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Ranking multi-criterion river basin planning alternatives using fuzzy numbers

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Abstract

The methodology proposed by Anand Raj and Nagesh Kumar [5] to rank the river basin planning and development alternatives under multi-criterion environment using fuzzy numbers is applied to a case study. The purpose is to find the most suitable planning of reservoirs with their associated purposes aimed at the development of one of the major peninsular river basins (Krishna river basin) in India. A set of 7 alternative systems with 8 main objectives, which are further subdivided into 18 criteria, are considered for ordering or ranking them employing the opinion (preference structure) of three experts: an acadamician, a field engineer and an official from Ministry of Water Resources, using fuzzy numbers. The fuzzy weights (w_i) of alternatives (A_i) are computed using standard fuzzy arithmetic. The concepts of maximizing set and minimizing set are introduced to decide total utility or order value of each of the alternatives. © 1998 Elsevier Science B.V. All rights reserved

Keywords: Basin planning; Decision making; Fuzzy numbers; Maximizing set; Minimizing set; Multicriteria analysis; Ordering systems; Ranking alternatives; Reservoir systems; Watershed planning

1. Introduction

Multi-objective analysis has developed in explicit form largely through Harvard Water Program (HWP) and its research findings were published by Mass et al. [17]. The political-decision process appropriate to many water resources (WR) problems was described by Major [18]. Haimes [13] set forth the principles of regional WR planning to assist the decision-making process at various hierarchical levels – local, state, regional and federal. On the international scene, the United Nations Industrial Development Organization (UNIDO) [21] has issued guidelines for project evaluation that take into account multiple objectives.

To consider both qualitative and quantitative criteria, Gershon et al. [12] have combined ELECTRE methodologies into an overall methodology of ranking alternative systems and applied to WR management

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study. This method was also applied to a large-scale WR system alternatives by David and Duckstein [9] and Anand Raj [3]. With the advent of fuzzy set theory [25] many methods have been developed in ranking the alternatives under fuzzy environment in the last two decades. Notable contributions include Jain's [14, 15] concept of maximizing set, membership level concept of Baas and Kwakernaak [6], three indices proposed by Yager [23, 24], index of strict preference defined by Watson et al. [22], four grades of dominance studied by Dubois and Prade [10, 11], Adamo's [2] preference rule and method of Kerre [16]. Cano et al. [7], Requena [19] and Requena et al. [20] have presented methods of ranking fuzzy numbers using Artificial Neural Networks (ANN).

Most of these methods suffer at least from one of the following drawbacks:

(i) The procedure is computationally complex and hence difficult in implementation.
(ii) Unintuitive which hinders implementation.
(iii) Assume one criteria or one expert.
(iv) Presupposes existence of some fuzzy relation, or other function, across the alternatives; or
(v) Produce a crisp ranking from fuzzy data.

To overcome these difficulties, the authors [5] have proposed a method which is intuitive, computationally simple and easy to implement. The proposed methodology is presented, in brief, in the following section.

2. Methodology

2.1. Problem

To rank m alternatives (A_i; $i = 1, 2, \dots, m$) by a Decision Maker (DM) with the help of information supplied by n experts (E_j; $j = 1, 2, \dots, n$) about the alternatives for each of k criteria (C_k; $k = 1, 2, \dots, K$) and also the relative importance of each criteria with respect to some overall objective.

2.2. Fuzzy numbers

Let $\tilde{a}_{ij}^k$ be the fuzzy number (see Fig. 1) assigned to alternative A_i by an expert E_j for criteria C_k and let $\tilde{c}_{kj}$ be the fuzzy number assigned to criteria C_k by expert E_j. Then these fuzzy numbers, a subset of F, are described by

$$\tilde{a}_{ij}^k = (\alpha_{ij}^k/\beta_{ij}^k, \gamma_{ij}^k/\delta_{ij}^k) \quad \text{and} \quad \tilde{c}_{kj} = (\varepsilon_{kj}/\zeta_{kj}, \eta_{kj}/\theta_{kj}), \tag{1}$$

where $\alpha < \beta < \gamma < \delta$ and $\varepsilon < \zeta < \eta < \theta \in \mathscr{L}(1, 2, \dots, L)$.

2.3. Membership functions

Let $\mu_{Ai}(x)$ and $\mu_{Ck}(x)$ be the membership (general triangular) function of $\tilde{a}_{ij}^k$ and $\tilde{c}_{kj}$, respectively. Then

$$\mu_{Ai}(x) = \begin{cases} 0 & x < \alpha_{ij}^k, \\ (x - \alpha_{ij}^k)/(\beta_{ij}^k - \alpha_{ij}^k) & \alpha_{ij}^k < x < \beta_{ij}^k, \\ v_i & \beta_{ij}^k < x < \gamma_{ij}^k, \\ (\delta_{ij}^k - x)/(\delta_{ij}^k - \gamma_{ij}^k) & \gamma_{ij}^k < x < \delta_{ij}^k, \\ 0 & x > \delta_{ij}^k. \end{cases} \tag{2}$$

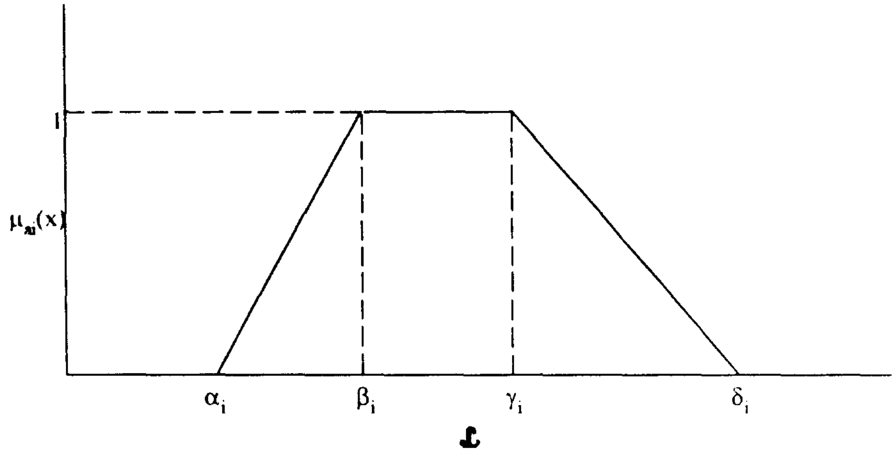


Fig. 1. Membership function of a fuzzy number.

A similar function can be written for $\mu_{Ck}(x)$. All this data can be summed up into the following matrices:

$$R_k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \overset{\begin{array}{cccc} E_1 & E_2 & \dots & E_n \end{array}}{\left[\quad \mu_{Ai}(x) = \tilde{a}_{ij}^k \in \mathscr{L} \quad \right]}, \tag{3}$$

$$R = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_K \end{array} \overset{\begin{array}{cccc} E_1 & E_2 & \dots & E_n \end{array}}{\left[\quad \mu_{Ck}(x) = \tilde{c}_{kj} \in \mathscr{L} \quad \right]}. \tag{4}$$

2.4. Fuzzy weights

Given the data of R and R_k, the DM computes the fuzzy weights ($\tilde{w}_i$; $i = 1, 2, \dots, m$) of all the alternatives using

$$\tilde{w}_i = (1/KL) \odot [(m_{i1} \odot n_1) \oplus (m_{i2} \odot n_2) \oplus \cdots \oplus (m_{ik} \odot n_k)], \tag{5}$$

where

$$\tilde{m}_{ik} = 1/n \odot [a_{i1}^k \oplus a_{i2}^k \oplus \cdots \oplus a_{in}^k] \quad \text{and} \quad \tilde{n}_k = 1/n \odot [c_{k1} \oplus c_{k2} \oplus \cdots \oplus c_{kn}].$$

$\oplus$ represents fuzzy addition, $\odot$ represents fuzzy multiplication. $\tilde{m}_{ik}$ and $\tilde{n}_k$ are simply the row averages of the matrices in Eqs. (3) and (4), respectively. For this purpose let us define

$$\alpha_{ik} = \sum \alpha_{ij}^k / n \quad \text{and} \quad \varepsilon_k = \sum \varepsilon_{kj}/n \quad \text{where } j = 1, 2, \dots, n. \tag{6}$$

Similar expressions can be written for $\beta_{ik}, \gamma_{ik}, \delta_{ik}, \zeta_k, \eta_k$ and θ_k. Then the fuzzy weight $\tilde{w}_i$ can be described as

$$\tilde{w}_i = (\alpha_i[L_{i1}, L_{i2}]/\beta_i, \gamma_i/\delta_i[U_{i1}, U_{i2}]), \tag{7}$$

where

$$\begin{aligned} &\alpha_i = \sum \alpha_{ik}\varepsilon_k/KL, \qquad \beta_i = \sum \beta_{ik}\zeta_k/KL,\\ &\gamma_i = \sum \gamma_{ik}\eta_k/KL, \qquad \delta_i = \sum \delta_{ik}\theta_k/KL, \quad k = 1, 2, \ldots, K, \end{aligned} \tag{8}$$

$$\begin{aligned} &L_{i1} = \sum (\beta_{ik} - \alpha_{ik})(\zeta_k - \varepsilon_k)/KL, \qquad L_{i2} = \sum \varepsilon_k(\beta_{ik} - \alpha_{ik}) + \alpha_{ik}(\zeta_k - \varepsilon_k)/KL,\\ &U_{i1} = \sum (\delta_{ik} - \gamma_{ik})(\theta_k - \eta_k)/KL, \qquad U_{i2} = -\sum \theta_k(\delta_{ik} - \gamma_{ik}) + \delta_{ik}(\theta_k - \eta_k)/KL, \quad k = 1, 2, \ldots, K. \end{aligned} \tag{9}$$

The membership function $\mu_{wi}(x)$ of $\tilde{w}_i$ is given by

$$\mu_{wi}(x) = \begin{cases} 0 & x < \alpha_i,\\ -L_{i2}/2L_{i1} + \{(L_{i2}/2L_{i1})^2 + (x - \alpha_i/L_{i1})\}^{1/2} & \alpha_i < x < L_{i1}y^2 + L_{i2}y + \alpha_i,\\ \omega_i & L_{i1}y^2 + L_{i2}y + \alpha_i < x < U_{i1}y^2 + U_{i2}y + \delta_i,\\ -U_{i2}/2U_{i1} - \{(-U_{i2}/2U_{i1})^2 + (x - \delta_i/U_{i1})\}^{1/2} & U_{i1}y^2 + U_{i2}y + \delta_i < x < \delta_i,\\ 0 & x > \delta_i. \end{cases} \tag{10}$$

The membership function of maximizing set, $\mu_{\mathrm{M}}(x)$ and minimizing set, $\mu_{\mathrm{m}}(x)$ are, respectively, given by

$$\begin{aligned} &\mu_{\mathrm{M}}(x) = \begin{cases} v\{(x - x_{\min})/(x_{\max} - x_{\min})\}^r & x_{\min} < x < x_{\max},\\ 0 & \text{otherwise}, \end{cases}\\ &\mu_{\mathrm{m}}(x) = \begin{cases} v\{(x - x_{\max})/(x_{\min} - x_{\max})\}^r & x_{\min} < x < x_{\max},\\ 0 & \text{otherwise}, \end{cases} \end{aligned} \tag{11}$$

where $v = \min_{1<i<m}(\omega_i)$; $x_{\max} = \sup_{1<i<m}(\delta_i)$; $x_{\min} = \inf_{1<i<m}(\alpha_i)$.

In the above equation sup represents supremum and inf refers to infimum. In case if $r = 1$ we get linear membership function; if $r = 2$ we get convex curved (risk-prone) membership function and if $r = \frac{1}{2}$ we get concave curved (risk-averse) membership function.

The total utility $U_{\mathrm{T}}(i)$ of the membership function $\mu_{wi}(x)$ is then defined as

$$U_{\mathrm{T}}(i) = \{U_{\mathrm{M}}(i) + 1 - U_{\mathrm{m}}(i)\}/2, \tag{12}$$

where $U_{\mathrm{M}}(i) = \sup_x\{\mu_{wi}(x) \cap \mu_{\mathrm{M}}(x)\}$ and $U_{\mathrm{m}}(i) = \sup_x\{\mu_{wi}(x) \cap \mu_{\mathrm{m}}(x)\}$.

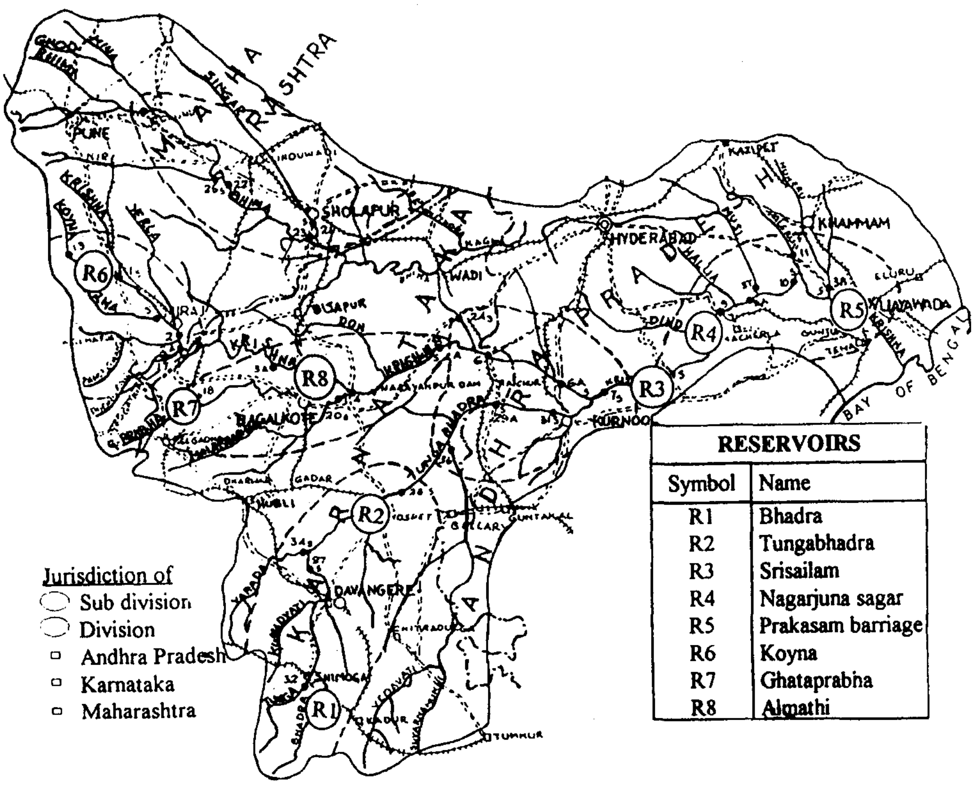
Using $U_{\mathrm{T}}(i)$ one can rank the alternatives. If two alternatives have the same utility values ($U_{\mathrm{T}}(1) = U_{\mathrm{T}}(2)$), one may use the vertices of the graphs of the corresponding membership functions to make the decision. That is, the vertex farther right is the largest, with decreasing size from right to left.

3. Application

The methodology developed is applied to Krishna river basin, which is one of the major peninsular river basins of India. River Krishna, the total length of which is 1400 km, rises from a spring at Mahabaleswar and flows through three states namely Maharastra, Karnataka and Andhra Pradesh. Its drainage area is about 260 000 sq km. Important tributaries are the Koyna, Ghataprabha, Malaprabha, Bhima and Tungabhadra. Salient features of these reservoirs are presented in Central Board of Irrigation and Power reports (CBIP, [8]).

Bhadra, Tungabhadra, Nagarjuna Sagar and Ghataprabha are dual purpose (irrigation and hydro power) projects while Srisailam and Koyna are hydro power projects and Almatti reservoir is an irrigation project. With increase in population densities and the number of industries in the river basin demand for water has increased enormously. This resulted in the need for the development of the existing and new reservoirs for the required water resource and to consider various objectives for the sustained development of the entire basin. This led to various problems in the basin such as waterlogging making a large portion of the irrigated areas unproductive, increase in alkalinity and salinity of subsoil resulting in health problems to live stock that consume the produce of the affected area, land submergence and associated rehabilitation problems, etc. A detailed assessment of these problems, both qualitative and quantitative, are presented by Abbasi [1] and in some of the reports of Government of India.

For the purpose of finding out the most suitable planning of the reservoirs with their associated purposes aimed at the development of the basin, Anand Raj and Nagesh Kumar [4] considered a total of 24 alternatives (i.e., various combinations of reservoirs, but do not represent all possible combinations) with 18 criteria falling under 8 main objectives. Of these 24 alternative systems, a subset of 7 alternatives were found to be preferred over others. These 7 alternatives (A_i; $i = 1, 2, \ldots, 7$) and 8 main objectives (C_k; $k = 1, 2, \ldots, 8$) are considered in this study. Krishna river basin, the location of the reservoirs, their names and the preferred alternatives are shown in Fig. 2. Table 1 gives the main objectives and the subcriteria considered.



Alternative Reservoir Systems: (A_1) R_3,R_4,R_5,R_8; (A_2) R_4,R_7,R_8; (A_3) R_2,R_3,R_8; (A_4) R_3,R_4,R_5; (A_5) R_1,R_6,R_7; (A_6) R_5,R_6,R_7,R_8; (A_7) R_4,R_5,R_8.

Fig. 2. Krishna river basin.

Table 1
Objectives of the study

Criteria	Main objective	Sub objective
C_1	National or regional development	Irrigation (lakh acres) Power generation (Th. MW) Relative regional techno, socio-economic improvement[a]
C_2	Water requirement	Quality of water[a] Annual sediment load (million tons) Gross storage capacity (Th. M. Cum.)
C_3	Flood protection	Max. flood discharge allowed (Th. Cumecs.) Expected frequency per year
C_4	Utilization of resources	Implementation costs (million Rs.) Operation and Maintenance costs (million Rs.) Natural resources[a]
C_5	Enhancement of environment[a]	Preservation of designated areas and existing facilities Effect on wildlife and vegetation Effect on land and environment Rehabilitation and submergence
C_6	Recreational enhancement	Tourism and recreational facilities[a]
C_7	Returns	Returns of the investment (million Rs.)
C_8	Flexibility	Flexibility of the system[a]

[a] Qualitative criteria (i.e., best; very good; good; average and worst).

Table 2
Evaluation of criteria by experts

Criteria	E_1	E_2	E_3
C_1	$(\frac{9}{10}, \frac{10}{10})$	$(\frac{10}{10}, \frac{10}{10})$	$(\frac{9}{9}, \frac{9}{10})$
C_2	$(\frac{5}{5}, \frac{6}{7})$	$(\frac{5}{6}, \frac{6}{6})$	$(\frac{6}{6}, \frac{6}{6})$
C_3	$(\frac{4}{5}, \frac{6}{6})$	$(\frac{4}{5}, \frac{5}{6})$	$(\frac{5}{5}, \frac{5}{5})$
C_4	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{2}{3}, \frac{3}{3})$	$(\frac{3}{3}, \frac{3}{3})$
C_5	$(\frac{5}{5}, \frac{5}{5})$	$(\frac{4}{5}, \frac{5}{6})$	$(\frac{5}{5}, \frac{5}{6})$
C_6	$(\frac{2}{2}, \frac{2}{2})$	$(\frac{2}{3}, \frac{3}{3})$	$(\frac{2}{2}, \frac{2}{2})$
C_7	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{8}{8}, \frac{8}{9})$	$(\frac{7}{8}, \frac{8}{8})$
C_8	$(\frac{1}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{3})$

For the evaluation of these alternatives, three experts (E_j; $j = 1, 2, 3$), one academician, one field engineer and an official from Ministry of WR are considered to give their opinion (preference structure) about these alternatives and criteria considered in the form of fuzzy numbers (fuzzy subsets) within a range 0–10 ($L = 10$). Experts are supplied with the required relevant information about the reservoirs, alternative systems and associated purposes, advantages and problems. They are also supplied with the relevant information about criteria before evaluation. These evaluations in the form of matrices (R and R_k; $k = 1, 2, \ldots, 8$) are given in Tables 2 and 3(a)–(h).

Table 3(a)
Evaluation of alternatives by experts for criteria C_1

Alternative	E_1	E_2	E_3
A_1	$(\frac{5}{6}, \frac{6}{7})$	$(\frac{6}{6}, \frac{6}{7})$	$(\frac{5}{6}, \frac{6}{6})$
A_2	$(\frac{2}{3}, \frac{3}{4})$	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{2}{3}, \frac{4}{4})$
A_3	$(\frac{5}{6}, \frac{6}{7})$	$(\frac{5}{6}, \frac{6}{6})$	$(\frac{5}{6}, \frac{7}{8})$
A_4	$(\frac{6}{7}, \frac{8}{9})$	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{6}{7}, \frac{7}{8})$
A_5	$(\frac{5}{5}, \frac{5}{6})$	$(\frac{5}{5}, \frac{5}{6})$	$(\frac{5}{5}, \frac{5}{5})$
A_6	$(\frac{3}{4}, \frac{4}{5})$	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{3}{4}, \frac{5}{6})$
A_7	$(\frac{4}{5}, \frac{5}{6})$	$(\frac{5}{5}, \frac{5}{6})$	$(\frac{5}{5}, \frac{5}{5})$

Table 3(b)
Evaluation of alternatives by experts for criteria C_2

Alternative	E_1	E_2	E_3
A_1	$(\frac{5}{6}, \frac{6}{7})$	$(\frac{5}{6}, \frac{6}{6})$	$(\frac{6}{6}, \frac{7}{7})$
A_2	$(\frac{6}{7}, \frac{7}{8})$	$(\frac{7}{7}, \frac{8}{8})$	$(\frac{8}{8}, \frac{8}{8})$
A_3	$(\frac{3}{4}, \frac{4}{5})$	$(\frac{4}{4}, \frac{5}{5})$	$(\frac{4}{4}, \frac{4}{4})$
A_4	$(\frac{6}{7}, \frac{8}{9})$	$(\frac{8}{8}, \frac{8}{8})$	$(\frac{8}{9}, \frac{9}{9})$
A_5	$(\frac{3}{3}, \frac{3}{3})$	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{2}{3}, \frac{3}{4})$
A_6	$(\frac{4}{4}, \frac{5}{5})$	$(\frac{4}{4}, \frac{4}{5})$	$(\frac{4}{5}, \frac{5}{6})$
A_7	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{7}{7}, \frac{7}{7})$	$(\frac{8}{8}, \frac{8}{8})$

Table 3(c)
Evaluation of alternatives by experts for criteria C_3

Alternative	E_1	E_2	E_3
A_1	$(\frac{6}{6}, \frac{7}{7})$	$(\frac{7}{7}, \frac{7}{8})$	$(\frac{6}{6}, \frac{7}{8})$
A_2	$(\frac{5}{6}, \frac{6}{7})$	$(\frac{5}{6}, \frac{6}{6})$	$(\frac{6}{7}, \frac{7}{7})$
A_3	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{2})$	$(\frac{3}{3}, \frac{3}{3})$
A_4	$(\frac{8}{9}, \frac{9}{9})$	$(\frac{8}{8}, \frac{8}{8})$	$(\frac{7}{8}, \frac{8}{9})$
A_5	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{3}{3}, \frac{4}{4})$	$(\frac{3}{3}, \frac{3}{4})$
A_6	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{4}{5}, \frac{5}{5})$	$(\frac{5}{5}, \frac{5}{5})$
A_7	$(\frac{7}{8}, \frac{8}{8})$	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{6}{7}, \frac{7}{7})$

Table 3(d)
Evaluation of alternatives by experts for criteria C_4

Alternative	E_1	E_2	E_3
A_1	$(\frac{4}{5}, \frac{5}{6})$	$(\frac{3}{4}, \frac{5}{6})$	$(\frac{4}{4}, \frac{5}{5})$
A_2	$(\frac{3}{4}, \frac{4}{5})$	$(\frac{4}{4}, \frac{4}{4})$	$(\frac{3}{4}, \frac{4}{4})$
A_3	$(\frac{2}{3}, \frac{3}{3})$	$(\frac{3}{3}, \frac{3}{3})$	$(\frac{2}{2}, \frac{2}{2})$
A_4	$(\frac{6}{7}, \frac{7}{8})$	$(\frac{7}{7}, \frac{7}{8})$	$(\frac{8}{8}, \frac{8}{8})$
A_5	$(\frac{8}{8}, \frac{9}{9})$	$(\frac{8}{8}, \frac{9}{10})$	$(\frac{8}{9}, \frac{9}{9})$
A_6	$(\frac{3}{3}, \frac{3}{3})$	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{3}{3}, \frac{3}{3})$
A_7	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{6}{7}, \frac{8}{9})$	$(\frac{7}{8}, \frac{8}{8})$

Table 3(e)
Evaluation of alternatives by experts for criteria C_5

Alternative	E_1	E_2	E_3
A_1	$(\frac{5}{6}, \frac{6}{7})$	$(\frac{4}{5}, \frac{6}{7})$	$(\frac{6}{6}, \frac{6}{6})$
A_2	$(\frac{3}{3}, \frac{4}{4})$	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{4}{4}, \frac{4}{4})$
A_3	$(\frac{6}{7}, \frac{7}{8})$	$(\frac{7}{7}, \frac{8}{8})$	$(\frac{6}{7}, \frac{8}{8})$
A_4	$(\frac{3}{4}, \frac{4}{5})$	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{3}{3}, \frac{4}{4})$
A_5	$(\frac{4}{5}, \frac{5}{6})$	$(\frac{4}{5}, \frac{5}{5})$	$(\frac{5}{6}, \frac{6}{6})$
A_6	$(\frac{6}{6}, \frac{7}{7})$	$(\frac{6}{7}, \frac{7}{8})$	$(\frac{6}{7}, \frac{7}{7})$
A_7	$(\frac{2}{3}, \frac{4}{5})$	$(\frac{2}{3}, \frac{3}{4})$	$(\frac{3}{3}, \frac{3}{3})$

Table 3(f)
Evaluation of alternatives by experts for criteria C_6

Alternative	E_1	E_2	E_3
A_1	$(\frac{2}{3}, \frac{3}{4})$	$(\frac{2}{3}, \frac{3}{4})$	$(\frac{3}{3}, \frac{3}{4})$
A_2	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{3}{3}, \frac{4}{4})$	$(\frac{3}{3}, \frac{3}{3})$
A_3	$(\frac{3}{3}, \frac{3}{3})$	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{2})$
A_4	$(\frac{3}{3}, \frac{4}{4})$	$(\frac{2}{2}, \frac{3}{3})$	$(\frac{3}{3}, \frac{3}{3})$
A_5	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{6}{7}, \frac{8}{9})$	$(\frac{7}{7}, \frac{8}{8})$
A_6	$(\frac{3}{3}, \frac{3}{4})$	$(\frac{3}{3}, \frac{3}{4})$	$(\frac{3}{4}, \frac{4}{4})$
A_7	$(\frac{3}{4}, \frac{4}{4})$	$(\frac{2}{3}, \frac{3}{3})$	$(\frac{3}{4}, \frac{4}{4})$

Table 3(g)
Evaluation of alternatives by experts for criteria C_7

Alternative	E_1	E_2	E_3
A_1	$(\frac{7}{7}, \frac{8}{8})$	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{6}{7}, \frac{7}{8})$
A_2	$(\frac{5}{6}, \frac{7}{8})$	$(\frac{5}{6}, \frac{6}{7})$	$(\frac{6}{6}, \frac{7}{8})$
A_3	$(\frac{2}{3}, \frac{3}{4})$	$(\frac{2}{3}, \frac{4}{5})$	$(\frac{2}{3}, \frac{3}{3})$
A_4	$(\frac{8}{8}, \frac{8}{9})$	$(\frac{7}{8}, \frac{8}{9})$	$(\frac{8}{8}, \frac{9}{9})$
A_5	$(\frac{6}{7}, \frac{7}{8})$	$(\frac{6}{6}, \frac{7}{7})$	$(\frac{6}{7}, \frac{7}{7})$
A_6	$(\frac{4}{5}, \frac{5}{6})$	$(\frac{4}{5}, \frac{6}{6})$	$(\frac{5}{5}, \frac{5}{5})$
A_7	$(\frac{8}{9}, \frac{9}{9})$	$(\frac{7}{8}, \frac{8}{8})$	$(\frac{6}{7}, \frac{8}{9})$

Table 3(h)
Evaluation of alternatives by experts for criteria C_8

Alternative	E_1	E_2	E_3
A_1	$(\frac{3}{3}, \frac{3}{3})$	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{2})$
A_2	$(\frac{2}{2}, \frac{2}{2})$	$(\frac{1}{2}, \frac{2}{3})$	$(\frac{1}{1}, \frac{1}{1})$
A_3	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{2})$	$(\frac{2}{2}, \frac{2}{2})$
A_4	$(\frac{2}{2}, \frac{2}{3})$	$(\frac{1}{1}, \frac{2}{2})$	$(\frac{2}{2}, \frac{2}{2})$
A_5	$(\frac{1}{2}, \frac{2}{3})$	$(\frac{1}{2}, \frac{2}{2})$	$(\frac{1}{2}, \frac{2}{2})$
A_6	$(\frac{2}{2}, \frac{2}{2})$	$(\frac{1}{2}, \frac{2}{3})$	$(\frac{2}{2}, \frac{2}{2})$
A_7	$(\frac{1}{1}, \frac{1}{2})$	$(\frac{2}{2}, \frac{2}{2})$	$(\frac{2}{3}, \frac{3}{3})$

Table 4
Pooling and averaging across experts

		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
(a) For all the alternatives of each criteria									
	A_1	$(\frac{5.33}{6.00},\frac{6.00}{6.67})$	$(\frac{5.33}{6.00},\frac{6.33}{6.67})$	$(\frac{6.33}{6.33},\frac{7.00}{7.67})$	$(\frac{3.67}{4.55},\frac{5.00}{5.67})$	$(\frac{5.00}{5.67},\frac{6.00}{6.67})$	$(\frac{2.33}{3.00},\frac{3.00}{4.00})$	$(\frac{6.67}{7.33},\frac{7.67}{8.33})$	$(\frac{2.33}{2.33},\frac{2.33}{2.67})$
	A_2	$(\frac{2.33}{3.33},\frac{3.67}{4.00})$	$(\frac{7.00}{7.33},\frac{7.67}{8.00})$	$(\frac{5.33}{6.33},\frac{6.33}{6.67})$	$(\frac{3.33}{4.00},\frac{4.00}{4.33})$	$(\frac{3.33}{3.67},\frac{4.00}{4.00})$	$(\frac{3.00}{3.33},\frac{3.67}{3.67})$	$(\frac{5.33}{6.00},\frac{6.67}{7.67})$	$(\frac{1.33}{1.67},\frac{1.67}{2.00})$
	A_3	$(\frac{5.00}{6.00},\frac{6.33}{7.00})$	$(\frac{3.67}{4.00},\frac{4.33}{4.67})$	$(\frac{2.33}{2.33},\frac{2.33}{2.67})$	$(\frac{2.33}{2.67},\frac{2.67}{2.67})$	$(\frac{6.33}{7.00},\frac{7.67}{8.00})$	$(\frac{2.33}{2.33},\frac{2.33}{2.67})$	$(\frac{2.00}{3.00},\frac{3.33}{4.00})$	$(\frac{2.00}{2.00},\frac{2.00}{2.33})$
$\tilde{m}_{ik}$	A_4	$(\frac{6.33}{7.33},\frac{7.67}{8.67})$	$(\frac{7.33}{8.00},\frac{8.33}{8.67})$	$(\frac{7.67}{8.33},\frac{8.33}{8.67})$	$(\frac{7.00}{7.33},\frac{7.33}{8.00})$	$(\frac{3.00}{3.67},\frac{4.00}{4.33})$	$(\frac{2.67}{2.67},\frac{3.33}{3.33})$	$(\frac{7.67}{8.00},\frac{8.33}{9.00})$	$(\frac{1.67}{1.67},\frac{2.00}{2.33})$
	A_5	$(\frac{5.00}{5.00},\frac{5.00}{5.67})$	$(\frac{2.67}{3.33},\frac{3.33}{3.67})$	$(\frac{3.00}{3.33},\frac{3.67}{4.00})$	$(\frac{8.00}{8.33},\frac{9.00}{9.33})$	$(\frac{4.33}{5.33},\frac{5.33}{5.67})$	$(\frac{6.67}{7.33},\frac{8.00}{8.67})$	$(\frac{6.00}{6.67},\frac{7.00}{7.33})$	$(\frac{1.00}{2.00},\frac{2.00}{2.33})$
	A_6	$(\frac{3.00}{4.00},\frac{4.33}{5.00})$	$(\frac{4.00}{4.33},\frac{4.67}{5.33})$	$(\frac{4.00}{4.67},\frac{4.67}{4.67})$	$(\frac{3.00}{3.33},\frac{3.33}{3.33})$	$(\frac{6.00}{6.67},\frac{7.00}{7.33})$	$(\frac{3.00}{3.33},\frac{3.33}{4.00})$	$(\frac{4.33}{5.00},\frac{5.33}{5.67})$	$(\frac{1.67}{2.00},\frac{2.00}{2.33})$
	A_7	$(\frac{4.67}{5.00},\frac{5.00}{5.67})$	$(\frac{7.33}{7.67},\frac{7.67}{8.00})$	$(\frac{6.67}{7.67}.\frac{7.67}{8.00})$	$(\frac{6.67}{7.67},\frac{8.00}{8.67})$	$(\frac{2.33}{3.00},\frac{3.33}{4.00})$	$(\frac{2.67}{3.67},\frac{3.67}{3.67})$	$(\frac{7.00}{8.00},\frac{8.33}{8.67})$	$(\frac{1.67}{2.00},\frac{2.00}{2.33})$
(b) For all the criteria									
$\tilde{n}_k$		$(\frac{9.33}{9.67},\frac{9.67}{10.00})$	$(\frac{5.33}{5.67},\frac{6.00}{6.33})$	$(\frac{4.33}{5.00},\frac{5.33}{5.67})$	$(\frac{2.33}{2.67},\frac{2.67}{3.00})$	$(\frac{4.67}{5.00},\frac{5.00}{5.67})$	$(\frac{2.00}{2.33},\frac{2.33}{2.33})$	$(\frac{7.33}{8.00},\frac{8.00}{8.67})$	$(\frac{1.66}{2.00},\frac{2.00}{3.00})$

4. Results and discussions

The importance of the problem lies in the fact that these fuzzy numbers (or fuzzy subsets) can be obtained incorporating the intuitive knowledge and experience of the experts to represent the overall utilities (or values or suitabilities) of a set of alternatives. A comparison between these subsets is therefore a comparison between the alternatives, i.e., a decision-making procedure. The idea is to select an alternative, given a set of fuzzy subsets that properly represent a group of them. To carry out this task we require information from the experts as to how well each alternative satisfies each criterion and also how important each criterion is to the overall objective. Each expert has a fuzzy number (fuzzy subset) defined over each alternative, for each criterion, with values in the linearly ordered set $\mathscr{L}$. Also, each expert has a fuzzy number defined over each of the criterion, with values in $\mathscr{L}$ expressing their relative importance. Then these fuzzy numbers are aggregated into a fuzzy set (fuzzy weight) on the alternatives with values in $\mathscr{L}$ so that the final utility values are determined. These utility values gives the ranking (or ordering) of the alternatives.

It can be seen from Table 2 that all the experts have given highest priority to C_1 and then to C_7, while least priority is given for C_6 and C_8. E_3 has given equal priority to alternatives A_5 and A_7, while E_1 gave equal priority to A_1 and A_3 (see Table 3(a)). Similarly, all the experts have given highest priority to A_4 for all the criteria except C_5 and C_6. For C_5 experts gave highest priority to A_3 and A_5, while for C_6, A_5 was given highest priority. All the experts are more or less consistent in giving low values for all the alternatives for criteria C_8, meaning that it is the criteria that has lowest priority (i.e., small weight). With the data given by experts $\tilde{m}_{ik}$ and $\tilde{n}_k$ are found from Eq. (5) and are presented in Table 4. Using Eqs. (6), (8) and (9) fuzzy weights of all the alternatives are determined and are given by

$$\tilde{w}_1 = (2.438[0.019, 0.467]/2.924, 3.121/3.708[0.025, \ -0.613]),$$

$$\tilde{w}_2 = (1.911[0.026, 0.482]/2.419, 2.639/3.064[0.018, \ -0.443]),$$

$$\tilde{w}_3 = (1.675[0.018, 0.406]/2.099, 2.265/2.699[0.018, \ -0.451]),$$

$$\tilde{w}_4 = (2.826[0.019, 0.521]/3.367, 3.582/4.150[0.022, \ -0.590]),$$

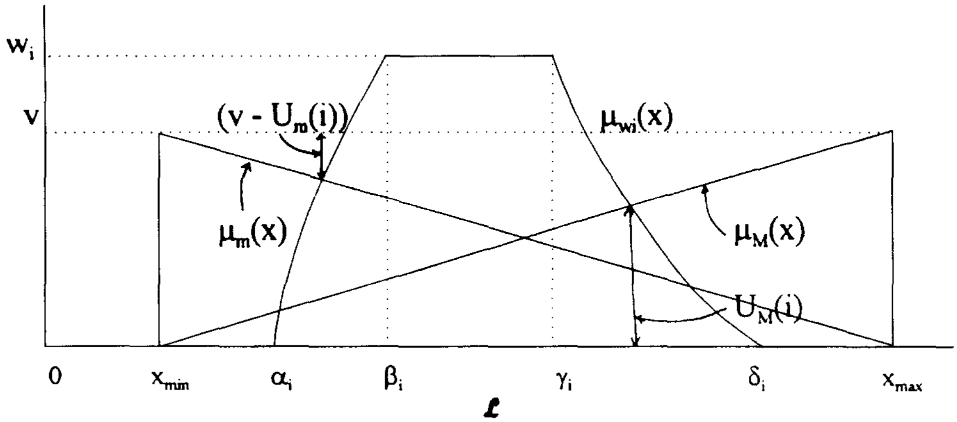


Fig. 3. Membership function of fuzzy weight, maximizing set and minimizing sets.

Table 5
Ranking and utility values of the alternatives

Rank	Utility value	Alternative
1	0.68583	A_4
2	0.55323	A_7
3	0.54109	A_1
4	0.41023	A_5
5	0.36263	A_2
6	0.29805	A_6
7	0.24796	A_3

$$\tilde{w}_5 = (2.147[0.024, 0.419]/2.590, 2.715/3.168[0.017, -0.469]),$$

$$\tilde{w}_6 = (1.778[0.024, 0.456]/2.257, 2.414/2.840[0.015, -0.442]),$$

$$\tilde{w}_7 = (2.468[0.032, 0.526]/3.026, 3.156/3.650[0.021, -0.515]),$$

and $x_{\min} = 1.675$; $x_{\max} = 4.150$ and $v = 1.0$.

The weights of the alternatives are in the form of a fuzzy subset (with a general triangular membership function). The relative importance (and so the difference in them) depends on the entire support but not on a single value. Considering $r = 1$, the membership functions of maximizing set and minimizing set are given by

$$\mu_{\mathrm{M}}(x) = \begin{cases} v\{(x - 1.675)/(4.150 - 1.675)\} & 1.675 < x < 4.150, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_{\mathrm{m}}(x) = \begin{cases} v\{(x - 4.150)/(1.675 - 4.150)\} & 1.675 < x < 4.150, \\ 0 & \text{otherwise.} \end{cases}$$

A typical graphical representation of the membership function of fuzzy weight, maximizing set and minimizing set are shown in Fig. 3. Using Eq. (12), total utility values are determined and the alternatives are ranked as shown in Table 5. Two utility values very close (i.e., differed by ε) indicate that the suitability (order) values of the two alternatives in consideration are more or less equal. The final ranking not only depends on the utility values but also on the vertices of the membership functions of the respective alternatives. Even if the two utility values are equal, the alternative that has the vertex of its membership function to the right (i.e., indicating that it has more weightage) is ranked superior over the other. To get clear distinction among such alternatives closer and rigorous analysis with some more information could be done on the respective alternatives and then the final decision could be taken.

5. Conclusions

In this paper an attempt is made to apply the methodology proposed by the authors to one of the major peninsular river basins (Krishna river basin) of India. Ranking of the river basin planning and development alternatives under multi-criterion environment using fuzzy numbers is presented. The purpose is to find the most suitable planning of the reservoirs with the associated purposes aimed at the sustained development of the basin. A set of 7 preferred alternative systems with 8 main objectives are considered. It is found that alternative A_4 is the best alternative, A_7 is the next while alternative A_3 is ranked as the last.

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