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EXISTENCE OF PERIODIC SOLUTIONS OF THE EQUATIONS OF INCOMPRESSIBLE MICROPOLAR FLUID FLOW

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Abstract—The equations of incompressible micropolar fluid flow are a coupled system of vector differential equations involving the two basic vectors, viz. the velocity $\bar{q}$ and the microrotation $\bar{\nu}$ of the fluid elements. Let $D = D(t)$ be a bounded region in space, and let a flow velocity and a microrotation be prescribed at each point of the boundary of $D(t)$. Assume that $D(t)$ as well as the assigned velocity and microrotation vectors depend periodically on the time t and that the condition $(2\mu+k)j-4a \leq 0$ is satisfied (equation (25) in the text). Further assumptions are that (i) to every continuous initial distribution of the flow fields over D, there corresponds a solution of the field equations for all time $t \geq 0$ satisfying the prescribed boundary conditions; (ii) there is one solution for which the Reynolds numbers Re, Rm satisfy the condition $Re^2+Rm^2 < 80$ and this solution is equicontinuous in $\bar{x} = (x, y, z)$ for all t. Then there exists a unique, stable, periodic solution of the micropolar flow equations in $D(t)$ taking the prescribed values on the boundary. The proof of the theorem rests on a formula describing the rate of decay of the kinetic energy of the difference of two micropolar flows in the domain subject to the same boundary conditions.

IN THIS paper we employ the energy method to deduce from certain plausible hypotheses the existence of stable, periodic solutions of the equations of motion of incompressible, micropolar fluids, referred to below as Eringen fluids. The flow of these fluids is governed by the theory initiated by Eringen *et al.*[1], [2]. and differs from the classical or Navier–Stokes theory of fluid flow in two important features, viz. the sustenance of the couple stress and the nonsymmetry of the stress tensor. The constitutive equations of the linear micropolar flow involve six constants and the field equations consist of a coupled system of partial differential equations for the basic vectors of the flow describing the velocity and microrotation of the fluid elements.

The discussion on the existence of periodic and stable solutions hinges on a formula describing the rate of decay of the kinetic energy of the difference of two micropolar flows which take place in a domain and subject to the same conditions on the boundary of the domain. Such a formula has already been noticed[3]. However, it needs to be modified suitably to facilitate the discussion on the existence of periodic solutions and this is reported below in Theorem 1. Theorem 2 provides the criteria concerning the existence of stable, periodic solutions. The present discussion involving micropolar fluids is inspired by the similar study by James Serrin[4] on Navier–Stokes equations describing the flow of non-polar Newtonian fluids.

(1) *Energy criterion for stability of micropolar flow*

The field equations of micropolar fluid flow are given by [2]

$$\rho\left\{\frac{\partial}{\partial t}\bar{v}+(\bar{v}\,.\,\mathrm{grad})\bar{v}\right\} = -\mathrm{grad}\,p+k\,\mathrm{curl}\,\bar{\nu}-(\mu+k)\,\mathrm{curl}\,(\mathrm{curl}\,\bar{v}) + (\lambda_1+2\mu+k)\,\mathrm{grad}\,(\mathrm{div}\,\bar{v}), \tag{1}$$

$$\rho j\left\{\frac{\partial}{\partial t}\bar{\nu}+(\bar{v}\,.\,\mathrm{grad})\bar{\nu}\right\} = -2k\bar{\nu}+k\,\mathrm{curl}\,\bar{v}-\gamma\,\mathrm{curl}\,\mathrm{curl}\,\bar{\nu}+(\alpha+\beta+\gamma)\,\mathrm{grad}\,(\mathrm{div}\,\bar{\nu}) \tag{2}$$

in which the vectors $\bar{v}$, $\bar{\nu}$ denote respectively the velocity and microrotation. The terms representing the body force and body couple are omitted. The density ρ and the squared radius of gyration j are constants and the velocity vector $\bar{v}$ is solenoidal. The constants λ_1, $2\mu+k$, k are the viscosity coefficients while the constants α, β, γ are the gyroviscosity coefficients. From the entropy production inequality it is seen that

$$3\lambda_1+2\mu+k \geqslant 0, \qquad 2\mu+k \geqslant 0, \qquad k \geqslant 0 \tag{3}$$

$$3\alpha+\beta+\gamma \geqslant 0, \qquad -\gamma \leqslant \beta \leqslant \gamma, \qquad \gamma \geqslant 0. \tag{4}$$

Consider the motion $(\bar{v}, \bar{\nu})$ of the incompressible Eringen fluid in the region $D = D(t)$ of space subject to the adherence or hyperstick condition on the boundary. If $(\bar{v}^*, \bar{\nu}^*)$ is another flow in the domain satisfying the same boundary conditions as the unstarred flow and

$$\bar{u} = \bar{v}^* - \bar{v}, \qquad \bar{\vartheta} = \bar{\nu}^* - \bar{\nu} \tag{5}$$

denote the field vectors of the difference of the two flows, the kinetic energy of the difference flow is given by

$$T = T_1 + T_2 \tag{6}$$

with†

$$T_1 = \tfrac{1}{2} \int \rho(\bar{u})^2, \qquad T_2 = \tfrac{1}{2} \int \rho j (\bar{\vartheta})^2. \tag{7}$$

Both the vectors $\bar{u}$, $\bar{\vartheta}$ vanish on the boundary of the domain and the vector $\bar{u}$ is a solenoidal field. Following the procedure in [3] we have the formulae giving the time-rate of change of the energy functionals T_1 and T_2 in the form

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \int \rho\bar{u} \,.\, (\operatorname{grad} \bar{u}) \,.\, \bar{v} + k \int \bar{\vartheta} \,.\, \operatorname{curl} \bar{u} - (\mu+k) \int (\operatorname{curl} \bar{u})^2, \tag{8}$$

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \int \rho j\bar{u} \,.\, (\operatorname{grad} \bar{\vartheta}) \,.\, \bar{\nu} + k \int \bar{\vartheta} \,.\, \operatorname{curl} \bar{u} - 2k \int (\bar{\vartheta})^2 - \gamma \int (\operatorname{curl} \bar{\vartheta})^2 - (\alpha+\beta+\gamma) \int (\operatorname{div} \bar{\vartheta})^2. \tag{9}$$

The first term on the right hand side of each of the above equations will be majorized by the use of Schwarz's inequality.

Using the inequalities

$$2\bar{u} \,.\, (\operatorname{grad} \bar{u}) \,.\, \bar{v} \leqslant \frac{2\mu+k}{2\rho} (\operatorname{grad} \bar{u})^2 + \frac{2\rho}{2\mu+k} (\bar{u})^2 (\bar{v})^2, \tag{10}$$

$$2\bar{u} \,.\, (\operatorname{grad} \bar{\vartheta}) \,.\, \bar{\nu} \leqslant \frac{2\mu+k}{2\rho} (\operatorname{grad} \bar{\vartheta})^2 + \frac{2\rho}{2\mu+k} (\bar{u})^2 (\bar{\nu})^2 \tag{11}$$

†The conventional volume infinitesimal is omitted in the integrals throughout the paper. The integrals are extended over the volume of the domain $D(t)$.

to replace the first terms on the right side of (8) and (9), we see that

$$\frac{dT_1}{dt} \leqslant \frac{2\mu+k}{4}\int (\operatorname{grad}\bar{u})^2 + \frac{\rho^2}{2\mu+k}\int (\bar{u})^2(\bar{v})^2 + k\int \bar{\vartheta}\,.\operatorname{curl}\bar{u} - (\mu+k)\int (\operatorname{curl}\bar{u})^2, \tag{12}$$

$$\frac{dT_2}{dt} \leqslant \frac{2\mu+k}{4}\int j(\operatorname{grad}\bar{\vartheta})^2 + \frac{\rho^2}{2\mu+k}\int j(\bar{u})^2(\bar{\nu})^2 + k\int \bar{\vartheta}\,.\operatorname{curl}\bar{u} - 2k\int (\bar{\vartheta})^2$$
$$-\gamma\int (\operatorname{curl}\bar{\vartheta})^2 - (\alpha+\beta+\gamma)\int (\operatorname{div}\bar{\vartheta})^2. \tag{13}$$

If V_0 is the maximum speed of the unstarred flow and ν_0 is the maximum microrotation magnitude in the domain $D(t)$ during the whole time $0 \leqslant t < \infty$ the second terms on the right side of (12) and (13) can be majorized using the first mean value theorem for definite integrals and thereby we obtain

$$\frac{dT_1}{dt} \leqslant \frac{2\mu+k}{4}\int (\operatorname{grad}\bar{u})^2 + \frac{\rho^2 V_0^2}{2\mu+k}\int (\bar{u})^2 + k\int \bar{\vartheta}\,.\operatorname{curl}\bar{u} - (\mu+k)\int (\operatorname{curl}\bar{u})^2, \tag{14}$$

$$\frac{dT_2}{dt} \leqslant \frac{2\mu+k}{4}\int j(\operatorname{grad}\bar{\vartheta})^2 + \frac{\rho^2 j\nu_0^2}{2\mu+k}\int (\bar{u})^2 + k\int \bar{\vartheta}\,.\operatorname{curl}\bar{u} - 2k\int (\bar{\vartheta})^2$$
$$-\gamma\int (\operatorname{curl}\bar{\vartheta})^2 - (\alpha+\beta+\gamma)\int (\operatorname{div}\bar{\vartheta})^2. \tag{15}$$

$$\int (\operatorname{grad}\bar{u})^2 = \int (\operatorname{curl}\bar{u})^2 \tag{16}$$

and

$$\int (\operatorname{grad}\bar{\vartheta})^2 = \int \{(\operatorname{curl}\bar{\vartheta})^2 + (\operatorname{div}\bar{\vartheta})^2\}. \tag{17}$$

Addition of the inequalities in (14), (15) and use of (16) lead to the result

$$\frac{dT}{dt} \leqslant -\frac{2\mu+k}{4}\int (\operatorname{grad}\bar{u})^2 - \frac{1}{2}k\int (\operatorname{curl}\bar{u} - 2\bar{\vartheta})^2 + \frac{\rho^2(V_0^2 + j\nu_0^2)}{2\mu+k}\int (\bar{u})^2$$
$$+\frac{2\mu+k}{4}\int j(\operatorname{grad}\bar{\vartheta})^2 - \gamma\int (\operatorname{curl}\bar{\vartheta})^2 - (\alpha+\beta+\gamma)\int (\operatorname{div}\bar{\vartheta})^2. \tag{18}$$

If

$$a = \min(\gamma, \alpha+\beta+\gamma) \geqslant 0 \tag{19}$$

we see on using (17), that

$$\gamma\int (\operatorname{curl}\bar{\vartheta})^2 + (\alpha+\beta+\gamma)\int (\operatorname{div}\bar{\vartheta})^2 \geqslant a\int (\operatorname{grad}\bar{\vartheta})^2 \tag{20}$$

and so from (18), (19), (20) we have

$$\frac{dT}{dt} \leqslant \frac{\rho^2(V_0^2 + j\nu_0^2)}{2\mu+k}\int (\bar{u})^2 - \frac{2\mu+k}{4}\int (\operatorname{grad}\bar{u})^2 + \left\{\frac{(2\mu+k)j}{4} - a\right\}\int (\operatorname{grad}\bar{\vartheta})^2. \tag{21}$$

From [5] we know that

$$\int (\operatorname{grad}\bar{u})^2 \geqslant \frac{80}{d^2}\int (\bar{u})^2 \tag{22}$$

$$\int (\text{grad}\,\bar{\vartheta})^2 \geqslant \frac{3\pi^2}{d^2}\int (\bar{\vartheta})^2 \tag{23}$$

where $d=$ l.u.b. over $0 \leqslant t < \infty$ of the diameter of a ball in which the domain $D(t)$ is embeddable. To ensure the stability of the unstarred flow, we demand that

$$\text{Sgn}\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = -1 \tag{24}$$

and then from (21–24) we have the criterion for stability. It is worthy of note that to meet the requirement (24), we must have

$$(2\mu+k)j-4a \leqslant 0 \tag{25}$$

apart from any other conditions. The condition (25) involving only the viscosity coefficient $2\mu+k$, the gyroviscosity a and the microstructure gyration parameter j is in response to the insistence of the asymptotic stability of the micropolar flow $(\bar{v}, \bar{\nu})$ and is a restriction on the constants *besides* those in (3) and (4) which are forced by thermodynamics.

Introduce the numbers

$$Re = \frac{2\rho V_0 d}{2\mu+k}, \qquad Rm = \frac{2\rho\sqrt{j}\nu_0 d}{2\mu+k} \tag{26}$$

representing respectively the Reynolds and the microrotational Reynolds numbers of the unstarred flow. If

$$p = \frac{2\mu+k}{2\rho d^2}, \qquad q = \frac{3\pi^2}{2\rho d^2} \tag{27}$$

from (21–23) and (25–27) we see that

$$\frac{\mathrm{d}T}{\mathrm{d}t} \leqslant (Re^2+Rm^2-80)pT_1 + \left(2\mu+k-\frac{4a}{j}\right)qT_2 \tag{28}$$

and (24) holds whenever (25) is true and

$$Re^2+Rm^2 < 80. \tag{29}$$

Let

$$-\epsilon = \max\left\{(Re^2+Rm^2-80)p, \left(2\mu+k-\frac{4a}{j}\right)q\right\}. \tag{30}$$

We have then:

Theorem 1. Let $D=D(t)$ be a bounded region of space which can be included in a ball of diameter d and let $\bar{v}$, $\bar{\nu}$ be the velocity and microrotation vectors of an incompressible micropolar flow in D satisfying prescribed boundary values. Then the kinetic energy of an arbitrary disturbance ($\bar{u} = \bar{v}^* - \bar{v}$, $\bar{\vartheta} = \bar{\nu}^* - \bar{\nu}$) satisfies the inequality

$$T\{\bar{u}, \bar{\vartheta}\} \leqslant T_0 \exp(-\epsilon t) \tag{31}$$

in which $T_0 = (T\{\bar{u}, \bar{\vartheta}\})_{(t=0)}$ denotes the initial energy of the disturbance and ϵ has the value given by (30).

Theorem 2. (Existence of stable, periodic flow of Eringen fluids.)

(C0) Let the velocity $\bar{v} = \bar{v}(\bar{x}, t)$ and microrotation $\bar{\nu} = \bar{\nu}(\bar{x}, t)$ be prescribed at each point of the boundary of the domain $D(t)$.

(C1) The domain $D(t)$ and the assigned velocity and microrotation depend periodically on the time t.

(C2) Condition (25) is satisfied.

(C3) To every continuous initial distribution of the velocity and microrotation over D, there corresponds a solution of the micropolar flow equations valid for all time $t \geqslant 0$ and satisfying the prescribed boundary conditions.

(C4) There is one solution for which the Reynolds numbers Rm, Re satisfy the condition (29). This solution is equicontinuous in $\bar{x} = (x, y, z)$ for all t.

Then there exists a unique, stable, periodic solution $\bar{v}(\bar{x}, t)$, $\bar{\nu}(\bar{x}, t)$ of the micropolar flow equations in $D(t)$ which takes the prescribed values on the boundary of $D(t)$.

For convenience in writing, let us denote the two fields $\bar{v}(\bar{x}, t)$, $\sqrt{j}\bar{\nu}(\bar{x}, t)$ in conjunction as a single six component vector field $\bar{A}(\bar{x}, t)$ and refer to the quantity $(Re^2 + Rm^2)^{1/2} = R$ as the Reynolds number of the flow $\bar{A}(\bar{x}, t)$.

Let $\bar{A}(\bar{x}, t)$ be the vector field of the flow guaranteed by the condition (C4) above and let us suppose that the period of the assigned boundary values is one. The sequence of vector functions $\bar{\Phi}_n(\bar{x}) = \bar{A}(\bar{x}, n)$ $(n = 1, 2, 3, \ldots)$ is bounded and equicontinuous in $\bar{x}$. Hence by Arzela's theorem (cf. [6], p. 59) this sequence contains a *subsequence* which converges uniformly to a continuous vector function $\bar{A}(\bar{x})$ in the domain $D(t)$. We shall see now that indeed the entire sequence $\bar{A}(\bar{x}, n)$ converges to $\bar{A}(\bar{x})$. If this is not true, there would be another subsequence converging to the continuous vector function $\bar{B}(\bar{x})$. Put

$$\bar{A}'(\bar{x}, t) = \bar{A}(\bar{x}, t + m - n), t \geqslant 0 \tag{32}$$

and let $m > n$. The vector function $\bar{A}'(\bar{x}, t)$ is a solution of the micropolar flow equations and satisfies the prescribed boundary conditions.

Let

$$T\{\bar{A}(\bar{x}, t)\} = \tfrac{1}{2} \int \rho |\bar{A}(\bar{x}, t)|^2 = \tfrac{1}{2} \int \rho \{\bar{v}(\bar{x}, t)\}^2 + \tfrac{1}{2} \int \rho j \{\bar{\nu}(\bar{x}, t)\}^2 \tag{33}$$

denote the kinetic energy of the flow $\bar{A}(\bar{x}, t)$.

From the definitions of $\bar{A}(\bar{x}, t), \bar{A}'(\bar{x}, t)$ and condition (C4), both the above flows satisfy the condition $R^2 < 80$ and by Theorem 1 we see that

$$T\{\bar{A}' - \bar{A}\} \leqslant T_0 \exp(-\epsilon t) \tag{34}$$

where $T_0 = (T\{\bar{A}' - \bar{A}\})_{t=0}$ and ϵ is given by (30). We note T_0 is bounded above by the constant $2\rho d^3(v_0^2 + j\nu_0^2) = \Lambda$ and hence by putting $t = n$ in (34) we get

$$T\{\bar{\Phi}_m - \bar{\Phi}_n\} \leqslant \Lambda \exp(-\epsilon n). \tag{35}$$

Letting $n \to \infty$ and passing on to the limit in (35) we see that

$$\lim_{m,n\to\infty} T\{\bar{\Phi}_m - \bar{\Phi}_n\} = 0. \tag{36}$$

The domain of integration in (35) and (36) is $D(n) = D(0)$. Allow m, n to infinity through sequences of integers such that $\bar{\Phi}_m(\bar{x}) \to \bar{B}(\bar{x})$ and $\Phi_n(\bar{x}) \to \bar{A}(\bar{x})$.

In view of (36) we have a contradiction of our earlier supposition that $\bar{A}(\bar{x})$ and $\bar{B}(\bar{x})$ are different. Hence the assertion that the entire sequence $\bar{A}(\bar{x}, n)$ converges to the continuous function $\bar{A}(\bar{x})$. By condition (C3) there exists a flow $\bar{A}^*(\bar{x}, t)$ such that

$$\bar{A}^*(\bar{x}, 0) = \bar{A}(\bar{x}). \tag{37}$$

We shall now see that the solution $\bar{A}^*(\bar{x}, t)$ is periodic and also stable.

Let

$$\bar{A}''(\bar{x}, t) = \bar{A}(\bar{x}, t+n). \tag{38}$$

By (34) we have

$$T\{\bar{A}^* - \bar{A}''\} \leqslant T_1 \exp(-\epsilon\iota) \tag{39}$$

in which ϵ has the same meaning seen earlier and

$$T_1 = (T\{\bar{A}^* - \bar{A}''\})_{t=0} = T\{\bar{A}(\bar{x}) - \Phi_n(\bar{x})\}. \tag{40}$$

Setting $t = 1$ in (39) we get

$$T\{\bar{A}^*(\bar{x}, 1) - \Phi_{n+1}(\bar{x})\} \leqslant T_1 \exp(-\epsilon) < T_1 \tag{41}$$

and now allowing n to infinity, we get

$$T\{\bar{A}^*(\bar{x}, 1) - \bar{A}(\bar{x})\} = 0 \tag{42}$$

since T_1 has the limit zero when $n \to \infty$. From (42) we can conclude that

$$\bar{A}^*(\bar{x}, 1) = \bar{A}(\bar{x}) = \bar{A}^*(\bar{x}, 0) \tag{43}$$

and so $\bar{A}^*(\bar{x}, t)$ is periodic.

By Theorem 1 we know that

$$T\{\bar{A}^*(\bar{x}, t) - \bar{A}(\bar{x}, t)\} \to 0 \text{ as } t \to \infty. \tag{44}$$

Since both $\bar{A}^*(\bar{x}, t)$ and $\bar{A}(\bar{x}, t)$ are equicontinuous, we see that

$$\bar{A}^* - \bar{A} \to 0 \text{ as } t \to \infty \tag{45}$$

and so

$$\max_{(\bar{x})} \bar{A}^*(x, t) - \max_{\bar{x}} \bar{A}(\bar{x}, t) \leq o(t). \tag{46}$$

Since $\bar{A}^*(\bar{x}, t)$ is periodic, we see from the above that

$$\max_{\bar{x}} \bar{A}^*(\bar{x}, t) \leq \max_{\bar{x}} \bar{A}(\bar{x}, t). \tag{47}$$

Hence the Reynolds numbers R for these two flows are such that

$$R^2(\bar{A}^*) \leq R^2(\bar{A}). \tag{48}$$

Since $R^2(\bar{A}) < 80$, by Theorem 1 we conclude that the flow $\bar{A}^*(\bar{x}, t)$ is stable. This completes the proof of Theorem 2.

REMARKS

(a) The boundary conditions prescribed must be compatible with a flow for which the restriction $R^2 < 80$ holds. Thus for sufficiently low valued (periodic) boundary prescription of $\bar{A}(\bar{x}, t)$, there exists a periodic flow to which every other motion tends eventually.

(b) When the assigned conditions on the boundary are *steady*, Theorem 2 assures the existence of a unique, stable, time-independent solution of the micropolar flow equations, taking the prescribed values on the boundary.

(c) The condition (C3) is mathematically stringent as the flow is to be valid for all $t \geq 0$. However, it is enough if this condition holds for those initial data for which $R^2 < 80$.

(d) Theorem 2 is not to be deemed as the standard type of mathematical existence theorem. The conditions (C3) and (C4) may not hold for certain types of boundary data. The extent of applicability of the Theorem is not well-defined and this aspect of the problem needs investigation.

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Résumé – Les équations d'un liquide micropolaire incompressible forment un système couplé d'équations différentielles vectorielles comprenant les deux vecteurs de base, c'est à dire la vitesse $\bar{q}$ et la microrotation $\bar{\nu}$ des éléments liquides. Soit $D = D(t)$ un domaine borné dans l'espace, et supposons données une vitesse de l'écoulement et une microrotation en chaque point de la limite de $D(t)$. Supposons que $D(t)$, de même que la vitesse et la microrotation données soient des fonctions périodiques du temps t et que la condition $(2\mu+k)j-4a \leq 0$ soit satisfaite (équation (25) dans le texte). D'autres hypothèses sont: (i) qu'à toute distribution initiale continue des champs d'écoulement sur D, il corresponde une solution des équations de champ pour tout temps $t \geq 0$, satisfaisant aux conditions données aux limites; (ii) qu'il y ait une solution pour laquelle les nombres de Reynolds Re, Rm satisfont la condition $Re^2 + Rm^2 < 80$ et que cette solution soit équicontinue dans $\bar{x} = (x, y, z)$ pour tout t. Alors il existe une solution unique, stable et périodique des équations d'écoulement micropolaire dans $D(t)$, prenant les valeurs prescrites à la limite. La démonstration du théorème s'appuie sur une formule décrivant le taux de décroissance de l'énergie cinétique de la différence de deux écoulements micropolaires dans le domaine soumix aux mêmes conditions aux limites.

Zusammenfassung – Die Gleichungen inkompressibler, mikropolarer Flüssigkeitsströmung sind ein gekoppeltes System vektorieller Differentialgleichungen, die zwei Grundvektoren einschliessen, das sind die Geschwindigkeit $\bar{q}$ und die Mikrorotation $\bar{\nu}$ der Flüssigkeitselemente. Es soll $D = D(t)$ ein begrenzter Bereich im Raum sein und eine Strömungsgeschwindigkeit und eine Mikrorotation sollen an jedem Punkt der Grenze von $D(t)$ vorgeschrieben sein. Es wird angenommen, dass sowohl $D(t)$ als auch die zugeordneten Geschwindigkeits- und Mikrorotationsvektoren periodisch von der Zeit t abhängen und dass die Bedingung $(2\mu + k)j - 4a \leqslant 0$ befriedigt ist (Gleichung (25) im Text). Weitere Annahmen sind dass (i) jeder kontinuierlichen Anfangsverteilung der Strömungsfelder über D eine Lösung der Feldgleichungen für die ganze Zeit $t \geqslant 0$ entspricht, die die vorgeschriebenen Grenzbedingungen befriedigt; (ii) dass es eine Lösung gibt für welche die Reynolds-Zahlen Re, Rm die Bedingung $Re^2 + Rm^2 < 80$ befriedigen und diese Lösung gleich-kontinuierlich in $\bar{x} = (x, y, z)$ für alle t ist. Es existiert dann eine einzigartige, stetige, periodische Lösung der mikropolaren Strömungsgleichungen in $D(t)$, die vorgeschriebenen Werte an der Grenze nehmend. Der Beweis des Theorems beruht auf einer Formel, die die Rate des Verfalles der kinetischen Energie der Differenz zweier mikropolarer Strömungen in dem Bereich beschreibt, die den selben Grenzbedingungen unterliegen.

Sommario – Le equazioni di unflusso fluido micropolare incomprimibile sono un sistema accoppiato di equazioni differenziali dei vettori che interessano i due vettori fondamentali e cioè la velocità $\bar{q}$ e la microrotazione $\bar{\nu}$ degli elementi fluidi. Supponiamo che $D = D(t)$ sia un regione limitata nello spazio e che si prescrivano una velocità di flusso e una microrotazione a ciascun punto del limite di $D(t)$. Presumiamo che i vettori $D(t)$ al pari de quelli della velocità prescritta e dell microrotazione dipendano periodicamente dal tempo t e che si soddisfi la condizione $(2\mu + k)j - 4a \leqslant 0$ (eq. (25) nel testo). Si presuma anche che: (1) ad ogni distribuzione iniziale continua dei campi di flusso su D corrisponde una soluzione delle equazioni di campo per tutto il tempo $t \geqslant 0$ che soddisfa le condizioni limite prescritte; (ii) c'è una soluzione per cui i numeri di Reynolds Re e Rm soddisfano la condizione $Re^2 + Rm^2 < 80$ e questa soluzione e equicontinua in $\bar{x} = (x, y, z)$ per tutto T. Esiste allora una soluzione periodica, stabile e unica delle equazioni di flusso micropolare in $D(t)$ prendendo i valori prescritti sul limite. La prova del teorema sta in una formula che descrive il ritmo di decadimento della energia cinetica della differenza di due flussi micropolari nel campo sottoposto alle stesse condizioni limite.

Абстракт — Уравнения потока несжимаемой микрополярной жидкости составляют связанная система векторных дифференциальных уравнений, включающие в себе два основные вектора, т. е. скорость движения $\bar{q}$ и микровращение $\bar{\nu}$ данных элементов жидкости. Пусть $D = D(t)$ быть ограниченной областью в пространстве, заданы скорость потока и микровращение в каждой точке границы $D(t)$. Допустим, что $D(t)$ зависит от времени t периодически, а также заданные векторы скорости и микровращения. Выполнено условие $(2\mu + k)j - 4a \leq 0$ (см. уравнение (25) в тексте). Другие допущения: (i) каждое непрерывное начальное распределение полей потока по D имеет соответствующее решение уравнений полей для всего времени $t \geq 0$, удовлетворящей заданным граничным условиям, (ii) имеется одное решение, для которого числа Рейнольдса Re, Rm удовлетворят условию $Re^2 + Rm^2 < 80$, и решение есть равнонепрерывным внутри $\bar{x} = (x, y, z)$ для всех t. Используя заданные значения на границе, существует единственное, устойчивое, периодическое решение уравнений микрополярного потока относительно $D(t)$. Доказательство теорема основано на формуле скорости распада кинетической энергии разницы двух микрополярных потоков в области при одинаковых граничных условиях.