4. Primary system relief valve open.
5. Turbine trip valve closed.
6. Superheater tripped.
7. Manual tripping by operator.

The following conditions cause total shutdown of the reactor by inserting all 13 control rods, necessitating a complete restart:

1. Reactor high power level.
2. Reactor power period too short (used during start-up only).
3. Manual tripping by operator.

Summary

The Elk River Reactor Project represents one further step in the development of nuclear energy for the generation of electric power. Considerable contribution is made to the knowledge and experience required to design and construct such a plant in the most economical and safe manner.

The design and installation of the electric system generally follows that of a conventional plant. Equipment and materials used were standard manufactured items; however, in many cases application differs from that found in a conventional plant.

Transient Torques in 3-Phase Induction Motors During Switching Operations

M. R. CHIDAMBARA S. GANAPATHY

Synopsis: Transient torques developed in 3-phase induction motors, during switching operations, i.e., during starting, during reconnection to the line, and during plugging are studied. A mathematical analysis for torque is given for each case. The expressions are obtained in the normalized form and are studied for various numerical values of the parameters of the machine. An experimental verification is given for the case of starting. Based on theoretical and experimental studies conclusions are drawn.

WHEN AN INDUCTION motor is switched in any way, whether while at rest or while running, transient torques that are usually several times the final steady-state value, occur and must be considered in the design. The transient conditions in induction motors have been studied for a very long time. Almost all of these studies refer to the study of transient currents, only a very few, to the authors' knowledge, refer to the transient torques, but without a complete and satisfactory analysis of the problem.

R. E. Hellmund discusses the peak currents and overvoltages that are caused in an asynchronous machine immediately after certain changes in circuit connections are made.[1] W. V. Lyon investigates the transient currents in electric machinery by the vector method.[2] David L. Lindquist and E. W. Yearsley have given graphical methods of analysis for the study of transient performance of electric elevators.[3] Y. H. Ku has studied the transient currents in synchronous and induction machines.[4] H. E. Koenig obtains the performance equations of several types of machines, which hold good both for steady-state and transient performance.[5] Differential equations for the response of an induction motor have been derived by H. C. Stanley.[6] These equations are so complicated that their general solution is very difficult. Solutions have been worked out by special simplifying assumptions,[7] or through the use of a differential analyzer.[8,9] General solutions for the transient response of a 2-phase induction motor have been developed, based on close approximations.[10] Gilfillan and Kaplan have given a transformation to Stanley's equations and have studied the transient plugging torques with the help of a differential analyzer.[11] The transient analysis of certain types of machines can be made conveniently by the method of instantaneous symmetrical components.[12]

In this paper, from the equations of performance for a balanced 3-phase induction motor expressions for the transient torque are derived for several conditions, an "ideal" cylindrical rotor machine being assumed in the analysis. To the authors' knowledge, for the cases of plugging and reconnection to the line, the general expressions for the transient torque in a normalized form are not obtained elsewhere.

With the help of these expressions and the set of normalized curves given here, it is possible to predict the peak transient torque for any motor with known values of parameters.

Theoretical Solution

The differential equations for the performance of an induction motor, in terms of the instantaneous symmetrical components have been given by Lyon.[13] These equations reveal that the positive- and negative-sequence variables are separated. Furthermore, the positive-sequence equations are the complex conjugates of the corresponding negative-sequence equations. Hence, one needs to consider only the positive-sequence equations.

Case of Starting

When balanced 3-phase potentials are applied to the terminals of an induction motor, the rotor of which is held stationary, the positive-sequence equation for the rotor is given by

$$(r_\beta+x_\beta D)(i_{\beta_1})+x_m D(i_{\alpha 1}\epsilon^{-j\phi})=0 \qquad (1)$$

This equation can be multiplied throughout by $\epsilon^{-j\phi}$ since ϕ is assumed to be constant. Then

$$(r_\beta+x_\beta D)(i_{\beta 1}\epsilon^{j\phi})+x_m D(i_{\alpha 1})=0 \qquad (2)$$

The positive-sequence equation for the stator is

$$(r_\alpha+x_\alpha D)i_{\alpha 1}+x_m D(i_{\beta 1}\epsilon^{j\phi})=v_{\alpha 1} \qquad (3)$$

It should be noted that $v_{\alpha 1}=V_{\alpha 1}\epsilon^{j\tau}$ where $V_{\alpha 1}=1/2\,Va\epsilon^{j\theta}$, the potential v_a being given by

$$v_a=V_a\cos(\omega t+\theta)$$

Taking the Laplace Transforms of equations 2 and 3,

$$V_{\alpha 1}/(s-j)=(r_\alpha+sx_\alpha)I_{\alpha 1}(s)+sx_m\epsilon^{j\phi}I_{\beta 1}(s) \qquad (4)$$

$$0=sx_m I_{\alpha 1}(s)+(r_\beta+sx_\beta)\epsilon^{j\phi}I_{\beta 1}(s) \qquad (5)$$

Solving for $I_{\alpha 1}(s)$ and $I_{\beta 1}(s)$, one has

$$I_{\alpha 1}(s)=\frac{V_{\alpha 1}(s+\kappa_\beta)}{\sigma x_\alpha(s-j)(s-\alpha)(s-\beta)} \qquad (6)$$

$$I_{\beta 1}(s)=\frac{\epsilon^{-j\phi}V_{1\alpha}x_m}{\sigma x_\alpha x_\alpha(s-j)(s-\alpha)(s-\beta)} \qquad (7)$$

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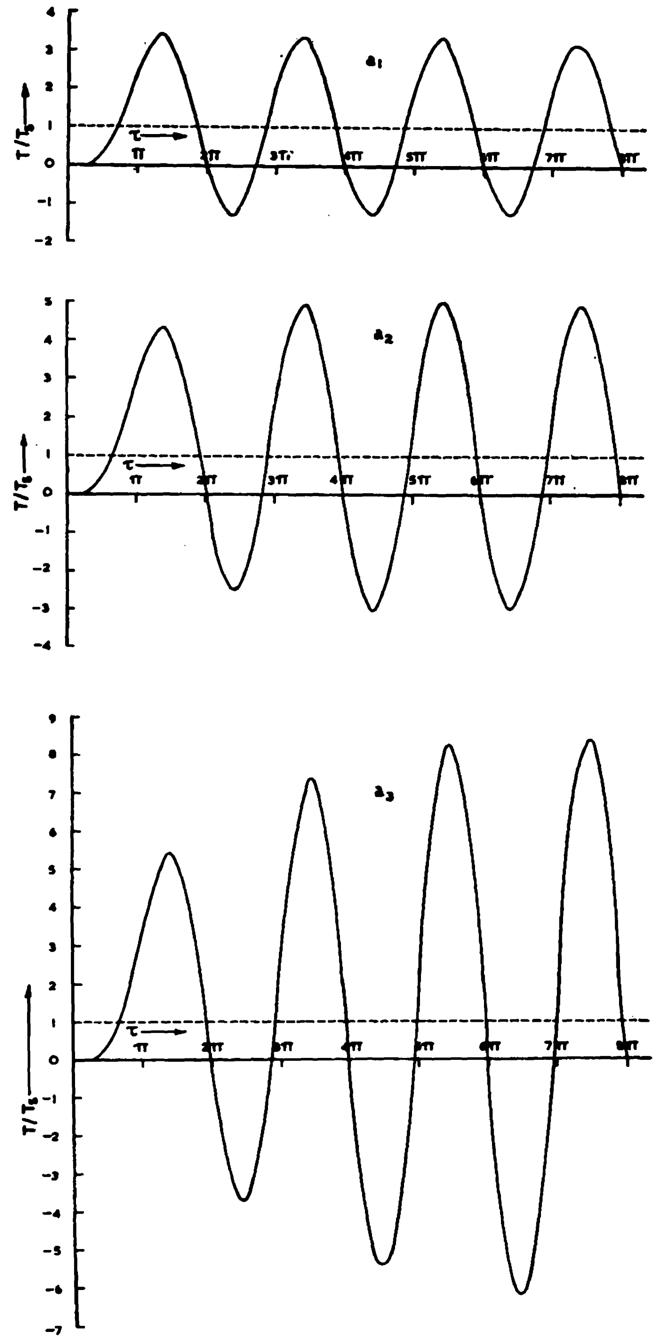

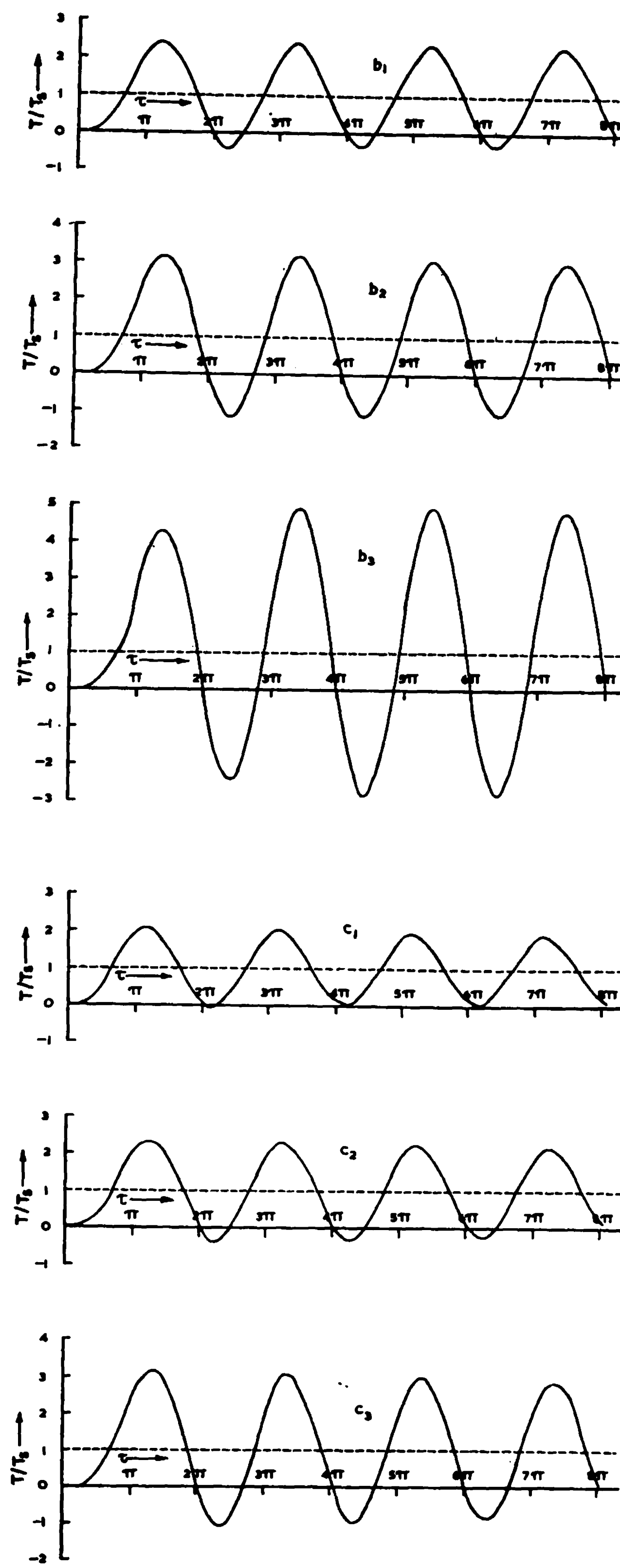

Fig. 1. Transient torques in 3-phase induction motors during starting

a1: $p_2=0.0025$, $\xi=0.005$	b3: $p_2=0.005$, $\xi=0.020$
a2: $p_2=0.0025$, $\xi=0.010$	c1: $p_2=0.010$, $\xi=0.005$
a3: $p_2=0.0025$, $\xi=0.020$	c2: $p_2=0.010$, $\xi=0.010$
b1: $p_2=0.005$, $\xi=0.005$	c3: $p_2=0.010$, $\xi=0.020$
b2: $p_2=0.005$, $\xi=0.010$	

where α, β are the roots of the characteristic equation $\sigma s^2+(\kappa_\alpha+\kappa_\beta)s+\kappa_\alpha\kappa_\beta = 0$ reducing *rhs* of equations 6 and 7 into partial fractions and taking inverse Laplace Transforms,

$$i_{\alpha 1}=\frac{V_{\alpha 1}}{\sigma x_\alpha}\left[\frac{(\kappa_\beta+j)\epsilon^{j\tau}}{(\alpha-j)(\beta-j)}+\frac{(\kappa_\beta+\alpha)\epsilon^{\alpha\tau}}{(\beta-\alpha)(j-\alpha)}+\frac{(\kappa_\beta+\beta)\epsilon^{\beta\tau}}{(\alpha-\beta)(j-\beta)}\right] \quad (8)$$

$$i_{\beta 1}=-\frac{x_m}{x_\beta}\epsilon^{-j\phi}\frac{V_{\alpha 1}}{\sigma x_\alpha}\left[\frac{j\epsilon^{j\tau}}{(\alpha-j)(\beta-j)}+\frac{\alpha\epsilon^{\alpha\tau}}{(\beta-\alpha)(j-\alpha)}+\frac{\beta\epsilon^{\beta\tau}}{(\alpha-\beta)(j-\beta)}\right] \quad (9)$$

The expression for torque is given by

$T=K\ P/2\omega\ Ar_\beta$ 2 real part of

$$\{j i_{\beta 2}\int i_{\beta 1}d\tau\} \quad (10)$$

$i_{\beta 2}$ is the complex conjugate of $i_{\beta 1}$.

Substituting and simplifying, the torque expression becomes

$$T=K\frac{P}{2\omega}Ar_\beta\frac{V^2}{(\sigma x_\alpha)^2}\left(\frac{x_m}{x_\beta}\right)^2\frac{1}{(1+\alpha^2)(1+\beta^2)}\times$$

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$$\left[1+\epsilon^{(\alpha+\beta)\tau}-\left(\cos\tau+\frac{1+\alpha\beta}{\alpha-\beta}\sin\tau\right)\epsilon^{\alpha\tau}-\left(\cos\tau-\frac{1+\alpha\beta}{\alpha-\beta}\sin\tau\right)\epsilon^{\beta\tau}\right] \quad (11)$$

The common coefficient in this expression is the steady-state torque. If T_s denotes the steady-state torque, the expression within big brackets in the *rhs* of equation 11 gives the ratio of the transient torque to the steady-state torque.

If α is represented by $-p_1$ and β by $-p_2$

$$p_1=\frac{\kappa_\alpha+\kappa_\beta}{\sigma}-\frac{\kappa_\alpha\kappa_\beta}{\kappa_\alpha+\kappa_\beta} \text{ and } p_2=\frac{\kappa_\alpha\kappa_\beta}{\kappa_\alpha+\kappa_\beta}$$

It can be seen that p_1 is very large compared with p_2 and thus the effect of the damping term p_2 persists longer than that due to p_1. Hence let the transient torque be expressed in terms of p_2 and another term ξ, which is defined by $\xi=p_2/p_1$.

Hence the torque expression becomes as follows:

$$\frac{T}{T_s}=\left[1+\epsilon^{-p_2(1+1/\xi)\tau}-\left(\cos\tau-\frac{\xi+p_2^2}{p_2(1-\xi)}\sin\tau\right)\epsilon^{-(p_2/\xi)\tau}-\left(\cos\tau+\frac{\xi+p_2^2}{p_2(1-\xi)}\sin\tau\right)\epsilon^{-p_2\tau}\right] \quad (12)$$

Having obtained the final expression for the ratio of transient torque to the steady-state torque, this expression could be plotted as a curve with respect to τ, the normalized time by substituting numerical values of the parameters of the induction motor. Furthermore, the effect of varying the parameters of the machine could be studied by varying the parameters within the range of values that are usually met with in practice. Fig. 1 shows a few typical curves.

Case of Reconnection to Line

Assume that the breakers are open, disconnecting the induction motor from the line, and that a moment later the breakers reclose when the speed has fallen to a given value.

The positive-sequence equations for the performance of the induction motor, when the rotor speed is constant are given by

$$(r_\alpha+x_\alpha D)i_{\alpha1}+x_m D(i_{\beta1}\epsilon^{jn\tau})=v_{\alpha1} \quad (13)$$

$$x_m(D-jn)i_{\alpha1}+[r_\beta+x_\beta(D-jn)]\times(i_{\beta1}\epsilon^{jn\tau})=0 \quad (14)$$

Solving these equations for $i_{\alpha1}$ and $i_{\beta1}$ by the method of Laplace Transforms, one gets

$$i_{\alpha1}=\frac{V_{\alpha1}}{\sigma x_\alpha}\left[\frac{\kappa_\beta+j(1-n)}{(j-p_1)(j-p_2)}\epsilon^{j\tau}+\frac{(\kappa_\beta+p_1-jn)}{(p_1-p_2)(p_1-j)}\epsilon^{p_1\tau}+\frac{(\kappa_\beta+p_2-jn)}{p_2-j)(p_2-p_1)}\epsilon^{p_2\tau}\right] \quad (15)$$

$$i_{\beta1}=-\frac{x_m}{x_\beta}\cdot\frac{V_{\alpha1}}{\sigma x_\alpha}\left[\frac{j(1-n)}{(j-p_1)(j-p_2)}\epsilon^{j(1-n)\tau}+\frac{(p_1-jn)}{(p_1-p_2)(p_1-j)}\epsilon^{(p_1-jn)\tau}+\frac{(p_2-jn)}{(p_2-j)(p_2-p_1)}\epsilon^{(p_2-jn)\tau}\right] \quad (16)$$

where p_1 and p_2 are the roots of the characteristic equation

$$s^2+\left(\frac{\kappa_\alpha+\kappa_\beta}{\sigma}-jn\right)s+\frac{\kappa_\alpha}{\sigma}(\kappa_\beta-jn)=0$$

$$p_1=-\frac{\kappa}{\sigma}+j\left[n-\frac{1-\sigma}{n}\cdot\kappa^2/\sigma^2\right]=-\gamma+j(n-\delta)$$

$$p_2=-\kappa/\sigma+j\left(\frac{1-\sigma}{n}\right)\kappa^2/\sigma^2=-\gamma+j\delta$$

The expression for torque is given by

$$T=K\frac{P}{2\omega}Ar_\beta 2Re\{ji_{\beta2}\textstyle\int i_{\beta1}d\tau\}$$

$i_{\beta2}$ is the complex conjugate of $i_{\beta1}$. Therefore, $j\,i_{\beta2}$ is given by

$$ji_{\beta2}=-\frac{x_m}{x_\beta}\cdot\frac{V_{\alpha2}}{x\sigma_\alpha}\left[\frac{(1-n)\epsilon^{-j(1-n)\tau}}{(j+p_1^*)(j+p_2^*)}+\frac{(p_1^*+jn)j}{(p_1^*-p_2^*)(p_1^*+j)}\epsilon^{(p_1^*+jn)\tau}+\frac{(p_2^*+jn)j}{(p_2^*+j)(p_2^*-p_1^*)}\epsilon^{(p_2^*+jn)\tau}\right] \quad (17)$$

Integrating equation 16,

$$\int i_{\beta1}d\tau=-\frac{x_m}{x_\beta}\cdot\frac{V_{\alpha1}}{\sigma x_\alpha}\times\left[\frac{1}{(j-p_1)(j-p_2)}\epsilon^{(1-n)\tau}+\frac{1}{(p_1-p_2))p_1-j)}\epsilon^{(1-n)\tau}+\frac{1}{(p_2-j)(p_2-p_1)}\epsilon^{(p_2-jn)\tau}\right] \quad (18)$$

Therefore, the torque is proportional to the product of equations 17 and 18, only the real parts of which contribute to the torque.

Twice the product of the common coefficients is

$$\left(\frac{x_m}{x_\beta}\right)^2\left(\frac{V}{\sigma x_\alpha}\right)^2$$

where $V^2=2|V_{\alpha1}|^2$, V being the effective value of the applied phase potential.

The product of the first terms is

$$\frac{(1-n)}{|j-p_1|^2\cdot|j-p_2|^2}=\frac{(1-n)}{[\gamma^2+(1+\delta-n)^2][\gamma^2+(1-\delta)^2]}$$

the sum of products of the first and second terms is

$$-\frac{(p_1^*+jn)j\epsilon^{(p_1^*+j)\tau}}{(|j-p_1|^2)(j-p_2)(p_1^*-p_2^*)}+\frac{(1-n)\epsilon^{(p_1-j)\tau}}{(p_1-p_2)|j-p_1|^2(j+p_2^*)}$$

$$=\frac{\epsilon^{-\gamma\tau}}{|j-p_1|^2|(j-p_2)|^2|p_1-p_2|^2}[j(p_1^*+jn)(p_1-p_2)(j+p_2^*)\epsilon^{j(1+\delta-n)\tau}+(1-n)(p_1^*-p_2^*)(p_2-j)\epsilon^{-j(1+\delta-n)\tau}]$$

The real part of this expression is

$$\frac{-\epsilon^{-\gamma\tau}}{|j-p_1|^2\cdot|j-p_1|^2\cdot|p_1-p_2|^2}(n-2\delta)\times[\{\gamma^2+(1-\delta)(1-n-\delta)\}\cos(1+\delta-n)\tau-\gamma n\sin(1+\delta-n)\tau]$$

the sum of the products of the first and third terms is

$$\frac{(p_2^*+jn)j\epsilon^{(p_2^*+j)\tau}}{(j-p_1)(|j-p_2|^2)(p_1^*-p_2^*)}-\frac{(1-n)\epsilon^{(p_2-j)\tau}}{|j-p_2|^2(j+p_1^*)(p_1-p_2)}$$

$$=\frac{\epsilon^{-\gamma\tau}}{|j-p_1|^2\cdot|j-p_2|^2\cdot|p_1-p_2|^2}\times[-(p_2^*+jn)j(p_1^*+j)(p_1-p_2)\epsilon^{j(1-\delta)\tau}-(1-n)(p_1^*-p_2^*)(p_1-)\epsilon j^{-j(1-\delta)\tau}]$$

Taking only the real parts, this becomes equal to

$$\frac{\epsilon^{-\gamma\tau}\cdot(n-2\delta)}{|j-p_1|^2\cdot|j-p_2|^2\cdot|p_1-p_2|^2}\times[\{\gamma^2-(1+\delta-n)(2n-1-1-\delta)\}\cos(1-\delta)\tau+\gamma n\sin(1-\delta)\tau]$$

and the sum of the products of the second terms and third terms

$$=\frac{(p_1^*+jn)j\epsilon^{(p_1+p_1^*)\tau}}{|p_1-p_2|^2\cdot|p_1-j|^2}+\frac{(p_2^*+nj)j\epsilon^{(p_2+p_2^*)\tau}}{|p_1-p_2|^2\cdot|p_2-j|^2}$$

When only the real parts are taken this becomes

$$=\frac{-\epsilon^{-2\gamma\tau}}{|j-p_1|^2|j-p_2|^2|p_1-p_2|^2}\times|(\delta)(2-n)(n-2\delta)+n\{\gamma^2+(1+\delta-n)\}^2]$$

The sum of the products of the second and third terms

$$=-\frac{(p_2^*+jn)j\epsilon^{(p_1-p_2^*)\tau}-}{|p_1-p_2|^2(p_1-j)(p_2^*+j)}-\frac{(p_1^*+jn)j\epsilon^{(p_2-p_1^*)\tau}}{|p_1-p_2|^2(p_2-j)(p_1^*+j)}$$

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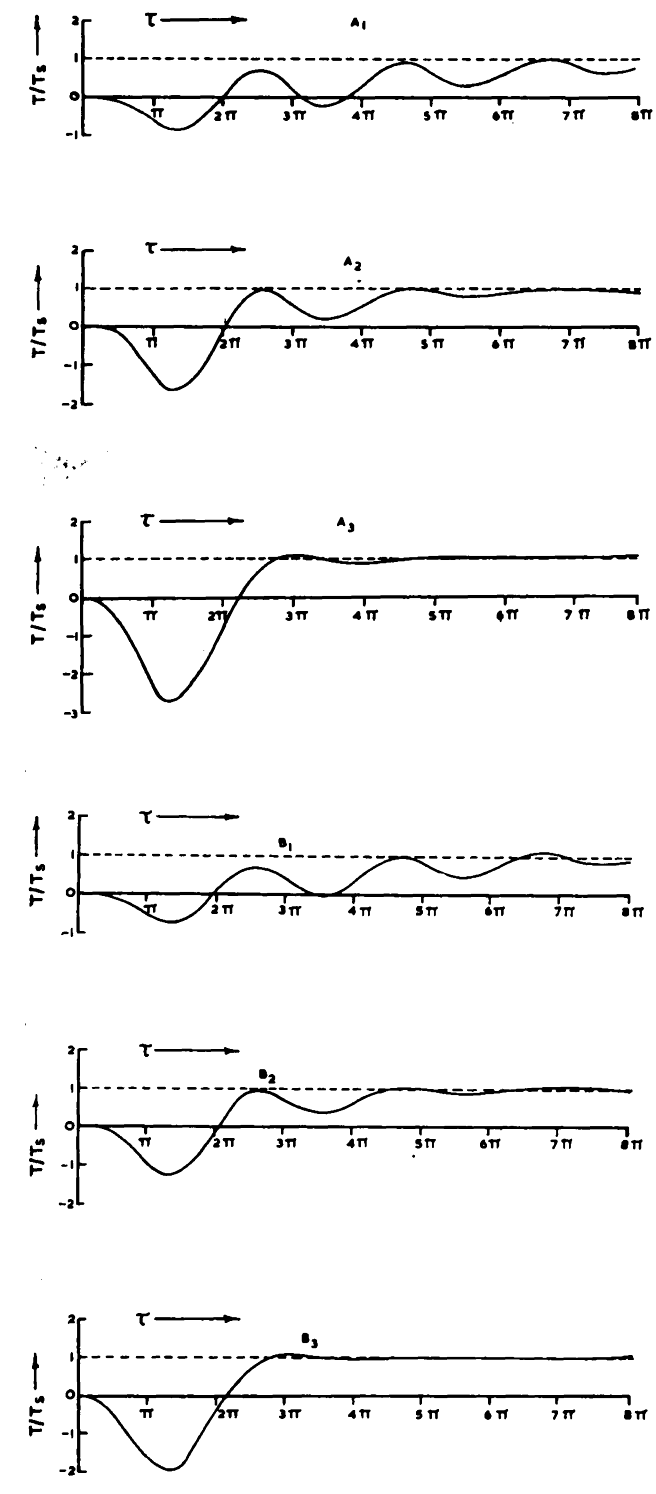

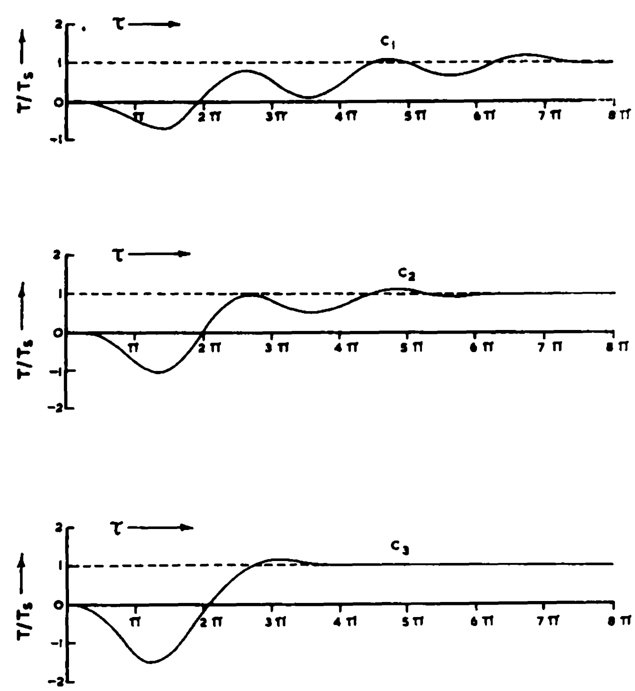

Fig. 2. Transient torques in 3-phase induction motors during reconnection to line

a_1: $n=0.94$, $\gamma=0.1$
a_2: $n=0.94$, $\gamma=0.2$
a_3: $n=0.94$, $\gamma=0.4$
b_1: $n=0.92$, $\gamma=0.1$
b_2: $n=0.92$, $\gamma=0.2$
b_3: $n=0.92$, $\gamma=0.4$
c_1: $n=0.90$, $\gamma=0.1$
c_2: $n=0.90$, $\gamma=0.2$
c_3: $n=0.90$, $\gamma=0.4$

Considering only the real parts, this becomes

$$=\frac{\epsilon^{-2\gamma\tau}}{|j-p_1|^2|j-p_2|^2|p_1-p_2|^2}\times [\{\gamma^2 n+(1-\delta)(1+\delta-n)n\}\cos(n-2\delta)\tau+(n-2\delta)\gamma n\sin(n-2\delta)\tau]$$

Therefore, the final torque expression, when reconnection is made to the supply following a momentary interruption of the supply, is given by

$$T=K\frac{P}{2\omega}Ar_\beta\left(\frac{x_m}{x_\beta}\right)^2\left(\frac{V}{\sigma x_\alpha}\right)^2\times\frac{(1-n)}{|j-p_1|^2|j-p_2|^2}\times$$

$$\left[1-\frac{n\{\gamma^2+(1+\delta-n)^2\}+\delta(2-n)(n-2\delta)}{(n-2\delta)^2(1-n)}\epsilon^{-2\gamma\tau}+\frac{\{\gamma^2 n+(1-\delta)(1+\delta-n)n\}\cos(n-2\delta)\tau-(n-2\delta)\gamma n\sin(n-2\delta)\tau}{(n-2\delta)^2(1-n)}\epsilon^{-2\gamma\tau}-\right.$$

$$\frac{\{-\gamma^2+(1+\delta-n)(2n-1-\delta)\}\cos(1-\delta)\tau-\gamma n\sin(1-\delta)\tau}{(n-2\delta)(1-n)}\epsilon^{-\gamma\tau}-$$

$$\left.\frac{\{\gamma^2+(1-\delta)(1-\delta-n)\}\cos(1+\delta-n)\tau+\gamma n\sin(1+\delta-n)}{(n-2\delta)(1-n)}\epsilon^{-\gamma\tau}\right] \quad (19)$$

The common coefficient will give the steady-state torque during reconnection to the line. Hence one can write the ratio of transient to steady-state torque in the form

$$T/T_s=[1+(-c_1+c_2\cos\phi_1\tau-c_3\sin\phi_1\tau)\epsilon^{-2\gamma\tau}+(-c_4\cos\phi_2\tau+c_5\sin\phi_2\tau)\epsilon^{-\gamma\tau}+(-c_6\cos\phi_3\tau-c_7\sin\phi_3\tau)\epsilon^{-\gamma\tau}] \quad (20)$$

It is very interesting to note here that γ, the damping factor, is not very different from that in the case of starting, since, when $\kappa_\beta=\kappa=\kappa_\beta$, the roots of the characteristic equation in the case of starting are

$$\alpha=-\frac{2\kappa}{\sigma}+\kappa/2\approx-\frac{2\kappa}{\sigma}=-2\gamma$$

and

$$\beta=-\kappa/2=-\frac{\sigma\gamma}{2}$$

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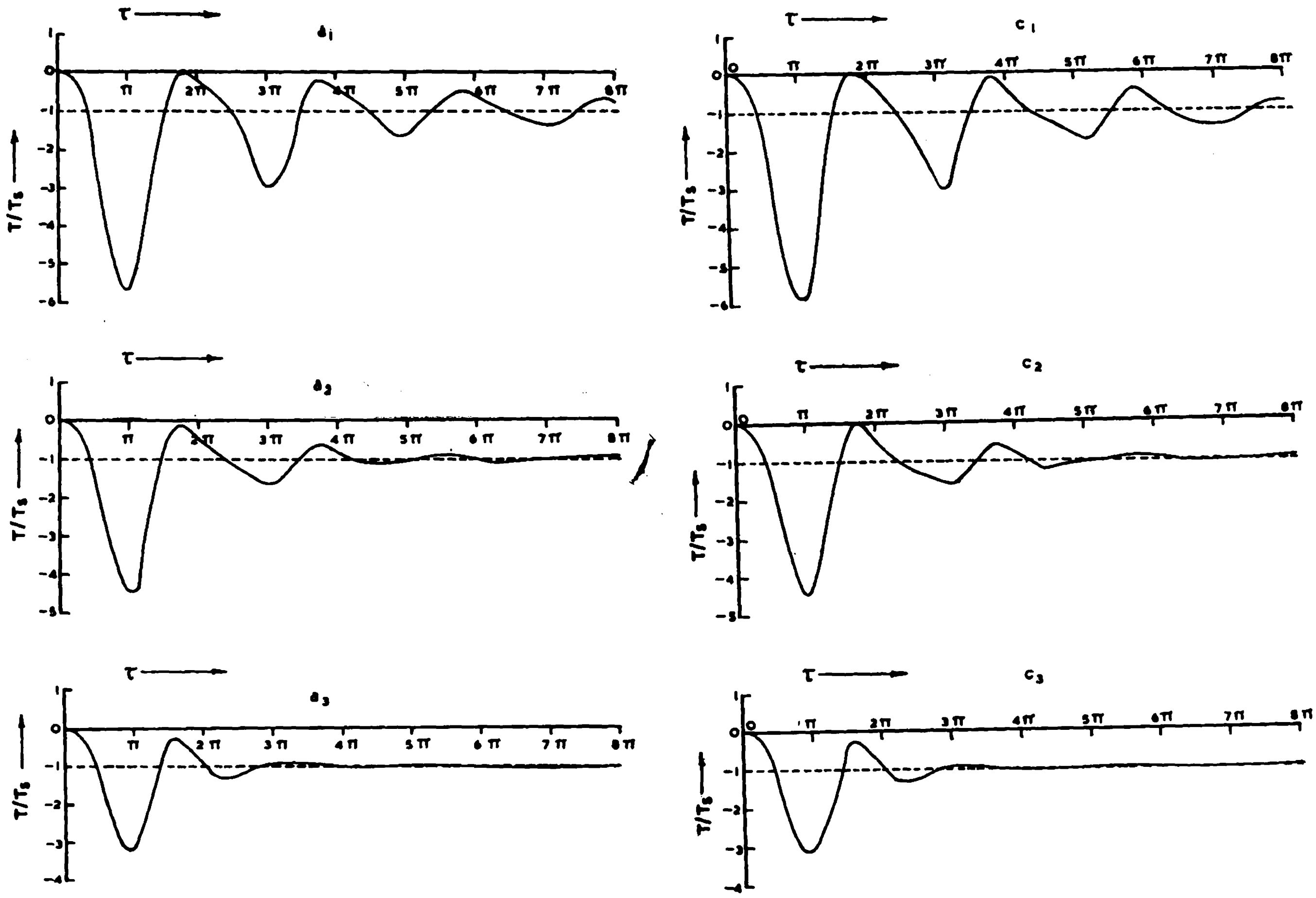

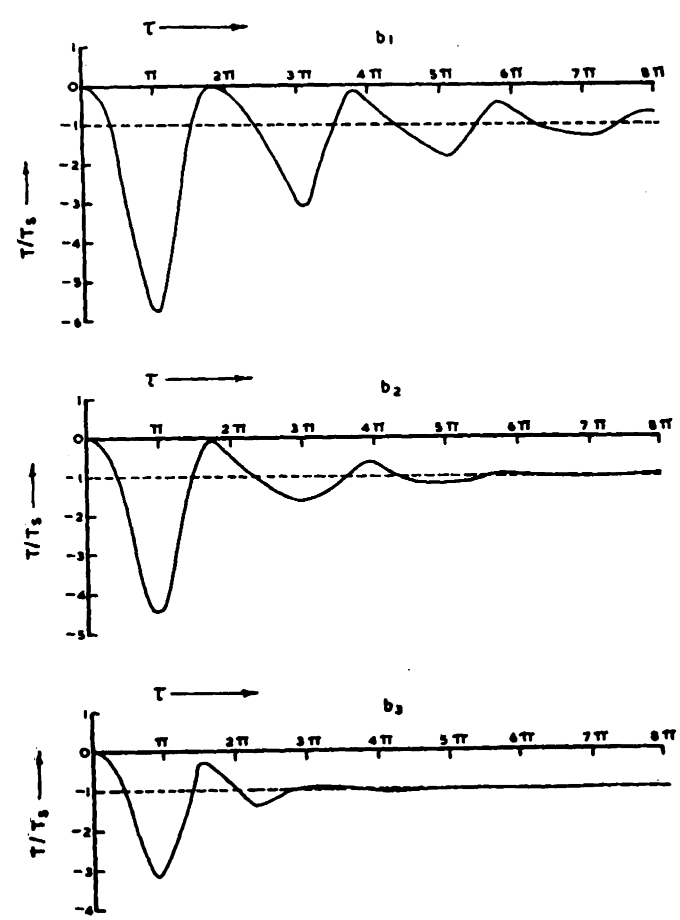

Fig. 3. Transient torques in induction motors during plugging

a_1: $n=0.94$, $\gamma=0.1$	b_3: $n=0.92$, $\gamma=0.4$
a_2: $n=0.94$, $\gamma=0.2$	c_1: $n=0.90$, $\gamma=0.1$
a_3: $n=0.94$, $\gamma=0.4$	c_2: $n=0.90$, $\gamma=0.2$
b_1: $n=0.92$, $\gamma=0.1$	c_3: $n=0.90$, $\gamma=0.4$
b_2: $n=0.92$, $\gamma=0.2$	

Hence, it can be concluded that the damping factor in the case of reconnection to the line could be obtained from those in the case of starting of the induction motor.

Further, it will be of interest to note the following:

1. The coefficients of sine terms in the torque expression are all equal.
2. The ratio of damping coefficients is 2.
3. $\phi_3=\phi_2-\phi_1$.
4. $C_2-C_1=1$.
5. $C_4+C_5=2$.

The torque expression contains only three variables, γ, n, and δ; strictly speaking, δ is not a variable, since

$$\delta=\frac{1-\sigma}{n}\cdot\frac{\kappa^2}{\sigma^2}\approx\gamma^2/n$$

The graphs of T/T_s versus τ for various numerical values of γ and n are shown in Fig. 2.

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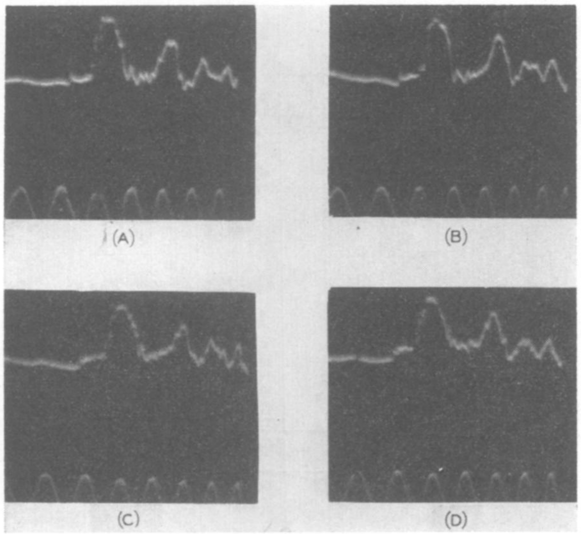

Fig. 4. Transient starting torques obtained by experiment

Voltage applied to stator terminals:

A—100 volts (l−n)
B—90 volts (l−n)
C—80 volts (l−n)
D—70 volts (l−n)

Case of Plugging

When a motor is required to be stopped more quickly, the motor is disconnected from the line and then reconnected by interchanging two of the line terminals. In this case, the applied phase potentials will be in the reversed phase order and, therefore,

$$\left.\begin{aligned} v_{\alpha 1} &= V_{\alpha 1}\epsilon^{-j\tau} \\ &\text{and} \\ v_{\alpha 2} &= V_{\alpha 2}\epsilon^{j\tau} \end{aligned}\right\} \quad (21)$$

The analysis of the performance during plugging is exactly similar to that during reconnection to the line except for the fact that applied phase potentials are now in reversed phase order. Hence equations 13 and 14 still hold good except that $V_{\alpha 1}$ is given by equation 21.

Solving for i_{α_1} and i_{β_1}, one has

$$i_{\alpha 1} = -\frac{V_{\alpha 1}}{\sigma x_\alpha}\left\{\frac{\kappa_\beta - j(1+n)}{(j+p_1)(j+p_2)}\epsilon^{-j\tau} + \frac{(\kappa_\beta + p_1 - jn)}{(j+p_1)(p_1-p_2)}\epsilon^{p_1\tau} + \frac{(\kappa_\beta + p_2 - jn)}{(j+p_2)(p_2-p_1)}\epsilon^{p_2\tau}\right\} \quad (22)$$

and

$$i_{\beta 1} = -\frac{x_m}{x_\beta}\cdot\frac{V_{\alpha 1}}{\sigma x_\alpha}\left\{\frac{-j(1+n)}{(j+p_1)(j+p_2)}\epsilon^{-j(1+n)\tau} + \frac{(p_1 - jn)}{(j+p_1)(p_1-p_2)}\epsilon^{(p_1-jn)\tau} + \frac{(p_2 - jn)}{(j+p_2)(p_2-p_1)}\epsilon^{(p_2-jn)\tau}\right\} \quad (23)$$

where p_1 and p_2 have the same values as those in the case of reconnection to the line.

The torque expression which could be calculated as in the case of reconnection to the line, is given by

$$T = K\frac{P}{2\omega}Ar_\beta\left(\frac{x_m}{x_\beta}\right)^2\left(\frac{V}{\sigma x_\alpha}\right)^2 \times \frac{(1+n)}{|j+p_1|^2\cdot|j+p_2|^2} \times \Bigg[-1 \frac{-n\{\gamma^2+(1+n-\delta)^2\} - \delta(2+n)(n-2\delta)\tau}{(1+n)(n-2\delta)^2}\epsilon^{-2\gamma\tau} + \frac{\{\gamma^2+(1+\delta)(1+n-\delta)\}n\cos(n-2\delta)\tau - \gamma n(n-2\delta)\sin(n-2\delta)\tau}{(1+n)(n-2\delta)^2}\epsilon^{-2\gamma\tau} + \frac{\{\gamma^2+(1+n-\delta)(1+2n-\delta)\}\times\cos(1+\delta)\tau - \gamma n\sin(1+\delta)\tau}{(1+n)(n-2\delta)}\epsilon^{-\gamma\tau} - \frac{\{\gamma^2+(1+\delta)(1+n+\delta)\}\cos(1+n-\delta)\tau - \gamma n\sin(1+n-\delta)\tau}{(1+n)(n-2\delta)}\epsilon^{-\gamma\tau}\Bigg] \quad (24)$$

The common coefficient gives the value of the steady-state plugging torque. The minus sign associated with this indicates that the plugging torque is a breaking torque. Equation 24 could be written in the form

$$\frac{T}{T_s} = [-1 + (-c^1 + c^2\cos\theta_1\tau - c^3\sin\theta_1\tau)e^{-2\gamma\tau} + (c^4\cos\theta_2\tau - c^3\sin\theta_2\tau)\epsilon^{-\gamma\tau} + (-c^5\cos\theta_3\tau + c^3\sin\theta_3\tau)\epsilon^{-\gamma\tau}] \quad (25)$$

As was the case with reconnection to the line, the damping factor, γ, could be obtained from those in the case of starting of the induction motor. The following additional points are worth noting:

1. The coefficients of the sine terms in the torque expression are all the same.
2. The ratio of the damping coefficients is 2.
3. $\theta_3 = \theta_2 + \theta_1$.
4. $c^1 - c^2 = 1$.
5. $c^4 - c^5 = 2$.

The transient torque during plugging can also be treated as a special case of reconnection to the line wherein "n" has a negative sign. In this case, the torque is obtained as a positive quantity for obvious reasons.

Fig. 3 represents the curves of T/T_s versus τ for various values of γ and n.

Experimental Verification for the Starting Case

The experiment is conducted on a 3-phase 220-volt squirrel-cage induction motor and the transient starting torque, obtained as a trace on the oscilloscope, is photographed, using a capacitance strain gage.[14,15]

Fig. 4 shows the photographs of transient starting torques for various voltages applied to the stator terminals. The horizontal axis is calibrated by taking the wave form of the applied potential on the same frame containing the transient torque. Vertical calibration is not necessary because the transient torque is studied as a ratio of the instantaneous torque to the final steady-state torque.

To obtain a comparison between the experimental results and the calculated results, the parameters of the machine under investigation are measured and a graph of T/T_s versus τ is drawn. This is shown in Fig. 5. The transient starting torques obtained by experiment are projected on a graph paper and the curves traced by smoothing out the ripples. These are shown in Fig. 6. A comparison of Figs. 5 and 6 is made.

Conclusions

The expressions for the transient torque are derived in the normalized form for

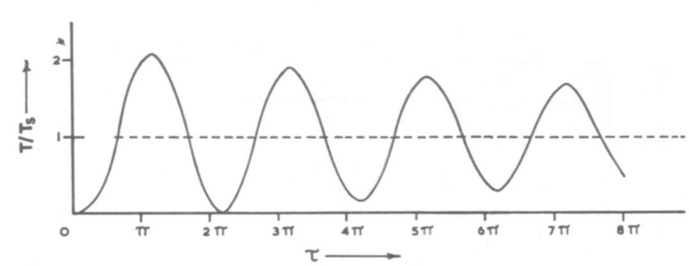

Fig. 5. Theoretical curve of the transient starting torque for the motor under test

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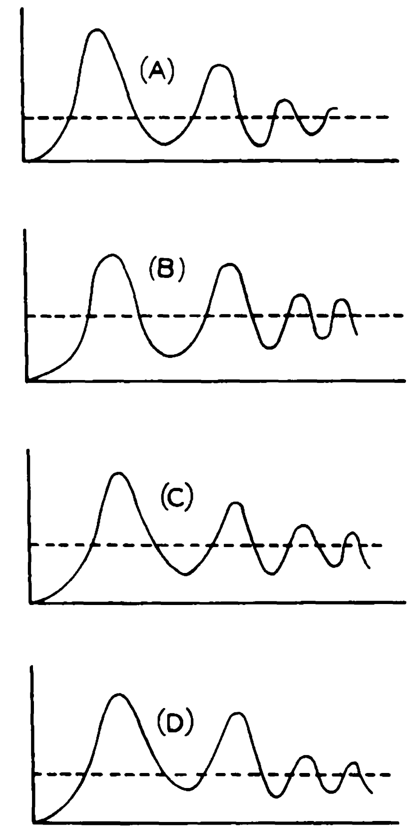

Fig. 6. Reproduction of the photographs shown in Fig. 4

several conditions and are studied for various numerical values of the parameters of the induction motor. With the help of these expressions and the set or normalized curves which are obtained for various numerical values of the parameters, it is possible to predict the peak transient torque for any motor with known values of the parameters.

Case of Starting

Fig. 6(B) agrees very closely with the theoretical curve shown in Fig. 5. The first transient starting torque peak is very nearly 2 times and the second peak is 1.8 times that in the experimental curve as compared with the 2.02 times and 1.92 times in the theoretical curve.

Figs. 6(B), (C), and (D) show that the values of the final steady-state torque are decreasing in magnitude, as is naturally to be expected with decreased values of the applied potentials.

The transient starting torque is a fundamental frequency torque and it may be as high as 7 to 8 times the final steady-state value.

Case of Reconnection to the Line

The highest transient torque is in the negative direction and is about 4 times the steady-state torque. The minimum peak is 0.75 times the steady-state torque and in the negative direction. The peak torque in the positive direction is never greater than 1.2 times the steady-state value.

Case of Plugging

It is observed from the curves that the transient plugging torque never becomes positive, although many of the curves touch the zero axis when passing through the maximum.

The highest value of the transient plugging torque is 5.8 times the steady-state value. It may be pointed out here that the previous two conclusions agree with the results obtained by Gilfillan and Kaplan.[11]

Nomenclature

a, b, c = subscript indicating phase of polyphase system
$0, 1, 2$ = subscript indicating zero, positive and negative sequence
α, β = subscripts indicating stator and rotor
v = instantaneous potential
i = instantaneous current
x_α, x_β = polyphase self or synchronous reactance
x_m = mutual magnetizing reactance
P = number of poles
ϕ = position angle of rotor with respect to stator
T = electromagnetic torque in the direction of rotation
K = torque unit constant
$\tau = \omega t$ ($2\pi \times$ frequency $\times$ seconds)
n = ratio of actual speed to synchronous speed
k = damping constant, ratio of resistance to synchronous reactance
σ = leakage coefficient ($= 1 - x_m^2/x_\alpha x_\beta$)
D = differential operator $d/d\tau$
ϵ = napierian base of logarithm (2.71828...)
j = imaginary number ($= \sqrt{-1}$)
$*$ = conjugate
A, B = number of stator and rotor phases

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Discussion

T. R. M. Szogyen (English Electric Company of Canada, St. Catherines, Ont., Canada): The authors are to be commended for their work in this interesting field of transient switching torques. High peak torques are of considerable importance from the point of view of design of mechanical linkages, couplings, and shafts subjected to such torque impulses.

The experimental verification of the authors is of particular interest. Some years ago, K. M. Chirgwin developed an apparatus for direct determination of induction motor torque. In his unpublished paper, Chirgwin shows transient torques, obtained experimentally, for both the locked rotor condition and for the plugging condition. The shape of these transient torque curves, Fig. 7, appears to be significantly different from those presented by Messrs. Chidambara and Ganapathy. Those obtained by Chirgwin have sharp-pointed peaks near the horizontal axis; they are similar to the square of a sine function modified by strong attenuation. I think that the method of obtaining the torque trace has a very important influence on the shape of the trace; artificial as well as inherent damping of the signal to be picked up can be decisive. Could the authors give a brief description of the arrangement which has been used for their experimental verification?

Also, the size and type of motor used in the experiment would be of interest to know. Could the authors furnish some guidance on how to select the "damping constants" of a motor whose rotor resistance changes greatly with changing rotor current

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