIS OF ENERGY STORAGE IN A THERMALLY STRATIFIED WATER TANK

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## SUMMARY

A one-dimensional transient heat conduction model to describe the decay of the thermocline in a stratified water tank is presented. The problem is formulated as an initial boundary value problem and the resulting governing equations in the fluid and in the storage wall are solved numerically to obtain the temperature profiles in the wall and the fluid. The formulation considers the axial conduction of heat, both in the fluid and in the solid wall. The mixing parameters introduced in the boundary conditions at the top and bottom of the tank in the fluid region account for mixing due to inlet and outlet streams with the stored fluid. The model is applicable to the storage of both hot and chilled water. The model is validated with experimental data from the literature. The parameters that influence the operation of a stratified thermal energy storage for both heat and cool storage are examined. © 1998 John Wiley & Sons, Ltd.

KEY WORDS stratified water tank; heat storage; cool storage; wall conduction; mixing parameter; transient analysis

## 1. INTRODUCTION

The need to use thermal energy storage (TES) for an intermittent source of energy such as solar is obvious. However, in recent years, TES has become increasingly important in conventional power systems to ensure optimal demand management. Another application has been energy storage as chilled water to ensure optimum 'time of day use' tariffs in air-conditioning plants. Chilled water storage also results in lower capital costs as smaller air-conditioning plants can be used to achieve the same cooling (Bhattacharya, 1991). There are numerous studies on energy storage techniques reported in the literature and these have been summarized by Kovach (1976), Turner (1978) and Wilson (1978). Thermal Energy Storage can be achieved by three distinct approaches: Sensible Energy Storage (SES), latent heat storage (using phase change materials or PCMs) and thermochemical reversible reactions. The present paper describes a study of SES in stratified tanks with water as storage medium. In these tanks heat is stored as sensible heat of water. Assuming that there is no mixing, hot water which is at lower density than the cold water will be in the upper part of the tank and the cold water in the lower part and thereby thermal stratification is achieved. This is extremely useful since hot water can be extracted from the top of the tank and supplied to the load at the design temperature. The same concept can also be used to store 'Cool' for air-conditioning applications—here the warm water will be in the upper part of the tank at a uniform temperature and the chilled water in the lower part of the tank. It is the chilled water that is led to the load at the coolest temperature possible due to stratification. It is seen that in both applications the thermal stratification or thermocline is extremely important.

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The stored energy of the upper layers of hot water degrades with time due to (i) heat loss to the environment (ii) thermal diffusion within the water body (iii) axial wall conduction between hot and cold fluid zones which induces convection (mixing) in the fluid body and (iv) mixing introduced in the charge and discharge cycle. The effect of the first and third mechanisms can be quantified and their effect can be minimized. The contribution of the second mechanism to degradation of thermodynamic availability depends upon the thermal diffusion between top layers of hot water and bottom layers of cold water. This mechanism is less significant owing to the low thermal conductivity of water. However, the evaluation of the degradation in stored energy due to the fourth mechanism is difficult as it depends on the method of distribution of water as it enters the tank.

Several investigators have described one-dimensional models in the literature. One of the earliest models describing a thermal storage tank is the TRNSYS model developed by Klein *et al.* (1978) and updated periodically. This model, extensively used in many solar thermal simulations, does not incorporate axial conduction in the container walls and dependence of thermophysical properties on temperature which is particularly important in chilled water storage. Gupta and Jaluria (1982a,b) developed a simple one-dimensional conduction model. Their model is limited to storage tanks in the static mode only. Ghaddar and Al-Marafie (1989) developed a one-dimensional numerical model to study the stratification in thermal storage tanks. This model makes use of a spatially dependent term to account for turbulent mixing at the inlet which can be calculated only from experiments. El-Nashar and Qamiyeh (1990) developed a one-dimensional unsteady-state heat transfer model to predict the temperature profiles in a heat accumulator for various modes of operation. Their model does not take into consideration the heat loss occurring through the top and mixing effects at the inlet and outlet of the tank at the top and bottom. Wu and Han (1978) formulated mathematically the liquid energy storage tank as an initial-boundary value problem. They studied the storage system in discharge cycle operation. Yoo and Pak (1993) developed a theoretical model of the charging process to study the performance of the stratified storage tanks, assuming perfect piston flow. Their model is characterized by only one parameter, the Peclet number. They obtained a closed-form solution for the temperature distribution using the Laplace transform technique. Al-Najem *et al.* (1993) investigated analytically the thermal stratification using a two-dimensional model. They validated their model with their experimental results. They concluded that the turbulent mixing factor has a significant effect on the temperature distribution. They observed the thermal degradation increases with increase in the turbulent mixing factor. Their model does not account for the wall effect, aspect ratio and length to wall-thickness ratio on thermal stratification. Cole and Bellinger (1982) developed a one-dimensional model with empirical constants to account for a few factors such as mixing during charging, the fluid wall thermal interaction, and wall heat capacity on thermal stratification. The normalized heat transfer coefficient used to account for fluid wall interaction is observed to be insignificant. They concluded that mixing has a greater significance in the management of thermoclines. Their model does not account for heat losses through the top and bottom and geometrical factors on the maintenance of thermoclines. Kleinbach *et al.* (1993) made a numerical study on several one-dimensional models and compared the experimental data for a wide range of conditions. They described the plug flow, plume entrainment and multinode models and recommended the conditions under which each model is to be used. Zurigat *et al.* (1989) made a comparative study of one-dimensional models for stratified thermal storage tanks. They validated six models available in the literature with their experimental data. They noted that the models show varying degree of agreement with the thermocline test data. They concluded that the models of Wildin and Truman (1985), Cole and Bellinger (1982), and Zurigat *et al.* (1988) show good agreement with the experimental data, with slight adjustments in the mixing parameters. Wildin and Truman (1989) have used modified wall resistance values to match their experimental results. Further they used explicit finite difference scheme which requires optimization of time step to avoid numerical instabilities. Zurigat *et al.* (1988) used effective diffusivity factor to account for mixing in the tank. Their model does not account for wall effect and heat losses to the environment on thermal stratification. Zurigat *et al.* (1991) also studied the influence of inlet geometry on the degree of thermal stratification in thermocline thermal energy storage devices. They concluded that the position and sharpness

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of the thermocline are functions of Peclet number and Richardson number. They also observed that stratification occurs above a critical value of Richardson number of 0·241.

In most cases, the storage tanks were analysed using one-dimensional heat transfer models assuming that there is no other fluid motion except the average bulk fluid flow. Many of these models do not take into consideration the axial conduction through the tank walls and mixing effects at the inlet and outlet of the fluid flow. Though there are two-dimensional and three-dimensional models available for predicting the performance of the stratified storage systems, they are first of all time-consuming and tedious. Secondly the flow becomes one-dimensional after the formation of the thermocline zone (Wildin and Truman, 1989). Hence, in the present study, a one-dimensional transient heat conduction model is proposed for the stratified system. Further most of the one-dimensional models do not take into account the effects of all the factors responsible for the degradation of thermoclines. So an attempt is made to formulate a one-dimensional numerical model with all the major factors of thermocline degradation included in it. The present model takes into consideration the effects of axial wall conduction, heat capacity and thermal conductivity ratios of the wall and fluid, the aspect ratio, the length to wall thickness ratio of the storage tank and the mixing effects at the inlet region due to in-flow and out-flow of stored fluid in the tank, and the variations in the properties of water and heat transfer coefficients with temperature. The following assumptions are made:

(i) There is no other fluid motion except the bulk fluid flow in the storage tank.
(ii) The viscous dissipation effects are neglected.

## 2. ANALYSIS

The storage system to be considered in this analysis is a vertical cylinder of diameter $D$ and length $L$ as shown in Figure 1. The tank is divided into $N$ equal elements in the longitudinal direction and each element is assumed to be at uniform temperature. The initial temperature profile of the tank is known. Figures 2(a) and

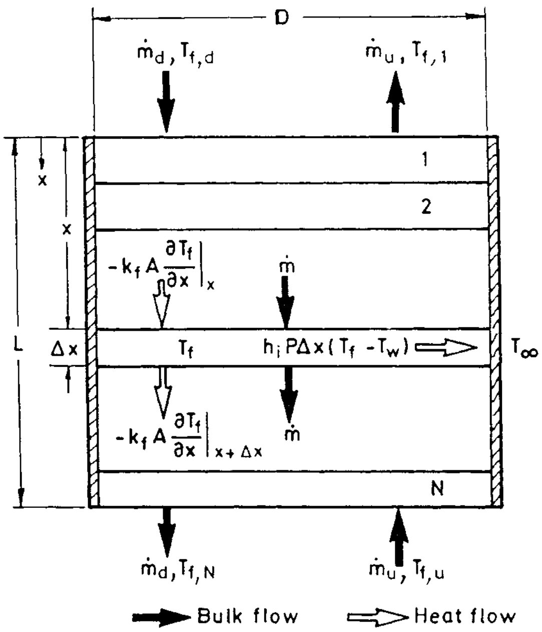


Figure 1. Stratified storage tank

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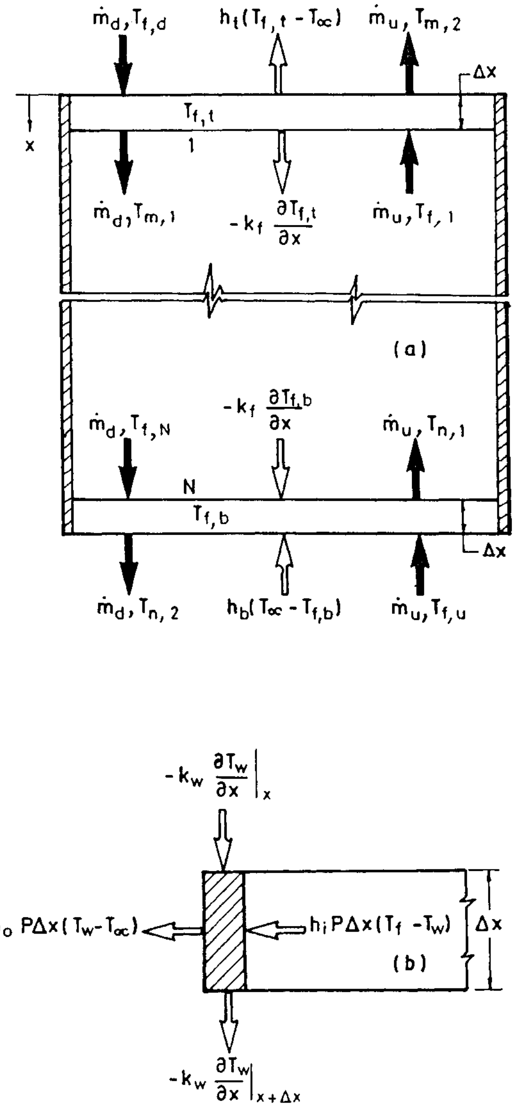


Figure 2. (a) Energy flows in the fluid; (b) energy flows in the tank wall

2(b) represent the energy flows into and out of the storage system under various modes of operation. The energy balance on an elemental control volume of the tank at distance $x$ from the top of the tank is shown in Figure 1 gives the governing differential equation for this transient conduction problem:

$$\frac{\partial T_f}{\partial t} = \frac{k_f}{\rho_f c_f}\frac{\partial^2 T_f}{\partial x^2} - \frac{\dot{m}}{\rho_f A}\frac{\partial T_f}{\partial x} - \frac{h_i P}{A \rho_f c_f}(T_f - T_w) \tag{1}$$

where

$$\dot{m} = \dot{m}_d - \dot{m}_u \tag{2}$$

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The governing equation (1) is a linear parabolic equation and needs two boundary conditions in addition to the initial condition. The boundary conditions in the fluid are obtained by making energy balances in the element 't' and 'b' very close to the top and bottom of the storage tank respectively, as shown in the Figure 2(a). Two fluid circuits, represented by the flow rates $\dot{m}_d$ and $\dot{m}_u$ (Figure 2(a)) are considered here. The former is called the collector or charge (heat storage)/discharge or load (cool storage) loop and the latter is called the load or discharge (heat storage)/charge (cool storage) loop. As the warm/cold water enters the storage tank through the top/bottom, the same volume of water is displaced from each element and finally leaves the tank through the bottom/top of the tank. The energy in the control volume either increases or decreases due to the inflow of fluid into the tank. In the case of charge loop (heat storage)/load loop (cool storage), as the fluid enters the tank through the inlet at the top the same volume of water is displaced in the tank and finally removed through the bottom of the tank. Similarly, the water entering through the bottom of the tank displaces the same volume of fluid in the tank and finally leaves through the top of the tank. Normally, any one of the above flow circuits will be operative at any time in both heat and cool storages. But when the heat storage is simultaneously charging and discharging both the flow circuits will be in operation. In a fully-stratified storage tank (FSST) model the mixing effects at the inlets/outlets and within the fluid can be neglected. However, even at very low velocities the inflow of water into the tank causes mixing of water in the element lying close to the inlet of the tank. The temperature of the fluid in this element will depend upon the quantity of fluid in the inlet stream and the amount of fluid in the element that mix with each other. If the entering stream has a higher temperature, energy is added to this control volume. Similarly, thermal energy is removed from the element when the fluid stream entering has a lower temperature. Furthermore, due to conduction between the elements, the temperature of the different regions get equalized, thereby reducing the effectiveness of the thermal storage and its ability to deliver energy at the maximum possible temperature in the case of heat storage and minimum possible temperature in the case of cool storage. The energy is gained/lost due to heat transfer between the fluid and environment through the top and bottom surfaces of the storage tank elements depending upon the temperature difference between the fluid in the element under consideration and the environment temperature.

Boundary condition in the fluid at $x = 0$ is given by

$$\frac{\partial T_{f,t}}{\partial x} - \frac{U_T}{k_f}(T_{f,t} - T_\infty) + \frac{\dot{m}_d c_f}{k_f A Z_1}(T_{f,d} - T_{f,t}) - \frac{\dot{m}_u c_f}{k_f A Z_2}(T_{f,t} - T_{f,1}) = 0 \tag{3}$$

Boundary condition in the fluid at $x = L$ is given by:

$$\frac{\partial T_{f,b}}{\partial x} - \frac{U_B}{k_f}(T_{f,b} - T_\infty) + \frac{\dot{m}_d c_f}{k_f A Z_4}(T_{f,N} - T_{f,b}) - \frac{\dot{m}_u c_f}{k_f A Z_3}(T_{f,b} - T_{f,u}) = 0 \tag{4}$$

The derivation of the boundary conditions are given in the appendix. Mixing parameters $Z_1$ to $Z_4$ are also defined in the appendix.

Equations (1)–(4) are non-dimensionalized using standard dimensionless parameters (given in the nomenclature). The dimensionless governing equations and boundary conditions in the fluid are as follows.

(1) *Fluid.*

Governing equation and boundary conditions:

$$\frac{\partial \theta_f}{\partial \tau} = \frac{\partial^2 \theta_f}{\partial X^2} - \mathrm{Pe}\frac{\partial \theta_f}{\partial X} - S(\theta_f - \theta_w) \tag{5}$$

$$\frac{\partial \theta_{f,t}}{\partial X} - \mathrm{Bi}_t \theta_{f,t} + \frac{\dot{m}_d \mathrm{Pe}}{\dot{m} Z_1}(\theta_{f,d} - \theta_{f,t}) - \frac{\dot{m}_u \mathrm{Pe}}{\dot{m} Z_2}(\theta_{f,t} - \theta_{f,1}) = 0 \tag{6}$$

$$\frac{\partial \theta_{f,b}}{\partial X} - \mathrm{Bi}_b \theta_{f,b} - \frac{\dot{m}_d \mathrm{Pe}}{\dot{m} Z_4}(\theta_{f,N} - \theta_{f,b}) + \frac{\dot{m}_u \mathrm{Pe}}{\dot{m} Z_3}(\theta_{f,b} - \theta_{f,u}) = 0 \tag{7}$$

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(2) *Applications of boundary conditions.*

(i) When there is no fluid flow into the tank $\dot{m}_d$ and $\dot{m}_u$ are equal to zero and these represent the boundary conditions of a fully stratified storage tank (FSST) in the stagnation mode.

(ii) When there is no flow in the collector (heat)/discharge (cool) loop, then $\dot{m}_d$ is equal to zero and equations (6) and (7) represent the boundary conditions of a stratified storage under charge (heat)/ discharge (cool) cycle.

(iii) Similarly when there is no flow in the load (heat)/charge (cool) loop, $\dot{m}_u$ is equal to zero and equations (6) and (7) represent boundary conditions of a storage tank in discharge (heat)/ charge (cool) cycle.

(iv) In equations (6) and (7) the first term represents the heat conducted and the second term represents the heat convected to the surroundings.

When $Z_1 = 1$, the inlet fluid from the top of the tank does not mix with the tank fluid. The value of $Z_1$ varies from 1 to a very large number, depending upon the degree of mixing. The larger the value of $Z_1$, the greater will be the degree of mixing. Similarly, $Z_2$ is a measure of mixing taking place in the top of the storage tank due to the disturbances created by the withdrawal of the fluid leaving the top of the tank. When the inlet stream and the bottom layers of the tank fluid do not mix with each during the upward flow from the bottom then the value of $Z_3$ is taken as 1. Similarly $Z_4 = 1$ represents the case of no mixing as the fluid leaves the bottom of the storage tank due to downward flow from the bottom of the storage tank. The larger the value of $Z_3$ or $Z_4$, the higher will be the degree of mixing. The momentum of the inlet jet causes mixing up to a certain region beyond the point of admission. The length of this disturbed region depends upon the flow rate and the type of diffuser system at the inlet. The mixing is an irreversible heat transfer occurring in the tank fluid and results in the degradation of available energy. So care has to be taken to minimize the mixing by properly designing the admission system so that the fluid entering the storage does not produce hydro-dynamic disturbances. The temperature of the fluid in the elements beyond the entrance region where mixing also can take place has to be evaluated. Hence, a separate mixing parameter, Zi, is defined to account for this mixing in the various positions in the tank beyond the inlet. (see the appendix). This number, Zi, represents the ratio of the amount of fluid from the node on the upstream side mixing with the fluid in the node under consideration. When the value of Zi is equal to zero there is no mixing in the node under consideration.

There is a heat transfer short circuit between the warm fluid zone and cold fluid zone through the conducting wall when the tank is insulated on its exterior surfaces. This energy transfer due to axial conduction depends on the thermal conductivity of the material and thickness of the wall of the storage tank.

The energy balance in the wall at distance $x$ from the top of the tank (Figure 2b) is given by:

$$\frac{\partial T_w}{\partial t} = \alpha_w \frac{\partial^2 T_w}{\partial x^2} + \frac{h_1 P}{A_w \rho_w c_w}(T_f - T_w) - \frac{h_0 P}{A_w \rho_w c_w}(T_w - T_\infty) = 0 \tag{8}$$

The boundary conditions in the wall at $x = 0$ and $x = L$ are given by

$$\frac{\partial T_w}{\partial x} = 0 \quad \text{at } x = 0 \tag{9}$$

$$\frac{\partial T_w}{\partial x} = 0 \quad \text{at } x = L \tag{10}$$

Equations (8)–(10) represent the governing and boundary condition equations in the wall. These are non-dimensionalized, using the dimensionless parameters given in the nomenclature.

(3) *Wall.*

Governing equation:

$$\frac{\partial \theta_w}{\partial \tau} = \frac{\alpha_w}{\alpha_f} \cdot \frac{\partial^2 \theta_w}{\partial X^2} + qr\mathrm{Nu}_i(\theta_f - \theta_w) - qr\mathrm{Nu}_o\theta_w \tag{11}$$

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Boundary conditions in the wall:

$$\frac{\partial \theta_w}{\partial X} = 0 \quad \text{at } X = 0 \tag{12}$$

$$\frac{\partial \theta_w}{\partial X} = 0 \quad \text{at } X = 1 \tag{13}$$

The initial temperature distribution in the fluid and wall are to be given:

$$\theta(0, X), \quad \theta_w(0, X) \quad \text{specified} \tag{14}$$

The above equations represent energy transfer in a stratified storage system for various operating conditions. The main parameters in the above equations are Peclet number, Pe, heat loss parameter $S$, heat capacity ratio, $r$, length to wall thickness ratio, $q$, conductivity ratio, and the wall conductance, $k_{eq}$. The Peclet number represents the ratio of energy withdrawal (or addition) from (or to) the tank by the fluid passing through it to the heat conduction across the thermoclines.

Equations (5) and (11) are linear parabolic partial differential equations. These are converted into finite difference equations and are solved, using the Chapeau higher-order discretization with Crank Nicolson averaging scheme subject to their initial and boundary conditions (equations (6), (7), (12)–(14)) at the top and bottom. Use of higher-order discretization scheme is necessary to reduce the error due to numerical diffusion. The spatial dimensionless temperatures at various intervals of time are obtained. The input parameters of the program are:

(i) Initial temperature distribution in the stored fluid and the enclosure wall.
(ii) Diameter, length and wall thickness of the storage tank.
(iii) Thermophysical properties of the tank material and insulation.
(iv) The flow rates in the charge and discharge loops.

The thermophysical properties of water at different temperatures are computed using subprograms. Some of the salient points of the present model are:

(i) It predicts the transient axial variation in temperature of the fluid and tank wall in both stratified heat and cool storage modes.
(ii) It is applicable to any mode of operation namely charge cycle, discharge cycle, simultaneous charge and discharge cycle and stagnation periods.
(iii) Aspect ratio, length to wall thickness ratio, heat capacity ratio, conductance ratio and wall conduction which affect thermal degradation in a stratified storage are accounted for in the model.
(iv) The variation in the convective heat transfer coefficient at the interface of the fluid and wall is considered.

The model is compared with the experimental and numerical data of Shyu *et al.* (1989) who used a two-dimensional laminar flow model to interpret the results of temperature and flow fields in a static stratified storage system. They have determined the temperature distribution in the fluid region near the wall by solving the vorticity transport equation, stream function and energy equations in the fluid and the energy equation in the wall. The computation is lengthy and requires considerable computer time. Further the model can be used for storages in the static mode of operation only.

The present model is also compared with the model of Wildin and Truman (1989), who proposed an explicit finite difference model to account for:

(i) Convective heat exchange between the water and walls and floor of the tank.
(ii) Heat exchange between the tank and surroundings.

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(iii) Two-dimensional heat conduction through the floor and wall.
(iv) One-dimensional heat conduction through the water.
(v) Mixing during thermocline formation during charging and discharging.

They have predicted their numerical results using a low value of wall resistance, which is almost half of the actual value to achieve close agreement with their experimental results. Thus, some amount of empiricism is involved in this model. Time and space steps are to be chosen carefully to avoid instabilities in computations using their model.

## 3. RESULTS

Shyu *et al.* (1989) have experimentally studied the stratification decay in cylindrical storage tanks of dimensions 200 mm (inside dia) × 400 mm (length) made of 3 and 6 mm thick stainless-steel sheets. These tanks were initially stratified and then allowed to cool. The initial temperature distribution of the fluid of

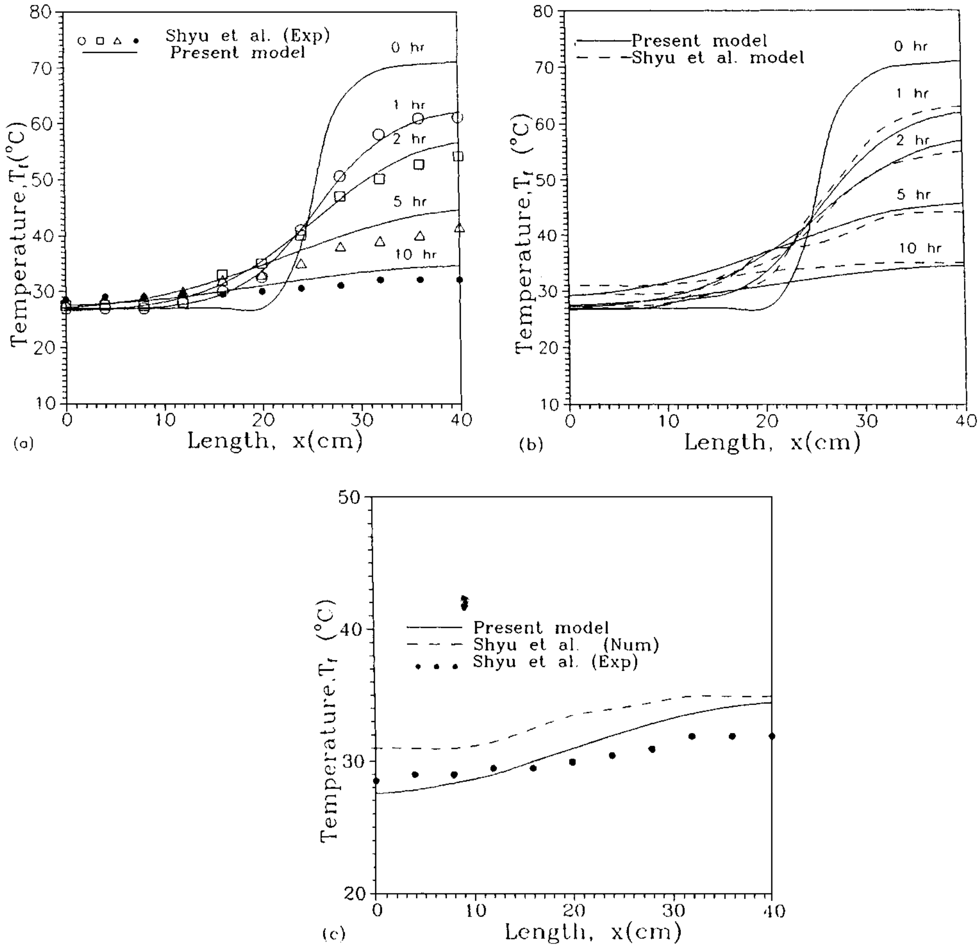


Figure 3. Comparison of present model with experimental and numerical data of Shyu *et al.* (1989), bare tank

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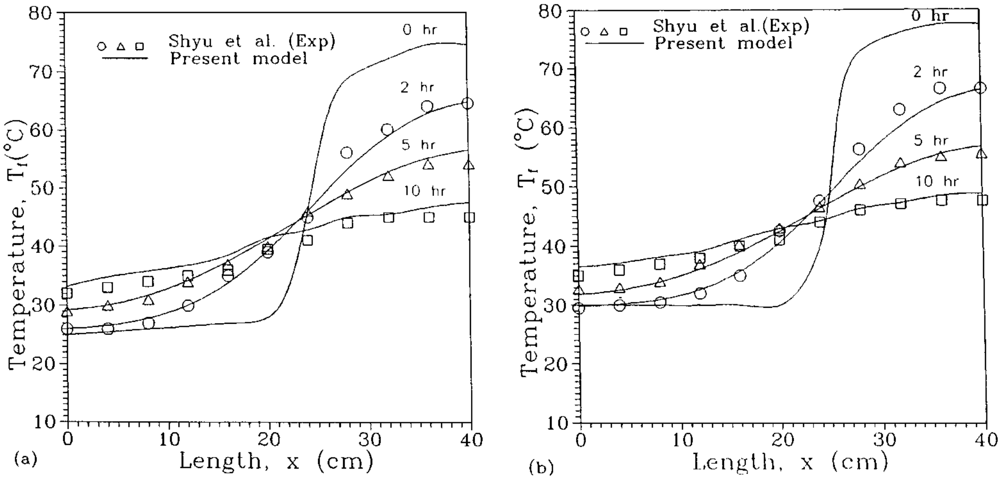


Figure 4. Comparison of present model with experimental data of Shyu *et al.* (1989): (a) Insulation thickness 65 mm; (b) insulation thickness 25 mm

their experiments is used to predict the spatial temperature distribution of the fluid at various instants of time using the present numerical model. The experimental and numerical results of the stratification decay for a 6 mm bare wall tank observed by Shyu *et al.* (1989) are compared with the numerical results of the present model in Figures 3(a) and 3(b), respectively. In Figure 3(c) the experimental and predicted temperature profiles of Shyu *et al.* (1989) are compared with the numerical results of the present model for a time interval of 10 h separately for the above case. Shyu *et al.* (1989) have also studied the stratification decay in storage tanks insulated with different thicknesses of insulation. The experimental temperature profiles of Shyu *et al.* (1989) in static stratified storage systems with an outside insulation of 65 and 25 mm thickness are compared with the numerical results of the present model in Figures 4(a) and 4(b), respectively. They had also performed their experiments on stratified storage tanks made of 3 mm thick stainless sheet, one insulated both on the exterior and interior surfaces and the other insulated on the exterior only. The inner insulation is of 5 mm thickness. The outside insulation in both the cases is 65 mm thick. The storage tanks are initially stratified and then left undisturbed. (stagnation period, static mode). The temperature profiles, using the present model, are compared with their experimental results in Figures 5(a) and 5(b), respectively. The effect of wall thickness on thermal stratification is shown by plotting predicted temperature profiles for two storage tanks of different wall thicknesses in Figure 5(c). The above storage tanks, made of stainless steel have the same length, diameter and thickness of glass wool insulation. Similarly, Figure 5(d) shows the numerically predicted temperature profiles in two geometrically similar stainless-steel storage tanks provided with different thicknesses of insulation.

The present model is also validated with the experimental and numerical results of Wildin and Truman (1989), who worked on cool storage. They performed their experiments on an acrylic scale model storage tank of 0·61 m dia and 0·91 m length. In charge cycle the storage tank which is initially at a temperature of about 15·5°C is charged with water at 5°C through an inlet at the bottom of the storage tank at the same rate as warm water (15·5°C) is withdrawn from the top. Similarly, in discharge cycle, the storage previously charged with chilled water at 5°C is discharged by admitting recycled warm water at 15°C through the top of the storage tank at the same rate as the chilled water is withdrawn through the bottom of the storage tank. They assumed no mixing and used wall resistances equal to half the actual values. Their numerical and experimental results are compared with the numerical results of the present model in Figures 6(a) and 6(b) assuming that mixing does not take place and using actual values of wall resistances. They also performed

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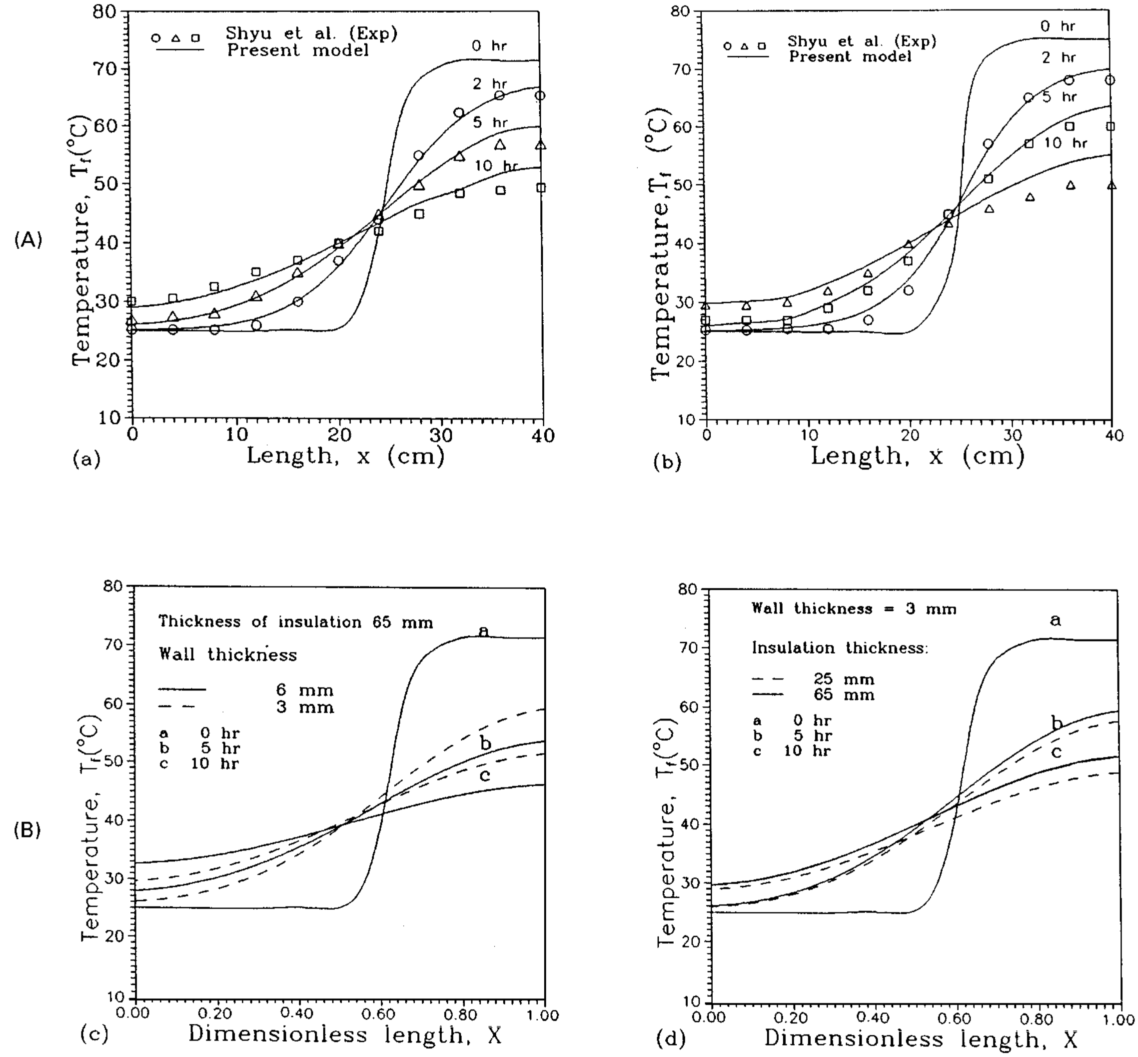


Figure 5(A). Comparison of present model with experimental data of Shyu *et al.* (1989): (a) insulation thickness 65 mm; (b) insulation thickness 25 mm. (B) Comparison of predicted temperature profiles of the present model: (c) insulation thickness 65 mm; (d) wall thickness 3 mm

another experiment on charge cycle with the same storage tank but of different external insulation resistance. They validated these experimental results with their numerical mixing model assuming that mixing takes place in the two nodes. The present model is used to predict the transient temperature profiles by first matching their temperature profile at 7 min by adjusting the value of Zi in the two nodes and use the same Zi to get the temperature values at other times. The numerical results of the present model are compared with their numerical and experimental results in Figure 6(c).

The model is also validated with the experimental results of Al-Najem *et al.* (1993). In their experiments a cylindrical tank, made of steel, having a diameter of 0·3 m, a thickness of 1·5 mm, height of 1·0 m and insulated with glass fibre of 50 mm thickness was used. The storage tank, initially filled with water at a temperature of 22°C was charged with hot water at 70°C temperature to simulate the charge cycle of the

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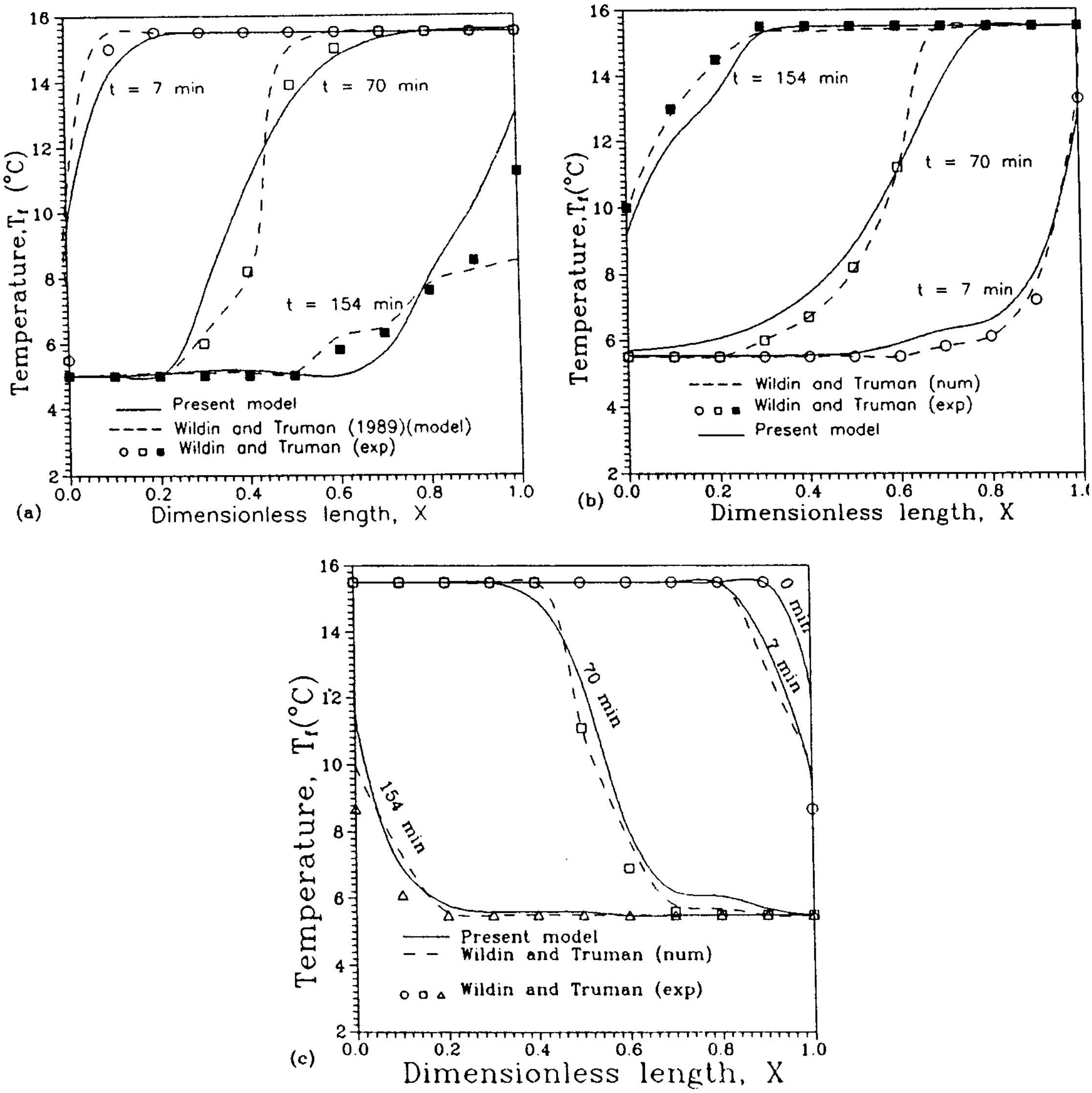


Figure 6. Comparison of present model with experimental and numerical data of Wildin and Truman (1989) for cool storage: (a) charging cycle (no mixing); (b) discharging cycle (no mixing); (c) charging cycle (mixing)

heat storage. The temperature profiles are obtained for two different flow rates and are plotted in Figures 7(a) and 7(b). Their experimental results were validated with their numerical model. They used varying turbulent mixing factors in their model to consider the effect of inlet jet on the stratification. There is close agreement between their numerical and the experimental results. The present model is used to predict the temperature profiles at various intervals of time assuming that mixing is taking place in the top three nodes. The mixing factors $Z_1$ at the inlet and Zi, at the top three nodes are adjusted to match their experimental values at one time interval and the subsequent temperature profiles are obtained using the same values.

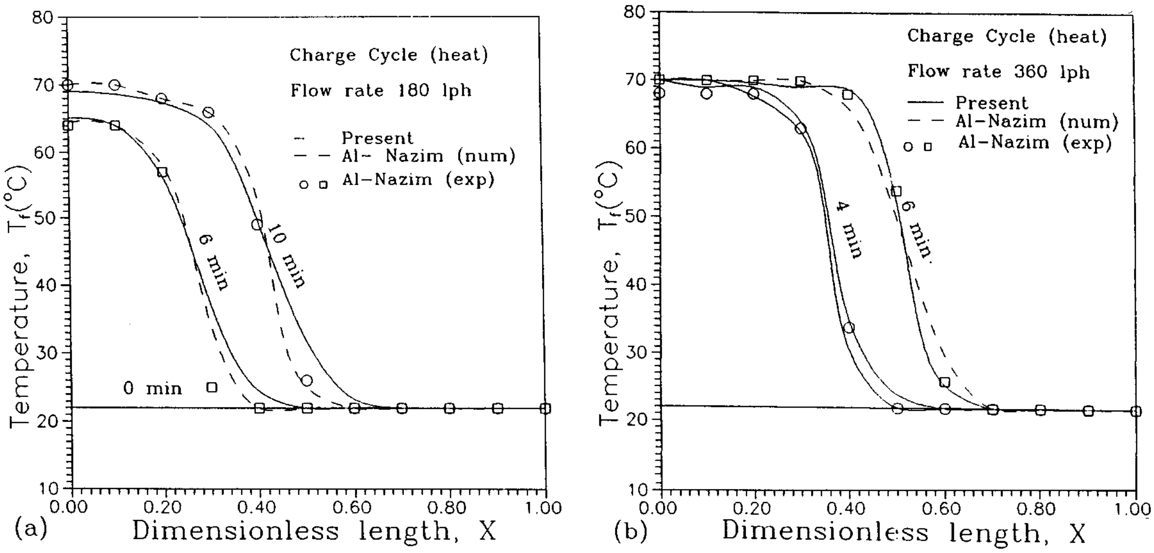


Figure 7. Comparison of present model with experimental and numerical data of Al-Najem *et al.* (1993) for charging cycle in heat storage: (a) flow rate 180 lph; (b) flow rate 360 lpn

## 4. DISCUSSION

In the experimental studies of Shyu *et al.* (1989) the storage tanks are initially stratified with hot water and are left undisturbed. The available energy stored degrades as can be seen from their experimental temperature profiles shown in Figures 3(a) and 3(b). The upper layers of hot water decrease in temperature due to irreversible heat transfer between storage tank and the environment. In addition, available energy is also degraded due to thermal diffusion within the tank fluid and axial conduction through the tank wall. These two effects result in the increase in temperature of bottom layers of cold water. But in the case of bare tanks heat loss to the environment is the major heat loss mechanism the bottom layers of cold water do not get heated up very much. Thus, the thermoclines degrade to such an extent that the fluid in the tank reaches an unstable temperature, necessitating the recharging of the tank. The region below the thermocline region heats up initially due to heat flow from the region above the thermocline, and then gradually decreases because, the heat losses from this region exceed the heat gain with increase in time. The temperatures are predicted, using the present model for a 6 mm bare (uninsulated) cylindrical tank used in the experiments of Shyu *et al.* (1989) for the same experimental conditions and the initial temperature distribution of the water body. Both the present model and the model of Shyu *et al.* (1989) assume that perfectly insulated conditions exist at the top and bottom. The predicted results of the present model are plotted along with the experimental and numerical results of Shyu *et al.* (1989) in Figures 3(a) and 3(b), respectively. The predictions of both the models in the hot fluid region are slightly higher than the experimentally observed values at large intervals of time, though there is good agreement between them at small intervals of time. The over prediction of temperatures in the hot fluid region may be due to actual top heat losses in the experiments which are neglected in both the models. In Figure 3(c) the numerical results and the experimental results of Shyu *et al.* (1989) are plotted along with the numerical results of the present model for a time interval of 10 h. It is observed that the agreement between the numerical and experimental results is better in the case of the present model. In the present approach the individual effects of all the major factors contributing to the stratification decay as mentioned earlier are included. The experimental results of two stainless-steel cylindrical storage tanks of 6 mm wall thickness, one insulated with 65 mm and the other with 25 mm

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thickness of exterior insulation are compared with the present numerical results in Figures 4(a) and 4(b). It is observed that the upper layers cool slowly and the bottom layers of water heat up more in these insulated tanks than in the bare wall tank (Figure 3). The time scale analysis shows that the heat losses to the environment is the major heat loss mechanism in a bare wall stratified storage tank and the thermal diffusion is the slower process. When there is an outside insulation, both the axial wall conduction and heat loss through the insulation assume equal importance in controlling the degradation of the thermocline. Thus, when the storage tank is insulated on the exterior, the rate of heat loss through the storage tank wall is reduced due to increased thermal resistance to radial heat transfer. But the heat is short-circuited through the conducting tank wall from the hot fluid zone to the cold fluid zone. Hence, the bottom layers of cold water heat up due to axial wall conduction. So the exterior insulation enhances the axial conduction. Both the axial wall conduction and heat exchange of the storage with the environment tend to reduce the available energy. Since the reduction in radial heat losses is much larger than the axial wall conduction heat transfer to the cold fluid layers in the insulated storage tanks, the rate of decay of thermoclines decreases with increase in insulation. The bottom layers of water heat up more and more as the thickness of insulation is increased (Figures 4(a) and 4(b)). In Figures 5(a) and 5(b) are predicted temperature profiles of the present model are plotted along with the experimental results of Shyu *et al.* (1989) for stainless-steel storage tanks of 3 mm wall thickness insulated on the exterior with 65 and 25 mm thick insulation, respectively. The predicted temperature profiles of storage tanks of 3 and 6 mm wall thicknesses with glasswool insulation thickness of 65 mm are compared in Figure 5(c) for two time intervals of 5 and 10 h, the other operating conditions of the storage tanks being the same in both the cases. The temperature profiles show that the thermocline degradation is more in storage tank of 6 mm wall thickness. The degradation of thermoclines is sensitive to the wall thickness. For the same thickness of exterior insulation thermal stratification increases with increase in the length to wall thickness ratio of the storage tank. In Figure 5(d) the temperature profiles obtained for two geometrically similar storage tanks having glass wool insulation of 25 and 65 mm thick are compared. It is seen from the above figure that the stratification improves slightly with increase in the thickness of insulation.

The numerical results of the present model are also compared with the experimental and numerical results of Wildin and Truman for a charging cycle (Figure 6(a)) and for a discharging cycle (Figure 6(b)) in a chilled water storage. The above figures show the vertical temperature distributions experimentally observed and predicted during the charging and discharging periods of the chilled water storage. Wildin and Truman (1989) used a 'no mixing model' to predict the temperature profiles. The present model is used to predict the temperature profiles for the above two cases assuming no mixing in the storage system. Figures 6(a) and 6(b) show that there is disagreement between the experimental and the numerical results of both the models. This is due to the mixing taking place in the storage system, which is not accounted for by the Wildin and Truman model and neglected in the computations using the present model. The thermoclines degrade when mixing occurs in the tank due to the turbulence created by the fluid entering the storage tank. Mixing reduces the available cooling capacity of a storage system. However, the mixing can be minimized by adopting carefully designed inlet and outlet manifold systems so that the fluid either enters or leaves the storage gently without disturbing the thermocline. However, the diffusion from the warmer fluid to the cold fluid cannot be avoided. So if the fluid layers in the storage are not disturbed, the diffusion will be the only heat transport mechanism. The temperature changes faster in the vicinity of interface with increase in the Peclet number. A large Peclet number implies a high velocity of tank inlet fluid or low thermal diffusivity or a tank with a large length to diameter ratio. A well designed inlet manifold system at the top and bottom of the tank and proper charging rates helps in minimizing mixing in the tank. The experimental and numerical data of Wildin and Truman (1989) for a cool storage in charge cycle is plotted in Figure 6(c) along with the predicted temperature profiles of the present model for the same. They have used the two node mixing model to predict the temperature profiles. The present model is used to predict the temperature profiles assuming mixing in the inlet and the first two nodes. The mixing parameters in the present model are adjusted to match the temperature profile at any particular instant and the predictions are continued with the same mixing factors till the charge cycle is completed. It is observed that there is close agreement between the experimental and numerical results in this

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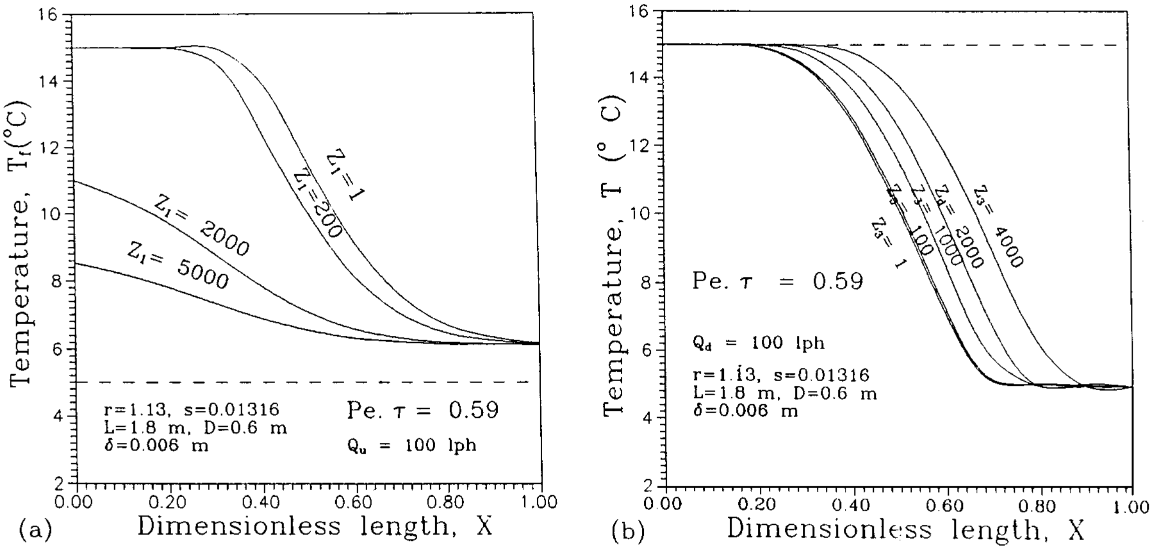


Figure 8. Effect of $Z$ parameter on thermal stratification in cool storage: (a) discharging cycle; (b) charging cycle

case. The present model is also used to validate the experimental and the numerical results of Al-Najim *et al.* (1993) for a heat storage in charging operation. The numerical values for the mixing parameters at inlet and the top three nodes are chosen by trial and error to match the temperature profiles at one time instant and the temperatures are predicted for the subsequent time intervals with the same mixing factors. The mixing parameters depend upon the flow rates and type of diffuser system. Figures 7(a) and 7(b) compare the experimental and numerical results of Al-Najim *et al.* (1993) with the numerical results of the present model for two different charging rates. There is good agreement between the numerical and experimental results in both the cases. The effect of mixing parameters $Z_1$ and $Z_3$ on stratification for a cool storage in discharging and charge cycle is shown in Figures 8(a) and 8(b), respectively. It is seen from Figure 8(a) that the thermocline degradation increases with the increase in the value of $Z_1$ in a discharge cycle. The thermocline degradation for a cool storage undergoing charging is shown in Figure 8(b). It is observed in the above figure that the thermocline degradation increases with increase in the value of $Z_3$. It is seen from both the figures that the rate of thermocline degradation is more in the discharge cycle (cool) compared to the charge (cool) cycle.

The numerical values of mixing parameters at inlet and in the nodes immediately after the inlets depend upon the type of diffuser, velocity of the inlet stream and the initial temperature difference between the warm and cold water. These are to be evaluated based on a large number of experiments at different flow rates and initial temperature differences for each diffuser and correlated.

## 5. CONCLUSIONS

A one-dimensional model incorporating axial wall conduction, thermal diffusion, heat transfer with the ambient and effects of fluid mixing on temperature has been proposed. The model has been compared with literature data on heat storage as well as chilled water storage satisfactorily. The present one-dimensional model compares favourably with the more complicated two dimensional models in the literature. The model can be used to examine the effects of parameters such as aspect ratio, length to thickness of the wall) ratio of the storage tank, heat capacity ratio of storage tank wall to stored fluid, mixing effects and the variations in thermophysical properties of the water due to changes in temperature.

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## NOMENCLATURE

| | |
|---|---|
| $A =$ | cross-sectional area of the storage tank ($m^2$) |
| Bi $=$ | Biot number ($hL/k_f$) |
| $c =$ | specific heat (kJ $kg^{-1}$ °C)$^{-1}$ |
| $C_{1 \cdots 8} =$ | heat capacity of storage fluid (kJ °C)$^{-1}$ |
| $D =$ | diameter of the tank (m) |
| $h =$ | heat transfer coefficient (W $m^{-2}$ °C)$^{-1}$ |
| $k =$ | thermal conductivity (W $m^{-1}$ °C)$^{-1}$ |
| $k_{eq} =$ | thermal conductance of the tank wall (W $m^{-1}$ °C)$^{-1}$ |
| $L =$ | length of the tank (m) |
| $\dot{m} =$ | mass flow rate of tank fluid (kg $s^{-1}$) |
| $N =$ | number of tank segments |
| Nu $=$ | Nusselt number ($h_i L/k_f$) |
| $Nu_o =$ | Nusselt number ($k_{eq}L/k_f$) |
| $P =$ | perimeter of the tank (m) |
| Pe $=$ | Peclet number ($v\mathrm{L}/\alpha_f$) |
| $q =$ | length to wall thickness ratio ($L/\delta$) |
| $r =$ | ratio of heat capacities of storage fluid and tank wall material ($\rho_f c_f/\rho_w c_w$) |
| $S =$ | heat loss parameter at the tank wall ($S = 4$ Nu($L/D$)) |
| $t =$ | time (s) |
| $T =$ | temperature (°C) |
| $\Delta T =$ | initial temperature difference between warm and cold water (°C) |
| $U_T =$ | overall heat transfer coefficient at the top of the tank (W $m^{-2}K^{-1}$) |
| $U_B =$ | overall heat transfer coefficient at the bottom of the tank (W $m^{-2}$ $K^{-1}$) |
| $v =$ | average bulk velocity ($\dot{m}/A\rho_f$)(m $s^{-1}$) |
| $x =$ | axial coordinate (m) |
| $X =$ | dimensionless axial co-ordinate ($x/L$) |
| $Z_{1 \cdots 4} =$ | mixing parameters |

*Greek symbols*

| | |
|---|---|
| $\alpha =$ | thermal diffusivity ($m^2$ $s^{-1}$) |
| $\delta =$ | wall thickness (m) |
| $\Delta =$ | infinitesimal or small |
| $\theta =$ | dimensionless temperature (($T - T_\infty$)/$\Delta T$) |
| $\rho =$ | density, kg $m^{-3}$ |
| $\tau =$ | dimensionless time (Fourier number), ($t\alpha_f/L^2$) |

*Subscripts*

| | |
|---|---|
| b $=$ | bottom |
| d $=$ | downward flow |
| f $=$ | fluid |
| i $=$ | inside |
| o $=$ | outside |
| t $=$ | top |
| u $=$ | upward flow |
| w $=$ | wall |
| $\propto$ $=$ | ambient |
| $1, 2, \ldots, N$ | element |

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## APPENDIX

*Energy balance in the element at 't'*:

Energy stored/degraded in the element = Energy entering − Energy leaving.

The energy balance in the fluid very close to the top of the storage tank (Figure 2(a)) gives

$$\rho_{\mathrm{f}} c_{\mathrm{f}} A\, \Delta x \frac{\partial T_{\mathrm{f,t}}}{\partial t} = k_{\mathrm{f}} A \frac{\partial T_{\mathrm{f,t}}}{\partial x} - h_t A (T_{\mathrm{f,t}} - T_\infty) + \dot{m}_{\mathrm{d}} c_{\mathrm{f}} (T_{\mathrm{f,d}} - T_{\mathrm{m,1}}) + \dot{m}_{\mathrm{u}} c_{\mathrm{f}} (T_{\mathrm{f,1}} - T_{\mathrm{m,2}}) \tag{15}$$

where $T_{\mathrm{m,1}}$ and $T_{\mathrm{m,2}}$ are the temperatures in the elemental volume after the inlet streams from the top adjacent element mix with the fluid in the element '$t$' under consideration.

$$T_{\mathrm{m,1}} = \frac{(C_1 = \dot{m}_{\mathrm{d}} c_{\mathrm{f}} \Delta t) T_{\mathrm{f,d}} + (C_2 = m_{1,\mathrm{d}} c_{\mathrm{f}}) T_{\mathrm{f,t}}}{C_1 + C_2} \tag{16}$$

where $m_{1,\mathrm{d}}$ represents the amount of fluid in the element '$t$' mixing with the inlet stream entering through the top of the tank during the time interval $\Delta t$.

$$T_{\mathrm{m,2}} = \frac{(C_3 = \dot{m}_{\mathrm{u}} c_{\mathrm{f}} \Delta t) T_{\mathrm{f,1}} + (C_4 = m_{1,\mathrm{u}} c_{\mathrm{f}}) T_{\mathrm{f,t}}}{C_3 + C_4} \tag{17}$$

where $m_{1,\mathrm{u}}$ represents the amount of fluid in the element '$t$' mixing with the inlet stream from the bottom of the tank during the period $\Delta t$.

In the limit $\Delta x \to 0$, equation (15) simplifies to:

$$\frac{\partial T_{\mathrm{f,t}}}{\partial x} - \frac{U_{\mathrm{T}}}{k_{\mathrm{f}}} (T_{\mathrm{f,t}} - T_\infty) + \frac{\dot{m}_{\mathrm{d}} c_{\mathrm{f}}}{K_{\mathrm{f}} A} (T_{\mathrm{f,d}} - T_{\mathrm{m,1}}) - \frac{\dot{m}_{\mathrm{u}} c_{\mathrm{f}}}{k_{\mathrm{f}} A} (T_{\mathrm{m,2}} - T_{\mathrm{f,1}}) = 0 \tag{18}$$

On substitution of equations (16) and (17) the above equation simplifies to

$$\frac{\partial T_{\mathrm{f,t}}}{\partial x} - \frac{U_{\mathrm{T}}}{k_{\mathrm{f}}} (T_{\mathrm{f,t}} - T_\infty) + \frac{\dot{m}_{\mathrm{d}} c_{\mathrm{f}}}{K_{\mathrm{f}} A Z_1} (T_{\mathrm{f,d}} - T_{\mathrm{f,t}}) - \frac{\dot{m}_{\mathrm{u}} c_{\mathrm{f}}}{k_{\mathrm{f}} A Z_2} (T_{\mathrm{f,t}} - T_{\mathrm{f,1}}) = 0 \tag{19}$$

where

$$Z_1 = \frac{C_1 + C_2}{C_2} \quad \text{and} \quad Z_2 = \frac{C_3 + C_4}{C_3}$$

*Energy balance in the element 'b' at $x = L$*:

Energy stored/degraded = Energy in − Energy out

$$\rho_{\mathrm{f}} A C_{\mathrm{f}} \frac{\partial T_{\mathrm{f,b}}}{\partial t} = -k_{\mathrm{f}} A \frac{\partial T_{\mathrm{f,b}}}{\partial x} + U_{\mathrm{B}} A (T_{\mathrm{f,b}} - T_\infty) + \dot{m}_{\mathrm{d}} C_{\mathrm{f}} (T_{\mathrm{f,n}} - T_{\mathrm{n,2}}) + \dot{m}_{\mathrm{u}} C_{\mathrm{f}} (T_{\mathrm{f,u}} - T_{\mathrm{n,1}}) \tag{20}$$

In the limit as $\Delta x \to 0$, equation (20) simplifies to

$$\frac{\partial T_{\mathrm{f,b}}}{\partial x} - \frac{U_{\mathrm{B}}}{k_{\mathrm{f}}} (T_{\mathrm{f,b}} - T_\infty) - \frac{\dot{m}_{\mathrm{d}} C_{\mathrm{f}}}{k_{\mathrm{f}} A} (T_{\mathrm{f,N}} - T_{\mathrm{n,2}}) - \frac{\dot{m}_{\mathrm{u}} C_{\mathrm{f}}}{k_{\mathrm{f}} A} (T_{\mathrm{f,u}} - T_{\mathrm{n,1}}) = 0 \tag{21}$$

$$T_{\mathrm{n,1}} = \frac{(C_5 = \dot{m}_{\mathrm{u}} c_{\mathrm{f}} \Delta t) T_{\mathrm{f,u}} + (C_6 = m_{2,\mathrm{u}} c_{\mathrm{f}}) T_{\mathrm{f,b}}}{C_5 + C_6} \tag{22}$$

where $m_{2,\mathrm{u}}$ represents the mass of water in the element 'b' mixing with the inlet stream entering through the inlet at the bottom of the tank.

$$T_{\mathrm{n,2}} = \frac{(C_7 = \dot{m}_{\mathrm{d}} c_{\mathrm{f}} \Delta t) T_{\mathrm{f,N}} + (C_8 = m_{2,\mathrm{d}} c_{\mathrm{f}}) T_{\mathrm{f,b}}}{C_7 + C_8} \tag{23}$$

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where $m_{2,d}$ represents the mass of water in the element 'b' mixing with the inlet stream from the top of the tank in time period $\Delta t$.

Substitution of equations (22) and (23) into equation (21) gives

$$\frac{\partial T_{f,b}}{\partial x} - \frac{U_B}{k_f}(T_{f,b} - T_\infty) - \frac{\dot{m}_d c_f}{k_f A Z_4}(T_{f,N} - T_{f,b}) + \frac{\dot{m}_u c_f}{k_f A Z_3}(T_{f,b} - T_{f,u}) = 0 \tag{24}$$

$$Z_3 = \frac{C_5 + C_6}{C_6} \quad \text{and} \quad Z_4 = \frac{C_7 + C_8}{C_8}$$

The above equations are non-dimensionalized using the dimensionless parameters given in the nomenclature. The mixing of the fluid may also take place in the nodes located next to the inlet on the downstream side depending on the velocity of the inlet jet. The temperature of mixing in the node under consideration is calculated from the following formula:

$$\theta_{f,j} = \frac{\theta_{f,j} + Zi \cdot \theta_{f,j\pm 1}}{Z_i + 1} \tag{25}$$

where '$j$' refers to the node in which mixing is taking place and '$j \pm 1$' refers to the node on the upstream of '$j$', and '$+1$' is used when the flow is from tank bottom to the top and '$-1$' when the flow is from the top to bottom.

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