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# Application of fuzzy nonlinear goal programming to a refinery model†

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**Abstract**

A simplified oil refinery model has been formulated as a fuzzy nonlinear goal programming problem, in which four non-compatible performance criteria (objective functions or goals) exist beside ten crisp constraints in the form of material balance equations. Total yearly profit of the refinery and the sensitivities of the profit to variations in refinery conditions have been assumed to be fuzzy goals. Linear and S-type membership functions have been assumed separately for all the fuzzy goals. Box's complex method has been used to solve the crisp equivalent of the fuzzy nonlinear goal programming problem. The "min" operator has been used as the aggregator. A software developed in C implements the model. The results show that the present methodology gives the decision maker a good flexibility in setting up the goals, in that he/she is not forced to specify goals crisply simply for mathematical reasons. Also, the present treatment of the problem in a fuzzy framework enables the decision maker to consider any number of goals in any combination. © 1998 Elsevier Science Ltd. All rights reserved

*Keywords:* fuzzy nonlinear goal programming; Box's complex method; refinery model; min operator

## 1. Introduction

From the time fuzzy set theory was first propounded by Zadeh (1965) it emerged as a new paradigm in which linguistic uncertainty could be modelled systematically. Literature on this exciting field grew by leaps and bounds and fuzzy logic has made inroads into many diverse fields, such as consumer electronic appliances, process control, decision making, economics, social sciences and, indeed, all other branches of science and engineering. Among other fields, optimization was one of the main beneficiaries of this "revolution". A number of researchers have contributed to fuzzy linear goal programming (Bellman and Zadeh, 1970; Zimmermann, 1991; Tiwari *et al.*, 1987, to mention but a few.). Sakawa and Yani (1991) introduced pareto optimality and augmented minimax methods to fuzzy nonlinear programming problems with fuzzy parameters. This is the only published report dealing with fuzzy nonlinear goal programming problems. As far as application to chemical engineering problems is concerned, Kraslawski *et al.*, 1989 first applied fuzzy dynamic programming to the synthesis of distillation columns. Very recently, Qian and Tessier (1995) applied fuzzy relational modelling to a product quality control problem in the optimization of a wood chip refining process. In this paper, a new model to solve the fuzzy nonlinear goal programming problems has been developed, in which the goals themselves and not the parameters are assumed to be fuzzy in nature. Also, the material balances have been treated in a crisp manner, unlike the approach of Qian and Tessier (1995). Here a simple, real life steady-state optimization problem in petroleum refineries, with multiple performance criteria, has been formulated and solved as a fuzzy nonlinear goal programming problem.

## 2. Fuzzy nonlinear goal programming model

Consider the conventional multi-objective nonlinear programming problem, with $n$ decision variables, $m$ nonlinear objective functions, and $k$ nonlinear constraints:

$$\text{Maximize } f_i(x_1, x_2, \ldots, x_n),\ i = 1, 2, \ldots, \mathrm{m} \tag{1}$$

subject to

$$\left.\begin{array}{l} C_i \equiv LL_i \leq g_i(x_1, x_2, \ldots, x_n) \leq UL_i,\ i = 1, 2, \ldots, \mathrm{k} \\ LB_i \leq x_i \leq UB_i,\ i = 1, 2, \ldots, \mathrm{n} \\ x_i \geq 0,\ i = 1, 2, \ldots, \mathrm{n} \end{array}\right\} \tag{2}$$

where, $LL_i$, $UL_i$ ($i$=1, 2,..., k) are respectively the lower

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and upper limits for the constraints and $LB_i$, $UB_i$ ($i$=1, 2,..., n) are respectively the lower and upper bounds for the decision variables. Let $Z_i$ be the maximized values of $f_i$ ($i$=1, 2,..., m).

There are several methods of finding "good" solutions to the above problem, such as scalar maximization, goal programming, etc. However, more often than not, the decision maker (DM) is confronted with the problem of not being able to fix the aspiration levels for the goals to be attained in a crisp fashion. Yet, he or she may be forced to give such levels in order to facilitate the proper modelling of the problem. However, since the advent of fuzzy set theory, this difficulty for the DM can easily be obviated (Tiwari *et al.*, 1987; Zimmermann, 1991).

Bellman and Zadeh (1970), while formulating their famous model, assumed that the objectives as well as constraints in an ill-structured situation can be represented by fuzzy sets. A decision is then defined as the intersection of all the fuzzy sets represented by objectives and the constraints and is represented by its membership function as follows (Zimmermann, 1991):

$$\mu_{\tilde{D}}(x)=(\mu_{\tilde{C}_1}(x)*.....*\mu_{\tilde{C}_k}(x))*(\mu_{\tilde{G}_1}(x)*.....*\mu_{\tilde{G}_m}(x))$$

where, $\tilde{D}$ represents fuzzy decision, $\tilde{C}_i$ represents the $i$th fuzzy constraint ($i$=1,...,k) and $\tilde{G}_j$ represents the $j$th fuzzy goal, $\mu_{\tilde{D}}(x)$ is the membership function of the decision and $\mu_{\tilde{C}_i}(x)$ is the membership function of the $i^{th}$ constraint and $\mu_{\tilde{G}_j}(x)$ is the membership function of the $j$th goal and * is an appropriate "aggregator" or connective. Depending on the context, the membership functions of some or all of the goals can be chosen to be linear or nonlinear. In view of the foregoing, assuming that only the goals are fuzzy and that constraints and parameters are crisp in nature, the fuzzified version of the model of (1) is as follows:

$$\left.\begin{array}{l}\text{Find } X=(x_1, x_2,..., x_n) \text{ such that}\\ G_i\equiv f_i(x)\lesssim Z_i \; i=1, 2,..., \text{m}\\ \text{subject to constraints and bounds given by Equation (2)}\end{array}\right] \quad (3)$$

where, "$\lesssim$" denotes the fuzzy version of $\leq$ and has the interpretation as "essentially smaller than or equal to" in the parlance of fuzzy set theory (Zimmermann, 1991). This means that though $f_i(X)$ is smaller than $Z_i$, a "leeway" has been allowed for $f_i(X)$ to go beyond $Z_i$. Similarly, when minimization is involved in some of the goals, we use $\gtrsim$ to mean "essentially greater than or equal to". Within the framework of the Bellman and Zadeh (1970) model, and following Zimmermann (1991), a crisp equivalent of the model of (3) is given as follows:

$$\left.\begin{array}{l}Maximize(\mu_1*\mu_2*......*\mu_m)\\ \text{subject to}\\ \text{constraints and bounds given by Equation (2) and}\\ 0\leq\mu_i\leq 1, \; i=1,2\ldots, m\end{array}\right] \quad (4)$$

where, $\mu_i$ is the membership function of the $i$th goal. The definitions of $\mu_i$ ($i$=1,..., m) are given later.

It is obvious that the symbol * is the aggregator which can be "addition operator" or "product operator" or "min operator". Also, it is quite clear from the above that the (4) is a crisp nonlinear programming problem. Thus the fuzzy nonlinear goal programming problem given by (3) has been converted to its crisp equivalent in (4). To solve it, Box's complex method has been resorted to and it has been found to be simple and powerful to solve problems of this nature.

## 3. Fuzzy formulation of the refinery model

First, fuzziness is introduced into the model through the four objective functions considered in Seinfeld and McBride (1970), viz. profit and the absolute values of the sensitivities in profit to variations in parameters $w_{10}$, $w_{11}$ and $h_2$. These sensitivities appear in the form of partial derivatives of profit with respect to $w_{10}$, $w_{11}$ and $h_2$, respectively, in the original model. This way of introducing fuzzification into the problem guarantees the DM the strict satisfaction of the material balances. For details of the refinery model the reader is referred to Seinfeld and McBride (1970).

In the present study membership functions are defined as follows. Using Box's complex method, the crisp maximum of the profit is found to be 105,392,664 dollars. Similarly, using the two-phase simplex method (Rao, 1985), the absolute values of the three sensitivities are minimized in the crisp sense (as these sensitivities turn out to be linear). Let $f$ be the profit, $f_1$ the absolute value of the sensitivity of $w_{10}$ on $f$, $f_2$ the absolute value of the sensitivity of $w_{11}$ on $f$ and $f_3$ the absolute value of the sensitivity of $h_2$ on $f$ respectively. The minimum values of $f_1$, $f_2$ and $f_3$ are found to be 51,268,410; 21,976,212 and 27,289,704,448, respectively.

Thus the refinery model is reformulated as the fuzzy nonlinear goal programming problem as

Find $X=(X_1,X_2,X_3,X_4,X_5,X_6,X_7,X_8,X_9)$ such that

$$f\lesssim 105{,}392{,}664$$
$$f_1\gtrsim 51{,}268{,}410$$
$$f_2\gtrsim 21{,}976{,}212$$
$$f_3\gtrsim 27{,}289{,}704{,}448$$

subject to constraints given in Seinfeld and McBride (1970).

This is done for illustrative purpose only. In practice, one need not find the crisp maximum of the profit and crisp minimum of the absolute values of the sensitivities in profit to variations in parameters $w_{10}$, $w_{11}$ and $h_2$. In general, the DM is aware of the profits and other quantities year-wise only and not the crisp maximum profit or crisp minimum of effects of the parameter sensitivities on the profit. Hence in such case, the DM can always use the best profit obtained so far or the worst sensitivities of the parameters on the profit as the aspiration levels, and during fuzzification a leeway can be given according to his/her choice. Then membership functions are defined for all the fuzzy goals as follows.

$$\mu_1=\begin{cases}1 & \text{if } f\le 105{,}392{,}664\\ \dfrac{105{,}392{,}664-f}{105{,}600{,}000-105{,}392{,}664} & \text{if } 105{,}392{,}664\le f\le 105{,}600{,}000\\ 0 & \text{if } f\ge 105{,}600{,}000\end{cases} \quad (5)$$

$$\mu_2=\begin{cases}1 & \text{if } f_1\ge 51{,}268{,}410\\ \dfrac{f_1-50{,}000{,}000}{51{,}268{,}410-50{,}000{,}000} & \text{if } 50{,}000{,}000\le f_1\le 51{,}268{,}410\\ 0 & \text{if } f_1\le 50{,}000{,}000\end{cases} \quad (6)$$

$$\mu_3=\begin{cases}1 & \text{if } f_2\ge 21{,}976{,}212\\ \dfrac{f_2-20{,}000{,}000}{21{,}976{,}212-20{,}000{,}000} & \text{if } 20{,}000{,}000\le f_2\le 21{,}976{,}212\\ 0 & \text{if } f_2\le 20{,}000{,}000\end{cases} \quad (7)$$

$$\mu_4=\begin{cases}1 & \text{if } f_3\ge -27{,}289{,}704{,}448\\ \dfrac{f_3-27{,}200{,}000{,}000}{27{,}289{,}704{,}448-27{,}200{,}000{,}000} & \text{if } 27{,}200{,}000{,}000\le f_3\le 27{,}289{,}704{,}448\\ 0 & \text{if } f\le 27{,}200{,}000{,}000\end{cases} \quad (8)$$

Apart from the linear membership functions, "S-type" (Zimmermann, 1991) (for goals involving minimization) and "mirror image of S-type" (for goals involving maximization) membership functions are also attempted. The equations for such membership functions are omitted here for the sake of brevity. Once membership functions are defined the crisp equivalent of the fuzzy goal nonlinear programming model is given as follows:

Maximize $\min(\mu_1,\mu_2,\mu_3,\mu_4)$
subject to
constraints given in Seinfeld and McBride, 1970)
and $0\le\mu_i\le 1$, i = 1 to 4 (9)

Now, Box's method cannot be used directly to solve the above problem, as most of the constraints (which are mass balances), are in the form of equalities. Hence the method suggested by Reklaitis *et al.*, 1983 has been resorted to. Accordingly, the entire problem has been reformulated with seven independent decision variables and seven constraints as follows. This is solved using Box's method.

Maximize $\lambda$
subject to
$-13{,}560\le 0.43x_1-0.678x_3\le 30{,}552.2$
$49{,}400\le 0.54x_1+1.596x_3\le 80{,}520$
$0.0\le x_1\le 90{,}000$
$14{,}700.6\le x_3\le 20{,}000$
$\lambda\le\mu_i\le 1$, $i=1$ *to* 4
and the constraint in the form of equations (5) to (8).

## 4. Results and discussion

The study has been carried out in three cases as follows: (1) $f$ and $f_1$ as fuzzy goals (2) $f$ and $f_2$ fuzzy goals and (3) $f$ and $f_3$ as fuzzy goals. The Table 1 shows that case (2) provided the best solution, from the DM's point of view, because, both $f$ and $f_2$ are very much near their respective crisp optimum values. This is particularly significant because we are no longer working in a "crisp-single-objective" environment. The next best solution is the case (1), because, here, the $f_1$ value did not go far away from its crisp minimum value and, moreover, there is only a marginal decrement in the value of profit $f$, compared with its crisp maximum value. Finally, in case (3), the marginal decrement in profit $f$ is accompanied by an enormous two hundred per cent increment in the value of $f_3$ compared with its crisp minimum value. In this way one can rank the parameters according the way they affect the profit.

Thus, in the methodology presented here, the DM has got good flexibility, in that, (1) he/she can consider as many cases as he/she wants, (2) the decision variables and/or parameters and/or the material balances can be considered as fuzzy goals along with the existing ones.

Table 1. Performance of the fuzzy nonlinear goal programming model

| Goals and decision vector* | Case (1) | Case (2) | Case (3) |
|---|---|---|---|
| $f$ | $0.980\times10^8$ | $1.050\times10^8$ | $1.001\times10^8$ |
| $f_1$ | $12.94\times10^7$ | | |
| $f_2$ | | $2.760\times10^7$ | |
| $f_3$ | | | $5.528\times10^{10}$ |
| $x_1$ | 46,650.473 | 18,683.166 | 74,963.039 |
| $x_3$ | 15,714.483 | 19,010.391 | 18,463.316 |
| CPU time (s) | 26 | 22 | 13 |
| Function evaluation | 80 | 79 | 53 |

* Here the decision vector does not include $\mu$*'s and* $\lambda$

In the present paper, however, only the four objectives defined by Seinfeld and McBride (1970) have been assumed to be fuzzy goals. This was decided mainly because only then would a comparison with their results be possible and meaningful. For this very reason, none of the cases produced a solution wherein the value of any goal went beyond the crisp maximum value (in the case of profit $f$) and crisp minimum value (in the case of $f_1, f_2$ and $f_3$). Our experience shows that both kinds of membership functions performed equally well.

## 5. Conclusions

A new methodology has been developed to solve fuzzy nonlinear goal programming problems. A realistic steady-state optimization problem occurring in refineries has been modelled as fuzzy nonlinear goal programming problem and solved successfully using the methodology developed. Linear and S-type membership functions have been considered for the fuzzy goals. Both performed equally well. The methodology presented was found to have tremendous flexibility in that fuzziness could be introduced not only via the goals but also through the independent variables and material balances. This treatment of the problem, within the framework of the fuzzy set theory, allows the DM to conduct a comprehensive study considering various combinations of the goals at varied aspiration levels.

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