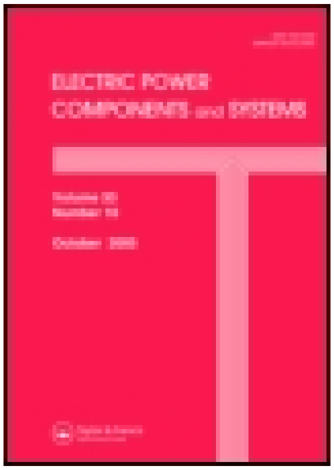

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A NEW ROBUST ALGORITHM FOR ESTIMATING THE STATE OF A POWER SYSTEM

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ABSTRACT

The paper presents a new algorithm to estimate the state of a power system. It is essentially an improved version of the Weighted Least Squares (WLS) whcih is widely used in practice. In order to obtain an optimal estimate, the weighting matrix is to be chosen as the error covariance matrix in the WLS algorithm. In practice the error covariance is largely unknown and the weighting matrix is suitably tuned. There are problems of convergence if the weighting matrix is chosen as the inverse of E [VV^t]. The new algorithm overcomes such problems and is found to have good convergence characteristics.

Although it is essentially an improvement of the WLS algorithm, it differes from it in the method of implementation. It is based upon a technique used to obtain the pseudo-inverse of a non-square matrix and is an extension of it to include weighting factors. The new algorithm does not call for any type of tuning of the weighting matrix and the number of iterations to converge do not depend upon the mix of measurements or size of the system. This is the main contribution of the paper.

INTRODUCTION

Weighted least-squares theory is widely used to estimate the static state of a power system [1 to 7]. The WLS theory is mathematically sound and it has very good error filtering capability. The measurement vector Z is related to the state vector X by

$$Z = f(X) + V \qquad (1)$$

Where V is the noise vector. The state vector X is obtained by minimising a quadratic cost function given by

$$C = [Z - f(X)]^t R^{-1} [Z - f(X)] \qquad (2)$$

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Minimising C with respect to the state vector, the algorithm for the solution of the state vector is given by equs. (3) and (4)

$$[\, J^t R^{-1} J \,]\Big|_{xi} \Delta X = J^t \Big|_{xi} R^{-1} \Delta Z \Big|_{xi} \qquad (3)$$

$$X^{i+1} = X^i + \Delta X \qquad (4)$$

where $\Delta Z = [\, Z - f(X^i) \,]$ and J is a matrix of partial derivatives of f(X) with respect to X.

The convergence of eqns. (3) and (4) depends heavily on the sensitivity matrix $[\, J^t R^{-1} J \,]$ which is influenced by the nature of the $[\, R^{-1} \,]$, the weighting matrix.

The WLS algorithm requires that the weighting matrix $[\, R^{-1} \,]$ be chosen as the inverse of $E[\, VV^t \,]$ [1,4] to obtain an optimal estimate. Such a choice for the weighting matrix is known to lead to ill-conditioning of the sensitivity matrix $[\, J^t R^{-1} J \,]$ of the WLS algorithm and leads to problems of its convergence [3, 4, 5, 6, 7]. In practice, the error covariance is largely unknown and the weighting matrix is tuned, the extent and the type of tuning depending upon the system size, its characteristics, etc,.

Several attempts are made to overcome the ill-conditioning problems. Sequential processing methods[4], methods of Levenberg and Marquadt[5] and orthogonalisation procedures[6] are some of these. In the Levenberg and Marquadt methods the diagonal elements of the sensitivity matrix are essentially strengthened or augmented.

The weighting factors matrix plays a significant role in the conditioning of the sensitivity matrix. It is shown[9] that whenever the weighting factors have a wide range, the net effect is equivalent to the numerical addition of small numbers to large numbers in the sensitivity matrix, leading to loss of vital information in the measurements that have smaller weighting factors due to truncation. All the methods discussed above do not contribute to decrease the said information loss.

Application of Golub's method[7] wherein a series of orthogonal transformations are used to convert the matrix $[\, \sqrt{R^{-1}} J \,]$ into an upper triangular matrix is said to be numerically stable but it involves additional programming effort.

A new algorithm, an improved version of WLS algorithm, is presented in this paper. It does not require any type of conditioning or tuning of the weighting matrix, or its entries, or the sensitivity matrix,and it converges even when the inverse of weighting factors matrix R is chosen as $E[\, VV^t \,]$. Further, this algorithm is found to give results with an accuracy superior to that obtained with the WLS algorithm. This new algorithm is based upon a technique used to obtain the pseudo-inverse of a non-square matrix and is an extension of the method of Ben Noble [8] to facilitate the inclusion of weighing factors.

THEORY

In the WLS algorithm, the state vector X is obtained by minimising a quadratic cost function given by eqn. (2), wherein

Z = measurement vector given by eqn. (1),

X= state vector

f(.) = non-linear function operator,

$R = E[\ VV^t\ ]$,

V = noise vector and

E = expectation operation.

Minimisation of eqn. (2) yeilds an iterative procedure wherein eqn. (3) has to be solved in each iteration.

Rewriting eqn. (3)

$[\ J^t\ R^{-1}\ J\ ]\ \Delta X = J^t\ R^{-t}\ \Delta Z$ and

Substituting $J' = [\sqrt{R^{-1}}\ ]\ J$ and $\Delta Z' = \sqrt{R^{-1}}\ \ \Delta Z$

eqn.(3) can be written as:

$$[\ J'^t\ \ J'\ ]\ \ \Delta X = J'^t\ \Delta Z' \qquad (5)$$

Assuming that there is no cross correlation among the error statistics of the measuremwents, the elements of $\sqrt{R^{-1}}$ matrix are obtained as the square root of corresponding elements of R^{-1} matrix.The matric J' and the vector $\Delta Z'$ can be partitioned as shown in eqn. (6)

$$J' = \begin{bmatrix} J_1' \\ J_2' \end{bmatrix} \quad \text{and} \quad \Delta Z' = \begin{bmatrix} \Delta Z_1' \\ \Delta Z_2' \end{bmatrix} \qquad (6)$$

so that J_1' is a square and non-singular matrix.
At this stage it is suffice to say that the first N rows of J' may be termed as J_1' and it will be subsequently shown that these are the dominant rows of the matrix J'. In view of the redundancy of measurements, the elements of J_2' can be expressed as a linear combination of the elements of J_1'

$$\text{i.e.}\quad J_2' = PJ_1' \qquad (7)$$

and

$$J' = \begin{bmatrix} I \\ P \end{bmatrix} J_1' \qquad (8)$$

It will be shown that the matrix P is obtained in the process of a transformation shown below.

Using eqn. (8), eqn.(5) can be written as

$$J_1^{'t} [I + P^t P] J_1' \Delta X = J_1^{'t} \Delta Z_1' + J_1^{'t} P^t \Delta Z_2' \tag{9}$$

i.e., $[I + P^t P] J_1' \Delta X = \Delta Z_1' + P^t \Delta Z_2'$ (10)

Equ. (10) can be simplified as

$$\Delta X = J_1^{'-1} \Delta Z_1' + J_1^{'-1} [I + P^t P]^{-1} P^t [\Delta Z_2' - P \Delta Z'] \tag{11}$$

The terms on the RHS of eqn. (11) can be obtained as shown below. Consider the augmented matrix

$$\begin{bmatrix} J_1' & \Delta Z_1' & I \\ J_2' & \Delta Z_2' & 0 \end{bmatrix} \tag{12}$$

Using Gauss - Jordan procedure with partial or full pivoting, the above matrix is transformed to:

$$\begin{bmatrix} I & (J_1^{'-1} \Delta Z_1) & J_1^{'-1} \\ 0 & (\Delta Z_2' - P \Delta Z_1') & -P \end{bmatrix} \tag{13}$$

The above procedure automatically transfers the N dominant rows of J' to form J_1'. The first N rows of J' are termed as J_1' as a result of the above transformation, in addition to the required terms $J_1^{'-1}$, $J_1^{'-1}$ $\Delta Z1'$, and $[\Delta Z_2' - P \Delta Z_1']$ in eqn. (11). The RHS of eqn. (11) is in two parts and may be written as

$$\Delta X = \Delta X_o + \Delta X_c \tag{14}$$

The terms ΔX_o and ΔX_c are interpreted as follows.

ΔX_o may be called the basic solution and is given by

$$\Delta X_o = [J_1']^{-1} \Delta Z_1 \tag{15}$$

The term ΔX_c may be called the correction term and is computed as

$$\Delta X_c = J_1^{'-1} [I + P^t P]^{-1} P^t [\Delta Z_2' - P \Delta Z_1'] \tag{16}$$

Consider the expression $(\Delta Z_2' - P \Delta Z_1')$. Using eqn.(7) and eqn. (15).

$$P \Delta Z_1' = J_2' (J_1')^{-1} \Delta Z_1' = J_2 \Delta X_o = \Delta Z_2' \tag{17}$$

Thus the matrix P may be considered as the transformation matrix which maps the vector Z_1' into the vector $\Delta Z_2'$ so that $P \Delta Z_1'$ would be equal to $\Delta Z_2'$. If there are no uncertainities in the form of errors in the measurement vector and hence in Z the term $[\Delta Z_2' - P \Delta Z_1']$. should be equal to zero. When the measurements are not error free, the vector $[\Delta Z_2' - P\Delta Z_1']$ will not be zero and hence the second term in eqn.(14) may be considered as the correction term to ΔX_o.

A few important aspects of this algorithm are given below.

a) The weighting factors have no effect in obtaining the basic solution ΔX_o and consequently calculation of the basic solution vector ΔX_o would not present any problems.

b) The inverses of $[I + P^t P]$ and J_1 are needed to compute the correction term ΔX_c . The matrix $[I + P^t P]$ will be shown to be a well conditioned matrix.

c) Rewriting P in terms of Jacobian J_1 and J_2 using eqn. (5)

$$P = \sqrt{R^{-1}} \; J_2 \; J_1^{-1} \; \sqrt{R} \tag{18}$$

An examination of eqn. (18) reveals that the formation of P while transforming eqn.(12) into (13) is a stable process. The matrices $\sqrt{R_2^{-1}}$ and $\sqrt{R_1^{-1}}$ scale the matrix $[J_2 \; J_1^{-1}]$.

Analysis of conditioning of the matrix $[I + P^t P]$.

Consider the matrix $(J'^t J')$. Its determinant can be written as [8]

$$\det [J'^t \; J'] = \sum_p (\det[J_p']^2) \tag{19}$$

where J_p' is a N x N matrix obtained by selecting any N rows of J' and the sum is taken over the square of the determinant of all such matrices.

$$[J'^t \; J'] = J_1'^t \; [I + P^t \; P] \; J_1'$$

Hence

$$\det [I + P^t \; P] = \frac{\det (J'^t \; J')}{[\det (J_1')]^2}$$

Since the matrix J_1' is one of the submatrices J_p' of J used for summation it follows that

$$\det [I + P^t \; P] \geq 1$$

Thus the matrix $[I + P^t \; P]$ is a well conditioned matrix[8]

ALGORITHM

The algorithm consists of the execution of the following steps untill convergence

1) Assume a set of initial values for the state vector.
2) Evaluate the Jacobian J and the vector ΔZ using the most recent values for the state vector X.
3) Form the augumented matrix given by eqn.(12).
4) Using Gauss-Jordon procedure with partial or full pivoting transform the augumented matrix given by eqn. (12) into the matrix given by eqn. (13).
5) Obtain the vector ΔX using eqn. (11).
6) Update the state vector X using eqn. (4).
7) Repeat the procedure starting from step 2 untill convergence.

RESULTS

This new algorithm is implemented using the IEEE 14-bus, 30-bus and 57-bus test systems as sample systems[9]. For the sake of brevity results obtained on the IEEE 30-bus system are presented and compared with those obtained using the WLS algorithm. The measurement vector is simulated using a package to generate a random number and assuming a certain value for the standard deviation of error in the measurements. This value for the s.d. of error is varied over a wide range i.e., from 2 to 10 percent. Full redundancy in the measurements is assumed in all the case studies unless otherwise stated.

A flat start of 1 p.u. for voltage magnitudes and zero for angles are assumed in the iterative procedure. When the weighting matrix is simulated as the inverse of $E[VV^t]$ it is the author's experience that the system of eqns. in the WLS algorithm have a problem of convergence [9,12]. The weighting matrix is tuned to obtaine convergence and the results for the WLS algorithm shown in this paper belong to this category.

One of the measurements of ill conditioning of a matrix is given by the ratio of its largest to smallest eigen value, called the condition number. This is evaluated for the sensitivity matrix $[J^t R^{-1} J]$ of the WLS algorithm and also for the matrix $[I + P^t P]$ of the new algorithm for a set of given data. These values are found to be 1.471×10^{11} and 30.0 respectively. These figures clearly explain the ill conditioning of the WLS algorithm and superiority of the new algorithm, when the weighting matrix is chosen to be the error covariance matrix. In practice the error covariance is largely unknown and the weighting matrix has to be tuned. It is to be noted that the proposed method leads to a set of well conditioned equations, even in the worst case of chosing the error covariance as the weighting matrix.

Table 1 tabulates the number of iterations required for convergence. The number of iterations in the new algorithm is generally equal to or slightly less than that of the WLS algorithm.

The author's experience is that when line-flows only are used as measurements, the estimate over-shoots initially takes 12 iterations to converge. In some cases the voltage magnitude are found to converge to their negative values. Such convergence problems are not encountered while using this new algorithm and the iterations required to converge remain constant at a value of 5 or 6 even for a very fine tolerence of 0.0001, irrespective of system size. The mix of measurements is not found to lead to any problem with the new algorithm.

Table 2 presents data to compare the results of the new algorithm with those obtained using the WLS algorithm. Maximum, average values and the s.d. of errors in voltage magnitude and angles and maximum and average errors in injection and line-flows are considered for comparision. A careful examination of above table reveals that the new algorithm gives a closer fit to the state vector than WLS algorithm. For the case wherein the s.d. of error in simulated measurements is 2 percent, maximum and average errors in the voltage magnitudes using the new algorithm are 0.002267 p.u. and 0.002075 p.u. respectively. The corresponding errors in the WLS algorithm are 0.003403 p.u. and 0.003193 p.u. respectively. Thus the maximum and average errors in voltage magnitude in the new algorithm are less by 34% and 35% repectively. Similarly the maximum and average errors in angle in the new algorithm are less by 32% and 17% repectively. Similar observations are made in regard to the 10% errors in simulated measurements.

CONCLUSIONS

A new algorithm is presented to estimate the power system state vector. This is the direct result of the research work done to analyse the various ill conditioning problems that arise in the classical WLS algorithm. It is shown that this algorithm is very stable and results in a very reliable estimate. It is shown that the estimates obtained using this method are closer to their true values than those of the WLS algorithm. It is shown that this algorithm does not call for any type of tuning of the weighting matrix.

It is found to converge in about 5 to 6 iterations, irrespective of system size even for a fine tolerance of 0.0001 p.u. and the number of iterations to converge is found to be independent of the combinations of measurements. In practice a convergence tolerance would be much higher than 0.0001 p.u. and the weighting matrix is generally tuned, in the absence of precise knowledge of covariance. In all such cases, the proposed algorithm would lead to convergence and accuracy.

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Table 1

Number of iterations to converge

S.D. of error in simulated measurements	Tolerance	No. of iterations: WLS algorithm	No. of iterations: New algorithm
2%	0.001	5	5
	0.0001	6	5
10%	0.001	5	5
	0.0001	5	5

Table 2
Errors in estimated voltage magnitude and angles and line-flows.

Variables	Error in estimated variables	s.d. of errors in the simulated measurements 2%: New algorithm	2%: WLS algorithm	10%: New algorithm	10%: WLS algorithm
Voltage Mag. (p.u.)	Maximum	0.002267	0.003403	0.005813	0.017060
	Average	0.002075	0.003193	0.004527	0.016000
	s.d.	0.000099	0.000146	0.000632	0.000605
Angle (deg.)	Maximum	0.040560	0.075630	0.232700	0.386750
	Average	0.026297	0.046660	0.139500	0.238640
	s.d.	0.011660	0.013970	0.075900	0.071391
Real Injection (MW)	Maximum	0.9812	1.1368	2.1846	5.6630
	Average	0.0271	0.1011	0.4133	0.5050
Reactive Injection (MVar)	Maximum	0.2201	0.2182	0.1361	1.0890
	Average	0.0362	0.0462	0.2348	0.2321
Real Lineflows (MW)	Maximum	0.6907	0.7972	0.1926	3.9712
	Average	0.0776	0.0870	0.0318	0.4335

Reactive	Maximum	0.2201	0.1920	1.0818	0.9626
Lineflows	Average	0.0214	0.0259	0.1508	0.1298
(MVar)					

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