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INTERACTION OF MACHINE TOOL AND WORKPIECE RIGIDITIES

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Abstract—The form errors of cylindrical workpiece turned between centres caused due to machine tool compliance (reciprocal of rigidity or stiffness) and workpiece compliance compensate each other to some extent. The paper analyses the extent of mutual compensation of these form errors. It shows that the most rigid machine tool need not be the best one from the point of view of accuracy of turned workpieces and that there is an optimum combination of machine tool and workpiece rigidities. It establishes a quantitative criterion for an optimum use of stays in turning practice.

WHEN cylindrical jobs are turned between centres on a lathe as in Fig. 1, the final shape obtained will be that of a true cylinder only when neither the machine tool, nor the workpiece undergoes any deflection under the action of cutting forces, that is, when the machine tool and the workpiece are absolutely rigid (Fig. 2a). In practice, there will be deviations from true cylindrical shape because the machine tool and the workpiece are always compliant to

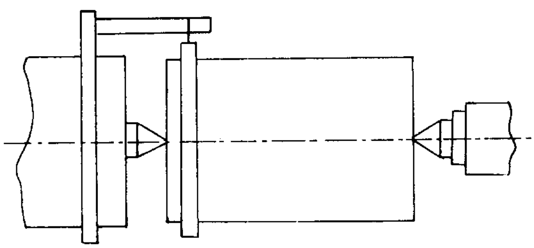

FIG. 1.

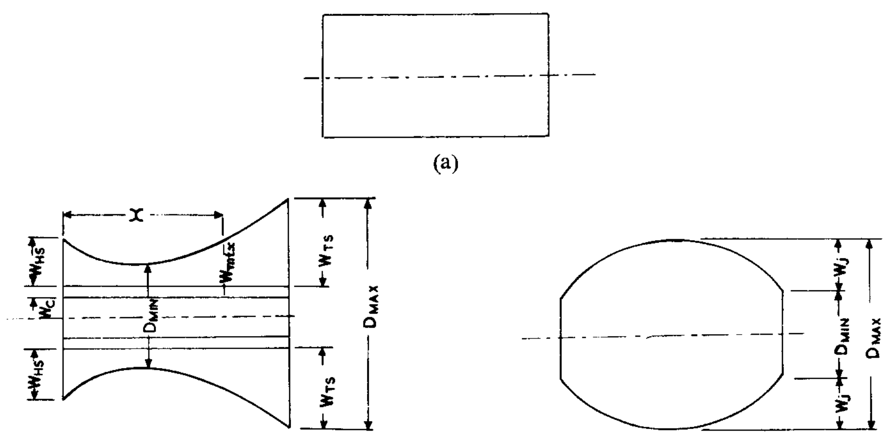

FIG. 2.

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some extent or other (compliance is the reciprocal of rigidity). When absolutely rigid jobs are turned between centres, the deviations from true cylindrical shape are caused by the compliance of the machine tool alone and the turned jobs are cradle shaped (Fig. 2b). The exact profile of the cradle shape can be found by calculating the deflections at various points along the length of the job caused due to the compliance of the machine tool.

The deflection of a point at a distance x from the head stock and on the job is given by the following expression [1, 2].

$$W_{mtx} = W_c + W_{hs}\left(\frac{L-x}{L}\right)^2 + W_{ts}\left(\frac{x}{L}\right)^2 \tag{1}$$

where W_{mtx} = the deflection caused due to the compliance of the machine tool of a point on the job at a distance x from the head stock end in mk/kg.

W_c = compliance of the carriage in mk/kg.

W_{hs} = compliance of the head stock in mk/kg.

W_{ts} = compliance of the tail stock in mk/kg.

L = length of the job in mm.

The compliance of the carriage W_c is mainly due to the deformation of the carriage and the deflection of the bed. As the deflection of the bed varies with the position of the tool and therefore x, W_c will not be, strictly speaking, constant over the full length of the bed. However, W_c in (1) is assumed to be constant over the length of the bed in view of the following:

(a) The maximum displacement of the tool with respect to the workpiece caused due to the deflection of the bed has been found not to exceed 10–15 per cent of the total displacement of the tool with respect to the workpiece for general purpose lathes [3, 4].

(b) In the work of a large number of authors on the problems of accuracy in metal cutting this assumption has been made [1, 2, 5, 6].

(c) As the component of the compliance caused by bed deflection is very small in comparison with the total machine tool compliance, the assumption that W_c = constant will not invalidate the basic or qualitative nature of conclusions arrived at. However, the quantitative conclusions or results obtained in the wake of such an assumption will not be 100 per cent accurate. But, still, the accuracy of these conclusions will be sufficiently high for all practical purposes.

(d) The treatment indicated will neither be basically altered, nor made wrong if the actual variation of W_c along the length of the bed is also considered.

Denoting the error in true cylindrical form due to compliance of machine tool under the action of 1 kg cutting force as Δ_m, we get

$$\Delta_m = 2\,(W_{mt\ \max} - W_{mt\ \min}) \quad \text{(see Fig. 2b)}$$

$$= 2\,[(W_{ts} + W_c) - W_{mt\ \min}]. \tag{2}$$

Equation (2) is obtained by assuming that W_{ts} is greater than W_{hs}, as it is normally observed in a lathe. (If W_{hs} is greater than W_{ts}, replace W_{ts} in the expression by W_{hs}.) $W_{mt\ \min}$ can be found to occur at $x = (W_{hs}/W_{hs} + W_{ts})L$ by differentiating (1). Substituting this value of x in (1), we get

$$W_{mt\ \min} = W_c + \frac{W_{hs}W_{ts}}{W_{hs} + W_{ts}}, \tag{3}$$

substituting (3) in (2) we get

$$\Delta_m = 2\left[W_{ts} - \frac{W_{hs}W_{ts}}{W_{hs}W_{ts}}\right]. \tag{4}$$

The diametrical variation Δ_m is reduced by reducing the compliance of the machine tool to as small a value as possible. In order to get an ideally cylindrical shape it may seem necessary to use an absolutely rigid machine tool. But it is never possible, in practice, to have absolutely rigid machine tools and increased rigidity is possible only at considerably higher costs.

Apart from the fact that higher rigidity is associated with increased costs, it is interesting to observe that an extremely rigid machine tool is rarely the optimum one even from the point of view of turning accuracy. This is because of the following: the work piece is never absolutely rigid and will deflect under the action of cutting forces. The deflection per kg of cutting force or compliance of the workpiece fixed between centres at any point x from the end is given by the well known formula of strength of materials [7].

$$W_j = \frac{1000\,(L-x)^2x^2}{3EIL} \tag{5}$$

where W_j = deflection per kg or compliance of the job in mk/kg

E = Young's modulus in kg/mm^2

I = Moment of inertia in mm^4; for circular cross section $I = \frac{\pi D^4}{64}$

L and D = length and diameter of the job or workpiece in mm.

If a compliant workpiece is turned between centres of an infinitely rigid machine tool, its final shape is that of a barrel with maximum convexity in the middle (Fig. 2c). The exact profile of the barrel shape can be determined by calculating the deflections at various points along the length of the job from the equation (5). Denoting the error in true cylindrical shape due to compliance of the workpiece under the action of 1 kg of cutting force as Δ_j, we get

$$\Delta_j = 2\,(W_{j\,\max} - W_{j\,\min})$$

= 2 (deflection of the workpiece at the centre—deflection of the workpiece at the ends)

$$= 2\left(\frac{250L^3}{3EI} - 0\right) = \frac{500L^3}{3EI}. \tag{6}$$

The two form errors Δ_m and Δ_j are opposite in direction and so tend to reduce each other. This fact can be restated to advantage in two ways:

(1) the form error of workpieces turned between centres caused due to compliance of the machine tool is reduced by the compliance of workpiece.

(2) the form error of workpieces turned between centres caused due to compliance of the workpiece is reduced by the compliance of the machine tool.

The above conclusions are rather simple but have immense significance, because they reveal that a machine tool with infinite rigidity or no compliance will not lead to maximum turning accuracy and there is an optimum combination of machine tool and workpiece rigidities.

TABLE 1. DEFLECTIONS AT VARIOUS POINTS ALONG THE LENGTH OF THE WORKPIECE

(dia. = 15 mm, [length (L)/dia. (D)] = 4)

Distance of the point from the head stock end (fraction of L)	0	0·1	0·2	0·3	0·39	0·4	0·5	0·6	0·7	0·8	0·9	1
W_{mt} (mk/kg)	1·143	1·055	0·992	0·955	0·940 (min)	0·945	0·961	1·002	1·060	1·153	1·284	1·429 (max)
W_j (mk/kg)	0 (min)	0·012	0·037	0·064	0·081	0·083	0·090 (max)	0·083	0·064	0·037	0·012	0 (min)
W_{total} (mk/kg)	1·143	1·067	1·029	1·019 (min)	1·021	1·028	1·051	1·085	1·124	1·190	1·296	1·429 (max)

A precise investigation of the increase of accuracy due to compliance of the machine tool or workpiece by strict mathematical methods is well-nigh impossible, as the equations obtained are cubic—first differentials of (1) and (5)—whose solutions are often imaginary and negative. Useful solutions of such equations are possible only after considerable simplification of the problem in which case its real nature may not be revealed.

In view of the above, the following numerical approach is adopted: the deflections of 12 points at equal intervals along the length of the workpiece caused due to a known compliance of the machine tool, viz., the head-stock, the tail-stock and the carriage, are calculated from (1). The deflections of the same points caused due to known compliance of the workpiece are calculated from (5). The addition of the two deflections of corresponding points will now yield the total deflections caused due to the combined compliance of the machine tool and the workpiece. The above procedure has been repeated for a constant machine tool and varying workpieces as given below:

Machine tool: $W_{hs} = 0{\cdot}508$ mk/kg
$W_{ts} = 0{\cdot}794$ mk/kg
$W_c = 0{\cdot}635$ mk/kg

Workpiece: material : M.S. (Young's Modulus
$E = 21{,}000$ kg/mm^2)
dia. = 5, 10, 15, 20, 25 mm
$L : D$ ratio = 1, 2, 3, 4, 5, 6, 7, 8, 9.

A model calculation as described above is shown in Table 1.

From Table 1

$$\Delta_m = 2\,(W_{mt\ \max} - W_{mt\ \min}) = 2\,(1{\cdot}429 - 0{\cdot}940) = 0{\cdot}978 \text{ mk/kg}$$

$$\Delta_j = 2\,(W_{j\ \max} - W_{j\ \min}) = 2 \times 0{\cdot}090 = 0{\cdot}180 \text{ mk/kg.}$$

Let Δ_T = error in true cylindrical form due to the total compliance of machine tool and workpiece.

$$\Delta_T = 2\,(W_{t\ \max} - W_{t\ \min}) = 2\,(1{\cdot}429 - 1{\cdot}019) = 0{\cdot}820 \text{ mk/kg.}$$

Let $\overline{\Delta}_j$ = compensation of diametral form error due to compliance of the job (mk)

$\overline{\Delta}_m$ = compensation of diametral form error due to the compliance of the machine tool (mk)

$$\overline{\Delta}_j = (\Delta_m - \Delta_T) = (0{\cdot}978 - 0{\cdot}820) = 0{\cdot}158 \text{ mk/kg}$$

$$\overline{\Delta}_m = (\Delta_j - \Delta_T) = 0{\cdot}180 - 0{\cdot}820 = -\,0{\cdot}640 \text{ mk/kg}$$

(the negative sign shows that there is a decrease of accuracy due to the compliance of the machine tool under the given conditions).

RESULTS AND DISCUSSION

(1) *Compensation of accuracy due to the compliance of the machine tool*

Figure 3 shows the variation of $\overline{\Delta}_m$ as the compliance characterized by $(L : D)$ ratio of the workpiece is varied under a constant compliance of the machine tool.

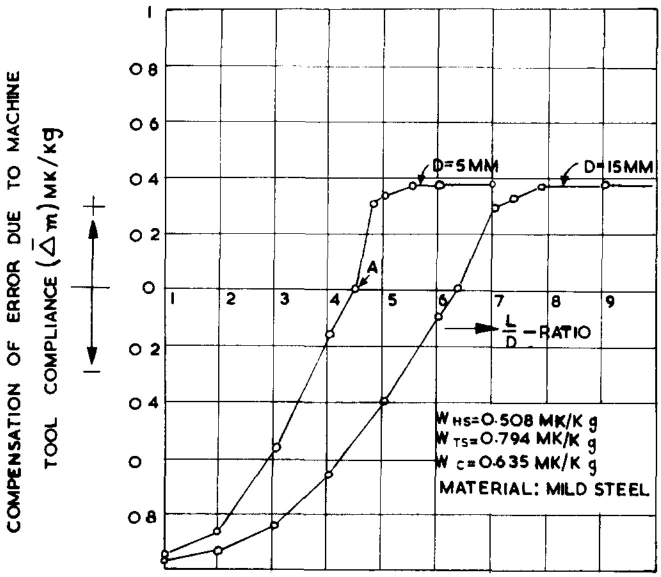

FIG. 3.

Compensation of form error due to machine tool compliance starts after a critical $L : D$ ratio of the workpiece (point A). Below this value, the machine tool compliance will only add to the inaccuracy. However, there is a limit to the compensation of accuracy due to the machine tool compliance. The maximum compensation of error due to machine tool compliance can be calculated from the following equation which can be derived easily.

$$\overline{\Delta}_{m\ (\max)} = 2\left[W_{mt\ (x=0)} - W_{mt\ (x=0\cdot 5L)}\right]. \tag{6}$$

(2) *Compensation of error due to the compliance of the workpiece*

Figure 4 shows the variation of $\overline{\Delta}_j$ as the $L : D$ ratio of the workpiece is increased. Accuracy is minimum or the compensation of error is zero when the workpiece is absolutely rigid $(L : D = 0)$. The extent of compensation increases as the compliance of the workpiece is increased. However, beyond a certain $L : D$ ratio of the workpiece (point C) the compensation of error ceases. The $L : D$ ratio at which the compensation of error is 0 can be calculated from the following equation:

$$(W_{ts} - W_{mt\ \min}) = W_{mt\ (x=0\cdot 5L)} + W_{j\ \max} - W_{hs}.$$

From the above,

$$W_{j\ \max} = (W_{hs} + W_{ts}) - W_{mt\ \min} - W_{mt\ (x=0\cdot 5L)}$$

or

$$(250L^3/3EI) = (W_{hs} + W_{ts}) - (W_{mt\ \min} + W_{mt\ (x=0\cdot5L)}). \tag{7}$$

For circular cross sections (7) can be re-written in the following form:

$$\frac{16640}{3E\pi D}\left(\frac{L^3}{D}\right) = (W_{hs} + W_{ts}) - (W_{mt\ \min} + W_{mt\ (x=0\cdot5L)}). \tag{8}$$

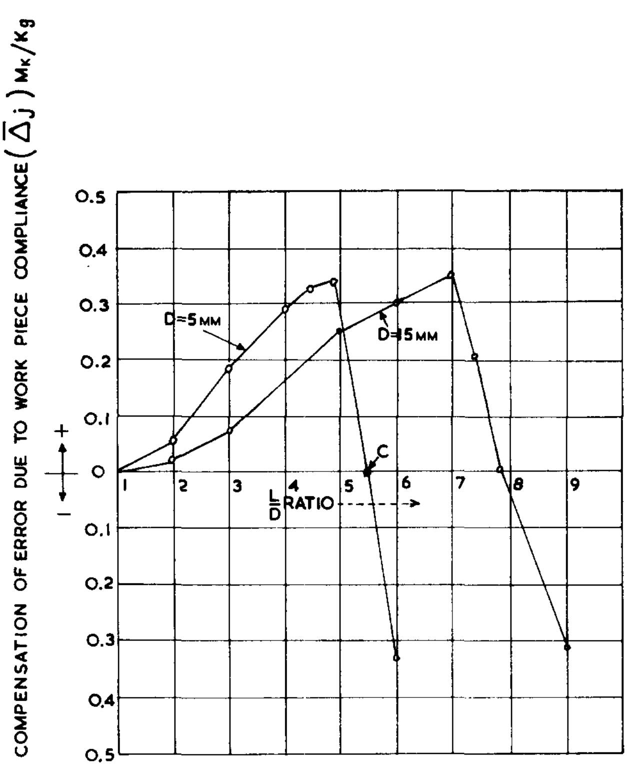

FIG. 4.

(3) *Optimum selection of stays*

In workshop practice it is recommended to use stays when $L : D$ ratios are large. But no scientific methods exists to arrive at the $L : D$ ratio at which stays may be used to advantage. Obviously this limiting $L : D$ ratio must bear a relation to the rigidity of the machine tool. Stays are useful obviously, when workpiece compliance exceeds a limiting value and ceases to contribute to turning accuracy, or when $\bar{\Delta}_j = 0$. This limiting workpiece compliance can be calculated from (8). It is interesting to note that the use of stays for $L : D$ ratios smaller than the limiting value will actually decrease accuracy and so should not be resorted to.

The graph in Fig. 5 is obtained by making use of equation (8) and will give the limiting $L : D$ ratio at which stays are recommended for steel jobs of dia. 5–25 mm.

(4) *Significance of optimum compliance*

Besides helping an optimum selection of stays, the concept of optimum compliance will be useful, mainly, in the following manner:

(a) given a number of lathes, they can be divided into a number of groups, each possessing a range of compliance. The workpieces too (especially the round ones, and the round ones are most common) can be divided according to compliance and fed to those machines whose compliance suits them best.

(b) In small lot production of accurate parts, it is economical to machine on lathes whose compliance is optimal with respect to that of the workpiece.

(c) The concept of optimum compliance is necessary in programming for a variable rigidity machine tool.

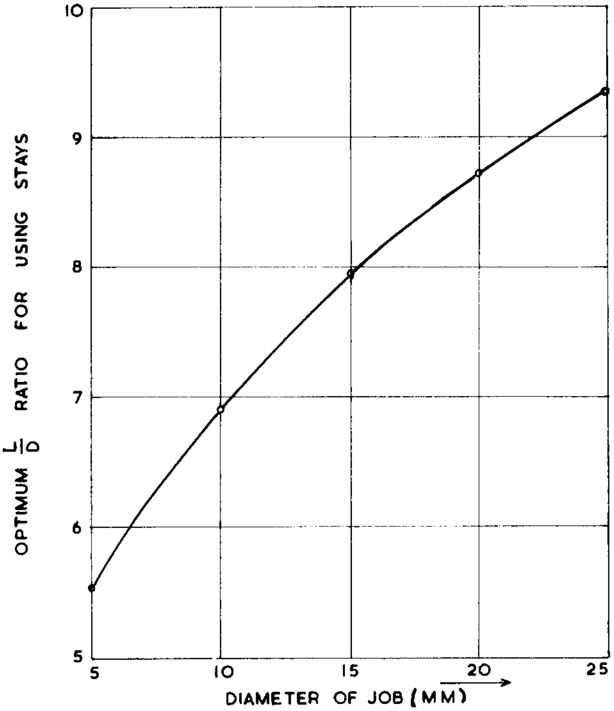

FIG. 5.

CONCLUSIONS

(1) The form errors of jobs turned between centres caused due to the machine tool compliance and workpiece compliance are in opposite directions and so will compensate each other to a certain extent.

(2) The form errors caused due to machine tool compliance are decreased by workpiece compliance in certain situations (or combinations of machine tool and workpiece compliances).

(3) The form errors caused due to the workpiece compliance are decreased by machine tool compliance in certain situations.

(4) The best machine tool from the point of view of accuracy is rarely the most rigid or least compliant one.

(5) Stays are useful only when the $L : D$ ratio of the workpiece exceeds a certain value as determined by the machine tool compliance. Use of stays below this limiting $L : D$ ratio will only decrease accuracy.

(6) There is an optimum combination of machine tool and workpiece compliances which will lead to highest accuracy.

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