ELSEVIER

Signal Processing 63 (1997) 229–240

SIGNAL PROCESSING

Mean convergence behavioural analysis of IIR adaptive filters

R.V. Raja Kumar[a,*], C.B. Rama Rao[b]

[a] Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur 721302, India
[b] Department of Electronics and Communication Engineering, Regional Engineering College, Warangal 506004, India

Received 9 December 1994; received in revised form 27 August 1997

Abstract

In spite of having several advantages, IIR adaptive filters have not been getting their due share in applications because of the need for stability monitoring during adaptation and uncertainty in convergence time for stochastic inputs which can be mainly attributed to the involved nonquadratic criterion function. Because of this type of criterion function, it has been very difficult to estimate the nature of convergence in the stochastic frame work. Recently, it is shown that the ensemble mean parameter updating equations of the IIR adaptive algorithms can be represented by the associated ordinary differential equations (ODEs). In this paper a method of solving the ODEs in order to analyse the mean convergence behaviour of these filters, given the mean description of the input in the form of power spectral density is presented. Further, this procedure is applied to study the convergence behaviour of general IIR adaptive filters. Effectiveness of this method is shown through several analytical and simulation results obtained from two adaptive filtering examples. © 1997 Elsevier Science B.V. All rights reserved.

Zusammenfassung

Adaptive IIR-Filter haben trotz ihrer verschiedenen Vorteile keine große Bedeutung in den Anwendungen gefunden. Dies ist begründet durch die Notwendigkeit, während der Adaption die Stabilität zu beobachten sowie durch die Unsicherheit bezüglich der Konvergenzdauer bei stochastischen Eingängen, was vorwiegend auf die verwendete nicht-quadratische Kriteriumsfunktion zurückzuführen ist. Wegen dieses Typs von kriteriumsfunktion war es sehr schwierig, die Art der Konvergenz im stochastischen Rahmen zu analysieren. Vor kurzem wurde gezeigt, daß die über das Ensemble gemittelten Parameterupdate-Gleichungen des adaptiven IIR-Algorithmus durch die zugeordneten gewöhnlichen Differentialgleichungen (ODE's) repräsentiert werden können. In der vorliegenden Arbeit wird eine Methode zur Lösung der ODE's vorgestellt. Diese Methode dient zur Analyse des mittleren Konvergenzverhaltens dieser Filter bei bekannter mittlerer Beschreibung des Eingangs durch die spektrale Leistungsdichte. Das Verfahren wird weiters auf die Untersuchung des Konvergenzverhaltens allgemeiner adaptiver IIR-Filter angewandt. Die Effektivität dieser Methode wird durch mehrere analytische und Simulationsergebnisse bewiesen, welche für zwei Beispiele adaptiven Filterns erhalten wurden. © 1997 Elsevier Science B.V. All rights reserved.

* Corresponding author. Tel.: + 91 3222 55221; fax: + 91 3222 55303; e-mail: rkumar@ece.iitkgp.ernet.in.

0165-1684/97/$17.00
PII S0165-1684(97)00159-X

Résumé

Bien que présentant de nombreux avantages, les filtres adaptatifs IIR ne sont guère utilisés dans les applications du f de la nécessité de contrôler leur stabilité durant l'adaptation et de l'incertitude sur le temps de convergence pour d entrées stochastiques, qui peut être principalement attribuée à leur fonction critère non quadratique complexe. Ce fonction critère rend très difficile l'estimation de la nature de la convergence dans un contexte stochastique. Il a é montré récemment que les équations de mise à jour des paramètres en moyenne d'ensemble des algorithmes adaptat IIR peuvent être représentées par les équations différentielles ordinaires (ODE) associées. Nous présentons dans c atricle une méthode de résolution de ces ODE permettant d'analyser le comportement de convergence moyen de c filtres, sur la base d'une description moyenne de l'entrée sous la forme de sa densité spectrale de puissance. Subséquemme cette procédure est appliquée à l'étude du comportement de convergence de filtres IIR adaptatifs généraux. L'efficacité cette méthode est montrée par plusieurs résultats analytiques et de simulation obtenus sur deux exemples de filtra adaptatif.

Keywords: Adaptive filter; Recursive algorithms; ODE; Convergence; Parameter trajectory

1. Introduction

The last decade has seen an increased interest in the area of IIR adaptive filters [1, 3, 5–8, 11, 19, 20]. It has been due to the following advantages of IIR adaptive filters over their FIR counter parts. In many situations, it is possible to approximate an FIR filter of a large order by an IIR filter of a much lower order. Further, an IIR filter may represent an optimum structure (ARMA or ARMAX) for many of the signals such as an AR signal process in noise which is commonly encountered in practice [7, 11]. The main drawbacks of an IIR adaptive filter are the need for stability monitoring during adaptation, and the uncertainty in the convergence time for stochastic inputs, which is as a result of the involved nonquadratic performance surface. The nature of convergence and convergence time may be different even for the same input signal when the noise sample set is different. In the case of IIR adaptive filters, while handling stochastic signals, it is very difficult to estimate the nature of convergence and convergence time. If the estimator dynamics is known, some idea regarding the stability and the convergence time required can be obtained. Therefore in such situations it is of interest to estimate the nature including the time of ensemble mean convergence, at least.

In this paper, we have presented a method of analysing the ensemble mean-convergence behaviour of recursive adaptive filters, given the mean description of the input. The basis for this analysis is the ordinary differential equation (ODE) repre entation of the recursive adaptive filters [10].

Originally, the ODE approach was presented l Ljung for the convergence analysis of recursi identification algorithms [10]. It was shown th the asymptotic behaviour of an identificati algorithm can be obtained by solving the OD corresponding to the identification algorith The recursive algorithms use an adaptation ste size which tends to zero as time tends to infini Whereas in an adaptive filter the step-si is a constant so that the algorithm can tra the variations in the input. Later, the ODE a proach was extended to IIR adaptive filte [2, 4, 13–16, 18]. In [14] it was shown that t ensemble mean behaviour of IIR adaptive filte can be represented by their coresponding ODE Earlier, we applied this approach to study t convergence behaviour of constrained IIR adapti filters [15, 16].

In this paper, we applied this approach to t general IIR adaptive filters and derived a procedu of obtaining the solution to the ordinary differenti equation representation of the adaptive algorithn given the power spectral density of the input. Fu ther, this procedure is applied to study the conve gence behaviour of general IIR adaptive filters. T ODEs representing an IIR adaptive filtering alg rithm are nonlinear and it is extremely difficult obtain a general closed-form expression for conve gence time and nature of convergence. It is or possible to evaluate and find solutions for speci filtering problems using numerical procedures. Hence, a method of solving the ODEs given the mean description of the input is derived in this paper. This analytical method involves computation of the ODE solutions numerically which can be applied to study the convergence behaviour. This analysis provides a means to obtain a good idea about the nature of parameter adjustment, stability and convergence time. Effectiveness of this method is shown through several analytical and simulation results obtained from adaptive filtering examples.

This paper is organised as follows. In Section 2 the formulation for an IIR adaptive predictor and the algorithm recursions of the recursive maximum likelihood (RML) algorithm are presented. In Section 3, the ODEs of the adaptive algorithms are given. A method of obtaining the ODE solution is also presented in this section. Section 4 describes some analytical and simulation results obtained from two problems of adaptive filtering. The important concluding remarks drawn from this work are given in Section 5.

2. The formulation of an IIR adaptive predictor

Let us consider the problem of predicting an ARMA process as shown in Fig. 1 for the derivation of the analytical method. When the ARMA process consists of a deterministic signal in noise, output of the prediction filter yields enhanced output [20]. Let the prediction filter of order $(p-1, r)$ be given as

$$H(z) = \frac{B(z)}{C(z)}, \tag{1}$$

where $B(z) = b_1 + b_2 z^{-1} + \cdots + b_p z^{-p+1}$ and $C(z) = 1 + c_1 z^{-1} + \cdots + c_r z^{-r}$.

Then the prediction error filter corresponding to $H(z)$ will be of order (p,r) which can be expressed as

$$G(z) = 1 - z^{-1} H(z). \tag{2}$$

In view of Eqs. (1) and (2) the outputs of the prediction and prediction error filters, respectively, can be expressed as

$$\hat{x}_t = \phi_t^{\mathrm{T}} \theta_{t-1} \tag{3}$$

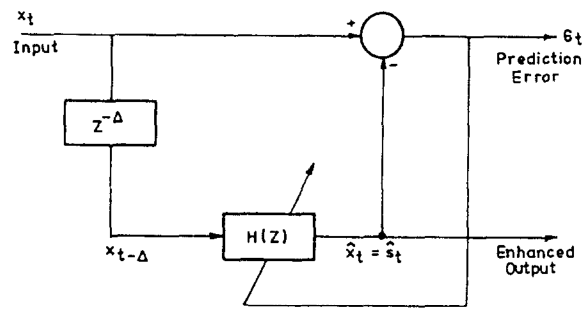

Fig. 1. Block diagram of the Δ-step predictor for the adaptive enhancement of a signal corrupted by white noise.

and

$$\varepsilon_t = x_t - \hat{x}_t, \tag{4}$$

where $\theta_t = [b_1 b_2 \ldots b_p c_1 c_2 \ldots c_r]^{\mathrm{T}}$ and $\phi_t = [x_{t-1} x_{t-2} \ldots x_{t-p} - \hat{x}_{t-1} \ldots - \hat{x}_{t-r}]^{\mathrm{T}}$. In a time-varying situation b_i's and c_i's are time-dependent. For simplicity of notation, the time-dependence is suppressed from the equations.

The RML algorithm recursions for estimating θ_t can be given as [7, 11]

$$\varepsilon_t = x_t - \phi_t^{\mathrm{T}} \hat{\theta}_{t-1}, \tag{5a}$$

$$\hat{P}_t = \frac{1}{\lambda}\left[\hat{P}_{t-1} - \frac{\hat{P}_{t-1}\psi_t\psi_t^{\mathrm{T}}\hat{P}_{t-1}}{\lambda + \psi_t^{\mathrm{T}}\hat{P}_{t-1}\psi_t}\right], \tag{5b}$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \mu \hat{P}_t \psi_t \varepsilon_t, \tag{5c}$$

where $\psi_t = \phi_t / D(q)$ and

$$D(q) = 1 + \hat{k}\hat{c}_1 q^{-1} + \cdots + \hat{k}^r \hat{c}_r q^{-r}. \tag{5d}$$

Here q is the delay operator and λ is the forgetting factor $(0 < \lambda < 1)$ used for computing the covarience matrix $\hat{P}_t$ recursively and μ is the adaptation step-size. $\hat{k}$ is an algorithm parameter which controls the transient behaviour of the algorithm. Usually $\hat{k}$ is taken as 1. Here Eq. 5(d) implies that ψ_t is a vector whose individual elements are obtained by filtering the corresponding elements of ϕ_t by the all pole filter with polynomial $[1/D(q)]$ whose z-transform is given by $1/D(z)$. When the adaptation is slow, however, ψ_t can be approximated to be consisting of the delayed samples of $x_t/D(q)$ and $\hat{x}_t/D(q)$. This algorithm is initialized by $\hat{P}(0) = \alpha I$ and $\hat{\theta}(0) = 0$ where α is a suitable scalar. In the Gauss–Newton algorithm form, (5) can be expressed as [7, 11]

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \mu \hat{R}_t^{-1} \psi_t \varepsilon_t,$$
$$\hat{R}_t = \lambda \hat{R}_{t-1} + \psi_t \psi_t^{\mathrm{T}}, \tag{6}$$

where $\hat{R}_t = P_t^{-1}$ is the Hessian matrix.

3. Convergence behavioural analysis using the ODE representation [9]

3.1. ODE representation

In [14], it is shown that the ensemble mean updating equations of Eqs. (5b) and (5c) can be represented in the following ordinary differential equation form:

$$\frac{\mathrm{d}\theta_t}{\mathrm{d}t} = \mu P_t E[\psi(\theta_t)\varepsilon(\theta_t)], \tag{7a}$$

$$\frac{\mathrm{d}P_t}{\mathrm{d}t} = \left(\frac{1}{\lambda} - 1\right)P_{t-1} - \frac{P_{t-1}E[\psi(\theta_t)\psi^{\mathrm{T}}(\theta_t)]P_{t-1}}{\lambda + E[\psi^{\mathrm{T}}(\theta_t)]P_{t-1}\psi(\theta_t)}. \tag{7b}$$

That is, the behaviour of the discrete-time algorithm is expressed in terms of the corresponding continuous-time ODE.

Here the denominator in Eq. (7b) involves computation of $E[\psi^{\mathrm{T}}(\theta_t)P_{t-1}\psi(\theta_t)]$ which can be expressed as

$$\sum_{i=1}^{p} \sum_{k=1}^{r} p_{ik} E[\psi(i,\theta_t)\psi(k,\theta_t)],$$

where p_{ik} is the (i,k)th element of the matrix P_{t-1} and $\psi(i,\theta_t)$ is the ith element of the vector $\psi(\theta_t)$.

While the ODEs corresponding to the recursive identification algorithms represent the asymptotic behaviour [10], the ODEs corresponding to the adaptive filters represent the ensemble mean behaviour [14]. The constant step-size used in adaptive filters is responsible for this. In addition to the regularity conditions required for the ODE representation given in [10], the ensemble mean representation requires an additional assumption on the input and the adaptive filter. It is necessary that the transient response of the adaptive filter dies down before the input changes appreciably. This means that there is a restriction on the rate at which the input signal characteristics can vary. That is, the rate of variation in the characteristics of the input should be much smaller than the inverse of the dominating time constant of the adaptive filter. Further in [14] it is shown that the solutions of the ODEs represent the mean trajectories of the parameter estimates, $\hat{\theta}_t$. It also means that the algorithms converge in mean to the stable stationary points of the ordinary differential equations.

3.2. Mean-convergence behavioural analysis

As explained in the Introduction, the convergence behaviour of an adaptive filter in a specific application can be obtained by solving the ODEs corresponding to the algorithm, given the ensemble-mean description of the input such as the power spectral density (psd). When the input to be estimated is an ARMA process, the expectations of the gradient and the covariance matrix are nonlinear functions of θ_t. Then the differential equations are nonlinear by nature. Therefore, to obtain analytical solutions, given a general input description, we have to resort to numerical procedures.

For solving these equations it is necessary to compute the expectations of $[\psi(\hat{\theta}_t)\psi^{\mathrm{T}}(\hat{\theta}_t)]$, $[\psi(\hat{\theta}_t)\varepsilon(\hat{\theta}_t)]$ and $[\psi(i,\hat{\theta}_t)\psi(k,\hat{\theta}_t)]$ for $1 \leqslant i \leqslant p$, $1 \leqslant k \leqslant r$. Expressions for these are derived in the following way.

Let the psd of the discrete input be given as

$$\Phi_{xx}(z) = X(z)X^{*}(z). \tag{8}$$

In view of Eqs. (2), (4) and (5d), the z-transforms of the error output of the prediction filter and the regression vector can be expressed as

$$\epsilon(z) = G(z)X(z), \tag{9}$$

$$\Psi(z) = \frac{1}{D(z)}\Phi(z), \tag{10}$$

where $\Phi(z) = [z^{-1}X(z) \ldots z^{-(p-1)}X(z) - z^{-1}H(z)X(z) \ldots -z^{-r}H(z)X(z)]$. Then the expectations required for solving Eqs. (7a) and (7b) can be expressed in terms of $\in(z)$ and $\Psi(z)$ as

$$E[\psi(\theta_t)\psi^{\mathrm{T}}(\theta_t)] = \frac{1}{2\pi \mathrm{j}}\oint \Psi(z)\psi^{+}(z)\frac{\mathrm{d}z}{z}, \tag{11}$$

$$E[\psi(\theta_t)\varepsilon(\theta_t)] = \frac{1}{2\pi \mathrm{j}}\oint \Psi(z)\in(z)\frac{\mathrm{d}z}{z} \tag{12}$$

and

$$E[\psi(i,\theta_t)\psi(k,\theta_t)] = \frac{1}{2\pi \mathrm{j}}\oint \psi(i,z)\psi^{+}(k,z)\frac{\mathrm{d}z}{z}. \tag{13}$$

For $1 \leqslant i \leqslant p$, $1 \leqslant k \leqslant r$, where the $+$ sign indicates the Hermition operation. These integrations can be computed using θ_t and $\Phi_{xx}(z)$ using Eqs. (8)–(10) and can be used to solve Eq. (7). Here, it is assumed that the power spectral density of the input is known.

The procedure to obtain the ODE solutions can be summarised as follows. At any given time t, given the parameter vector θ_t and the input power spectral description $\Phi_{xx}(z)$, expectations of $[\psi(\theta_t)\psi^{\mathrm{T}}(\theta_t)]$, $[\psi(\theta_t)\varepsilon(\theta_t)]$ and $[\psi(i,\theta_t)\psi(k,\theta_t)]$ can be obtained using Eqs. (11)–(13). Substituting these in the ODEs Eqs. (7a) and (7b), the increments to the ODEs at time t can be computed numerically. Using these increments θ_{t+1} can be obtained. Then using θ_{t+1} and input power spectral description, the same procedure can be repeated to get θ_{t+2}. Continuing this procedure until θ_t attains a value very close to the optimum, the parameter trajectories can be obtained.

Usually, the convergence behaviour of an adaptive algorithm is tested for a known general input. Then, the present analysis can be applied starting with the known psd of the input $\Phi_{xx}(z)$. On the other hand, when performance for specific input signals is to be analysed, as a preliminary step, it is necessary to first estimate the power spectral density $[\Phi_{xx}(z),\ z = \mathrm{e}^{\mathrm{j}\omega}]$ of the input signal using any one of the standard estimators such as the periodogram and the maximum-entropy methods [12] before applying the analytical procedure.

The parameter trajectories are the basis for the study of convergence behaviour. Speed of convergence, dynamics of the estimates during the process of adaptation including the initial, intermediate and final convergence, and the time taken for convergence, etc., can be obtained from the ODE solutions. Dependence of these on the input SNR and the algorithm parameters also can be obtained from these characteristics. Although this analytical method cannot provide closed-form solutions for the convergence analysis, it provides us with ensemble convergence behaviour. Individual convergence plots may deviate slightly from these characteristics for noisy inputs. When the input SNR is reasonably high, the deviation is not much and the ODE solutions give us a good idea about the nature of convergence including the convergence time for stochastic inputs. In view of Eqs. (8)–(13), it can be seen that the convergence is effected by $\phi_{xx}(z)$ which includes power spectral density of the input noise.

Since the extended least-squares (ELS) and recursive least-squares (RLS) algorithms can be treated as simplified and approximate versions of the RML algorithm [7], the analytical method presented here is applicable to these algorithms also. Although the hyperstable adaptive recursive filter (HARF) and its modified version (SHARF) [3, 17] are based on a different reasoning, recursions of these algorithms are also nearly the same as those of the RLS algorithm but for using a filtered gradient. Hence, the analytical procedure given here is applicable for these algorithms also.

4. Experimental results

To examine the validity of the present analysis, the analytical procedure derived in Section 3 is applied to several adaptive filtering problems using different algorithm parameters and SNRs. Here we present some of the representative results obtained from the following two examples of adaptive filtering.

Example 1. In this example the output of a second-order IIR system driven by pseudo-Gaussian noise is considered as the deterministic part of the noisy signal for prediction. The transfer function

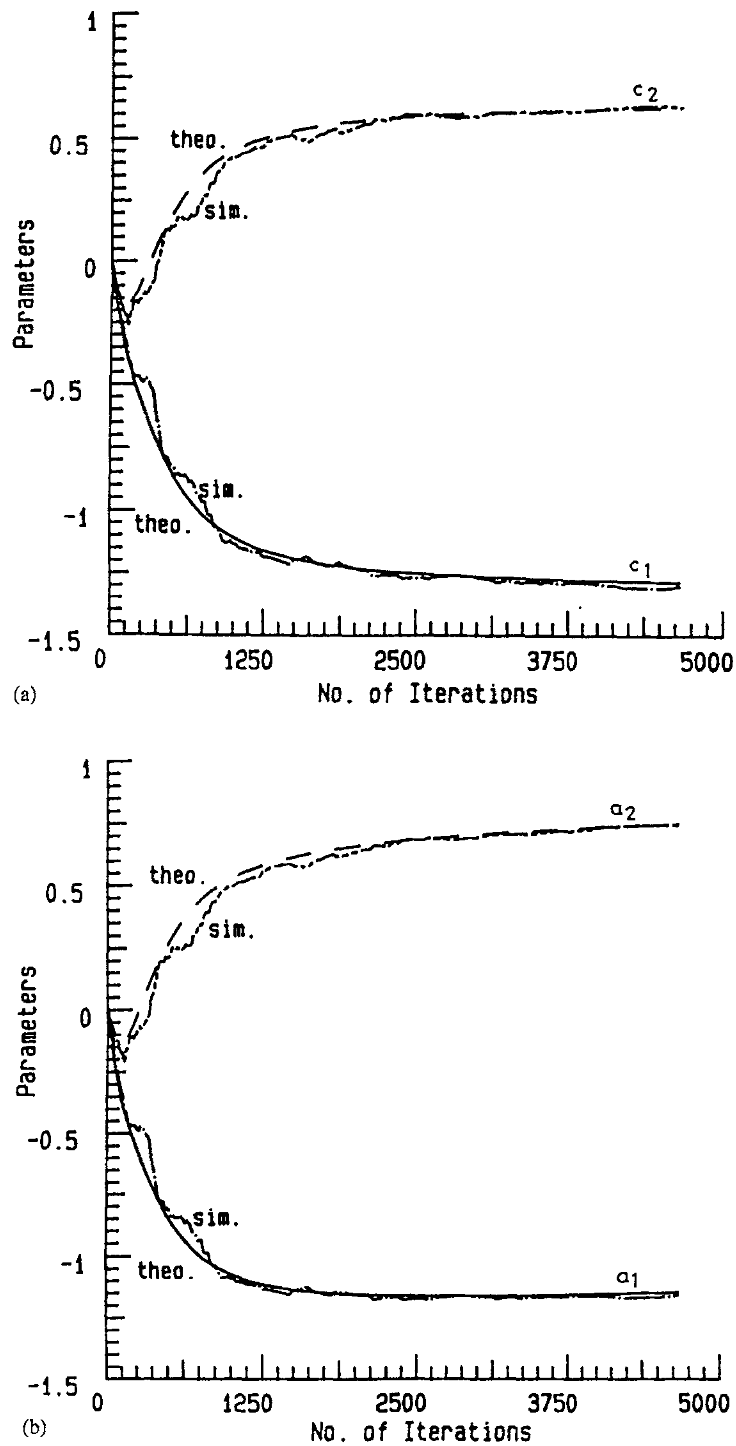

Fig. 2. Theoretical and simulated (a) pole and (b) zero parameter trajectories of the adaptive algorithm for $\mu = 0.02$, $\lambda = 0.98$ and SNR = 1000.

considered here is

$$G(z) = \frac{1 + c_1 z^{-1} + c_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{1 - [2 \times 0.8 \times \cos(0.2\pi)]z^{-1} + 0.64z^{-2}}{1 - [2 \times 0.9 \cos(0.3\pi)]\ z^{-1} + 0.81z^{-2}},$$

where the normalised frequency and radius of the conjugate poles are 0.15 and 0.9, respectively, and the normalised frequency and radius of the conjugate zeros are 0.1 and 0.8, respectively. Variance of the input noise = 1.0. Independent and appropriate amount of pseudo-white noise is added to this system output to form a noisy signal at different SNRs.

Example 2. The second example considered here consists of two-sinusoids of different frequencies in additive noise which may be expressed as

$$x_t = \sqrt{2S_1} \cos \omega_1 t + \sqrt{2S_2} \cos \omega_2 t + n_t,$$

where S_1 and S_2 are the powers of the individual sinusoides whose frequencies are ω_1 and ω_2, respectively.

Here $\omega_1 = 25\pi/128$ and $S_1 = 1.0$, and $\omega_2 = 50\pi/128$ and $S_2 = 0.5$. Variance of the input noise $= (S_1 + S_2)/$(the required SNR).

Fig. 2(a) and 2(b) illustrate the pole and zero parameter trajectories of the system identification problem. These figures include both the analytical and individual simulation plots. These are obtained for $\mu = 0.02$, $\lambda = 0.98$ and SNR $=$ 1000.0. It is seen from these figures that the theoretical plot which gives the ensemble mean behaviour is in close agreement even with the individual simulation plot. From Fig. 2(a) it can be seen that the parameter c_1 attains near optimum value in around 3000 iterations. Fig. 3 illustrates the simulated ensemble mean behaviour of the IIR adaptive algorithm along with the analytical results. It shows the trajectories of the first two parameters (θ_1 and θ_2) of the parameter vector, $\theta_t^{\mathrm{T}} = [\theta_1, \theta_2, \theta_3, \theta_4]$. This plot is also obtained by applying the RML algorithm to the system identification problem. The simulation plot is obtained by averaging 50 individual simulation plots. It can be seen from this plot that the ensemble mean of the simulation results is in very

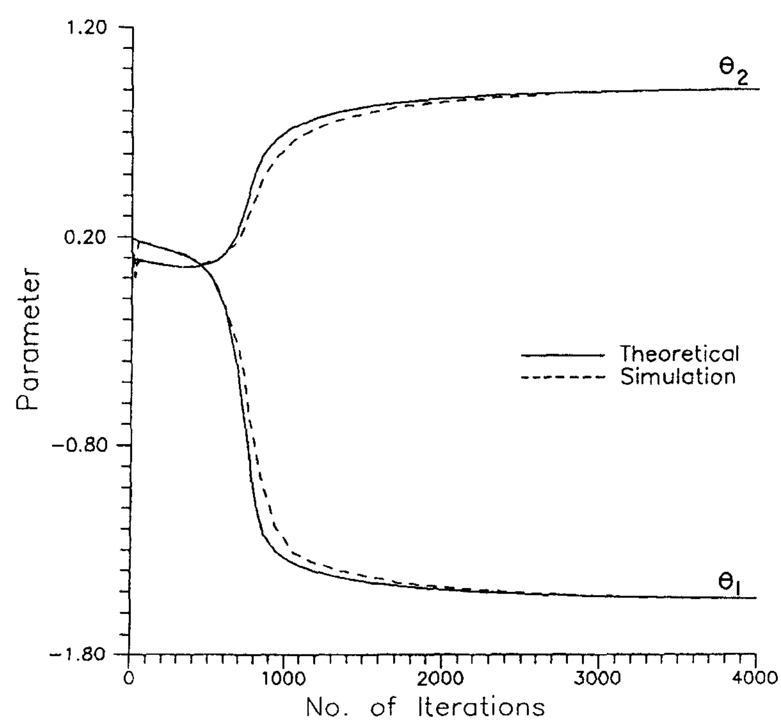

Fig. 3. Comparison between the theoretical and mean-simulation parameter trajectories of the adaptive algorithm.

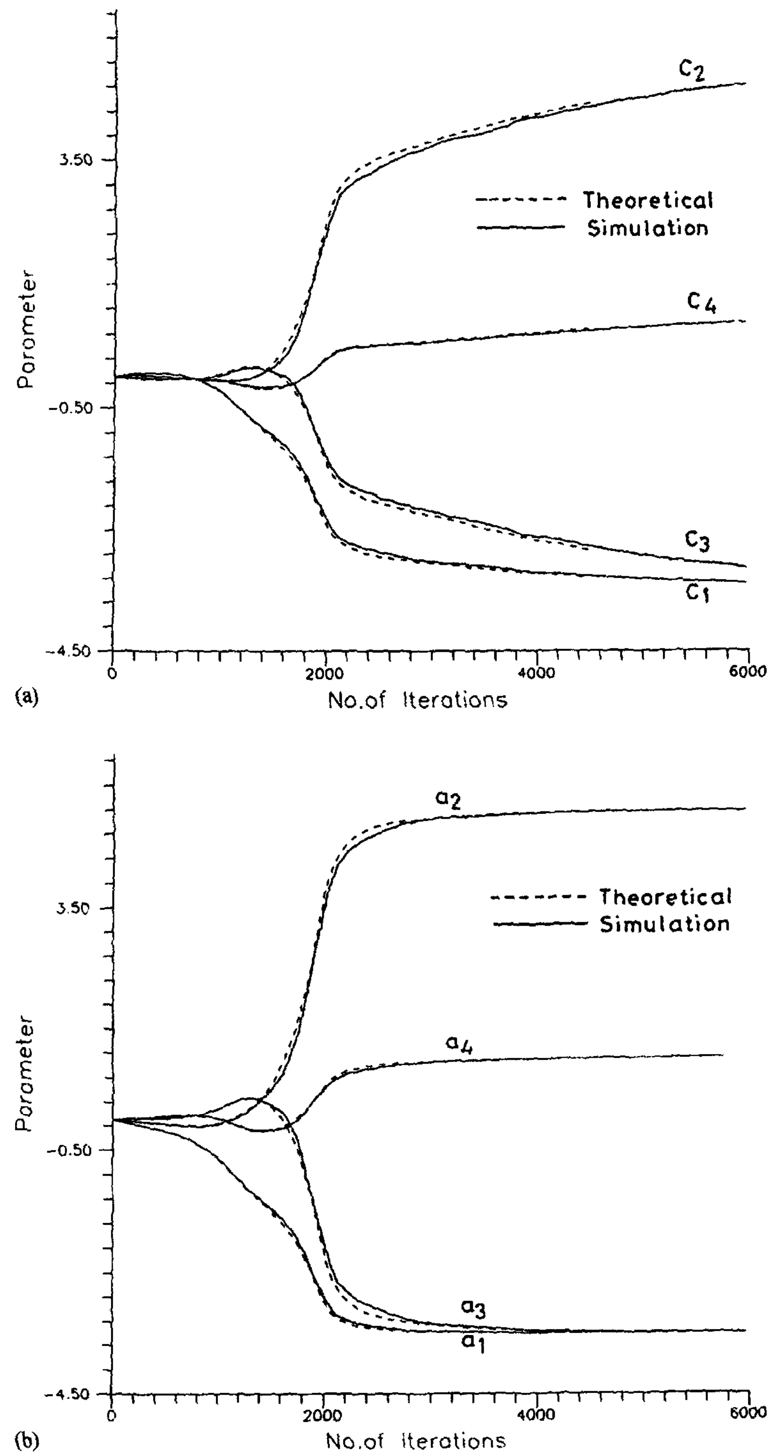

Fig. 4. The (a) pole and (b) zero parameter trajectories of the adaptive algorithm for the two sinusoidal input (input SNR = 10 dB, $\mu = 0.1$ and $\lambda = 0.995$).

close agreement with the mean parameter trajectory obtained by solving the ODEs. It clearly shows that the ODE solutions very closely yield the ensemble convergence behaviour of the IIR adaptive algorithms. To avoid redundancy, trajectories of rest of the parameters are not included here.

Fig. 4(a) and 4(b) illustrate the pole and zero parameter trajectories of the adaptive algorithm for Example 2. These are obtained for an input SNR of 10 dB keeping the values of μ and λ at 0.1 and 0.995, respectively. From these figures also it can be seen that there is a very good agreement between the theoretical and individual simulation results. Since a high-input SNR is used, the parameter noise variance is very small here. Fig. 5 presents the simulated and theoretical pole parameter trajectories for an SNR of 0 dB while the values of μ and λ are the same as those of Fig. 4. Since the parameter noise variance increases with additive noise power, Fig. 5 exhibits a higher noise variance than Fig. 4. Comparing these plots, it can be observed that the convergence speed increases with the input SNR.

Figs. 6 and 7 are obtained for different values of μ and λ while the input SNR = 0 dB which is same as that of Fig. 5. From all these figures, it can be seen that there is a good agreement between the theoretical and simulation results. These plots show that the parameter c_1 attains the optimum value in about 5000, 6000 and 2700 iterations, respectively. It can be observed that the convergence speed is proportional to μ and $(1 - \lambda)$. To reduce the redundancy, the zero parameter trajectories are not included here. These figures illustrate that during the initial convergence the adaptive algorithm is very slow while during the intermediate convergence it is very fast. The speed of final convergence falls in between those of the initial and intermediate

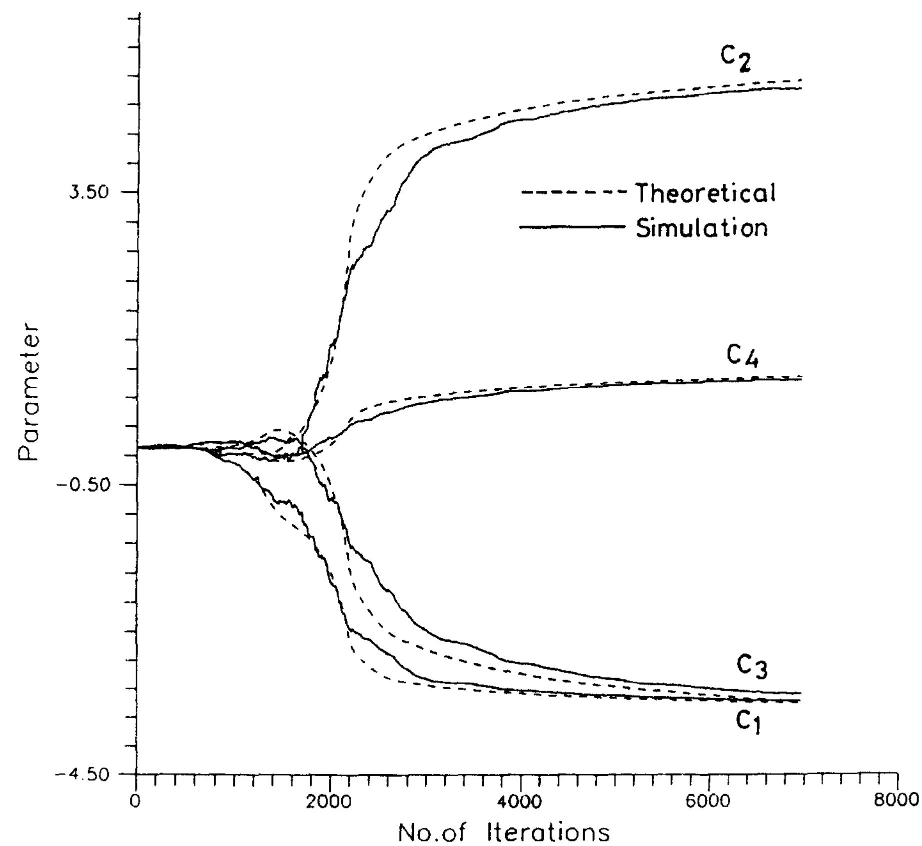

Fig. 5. The pole parameter trajectories of the adaptive algorithm for the two sinusoidal input (input SNR = 0 dB, $\mu = 0.1$ and $\lambda = 0.995$).

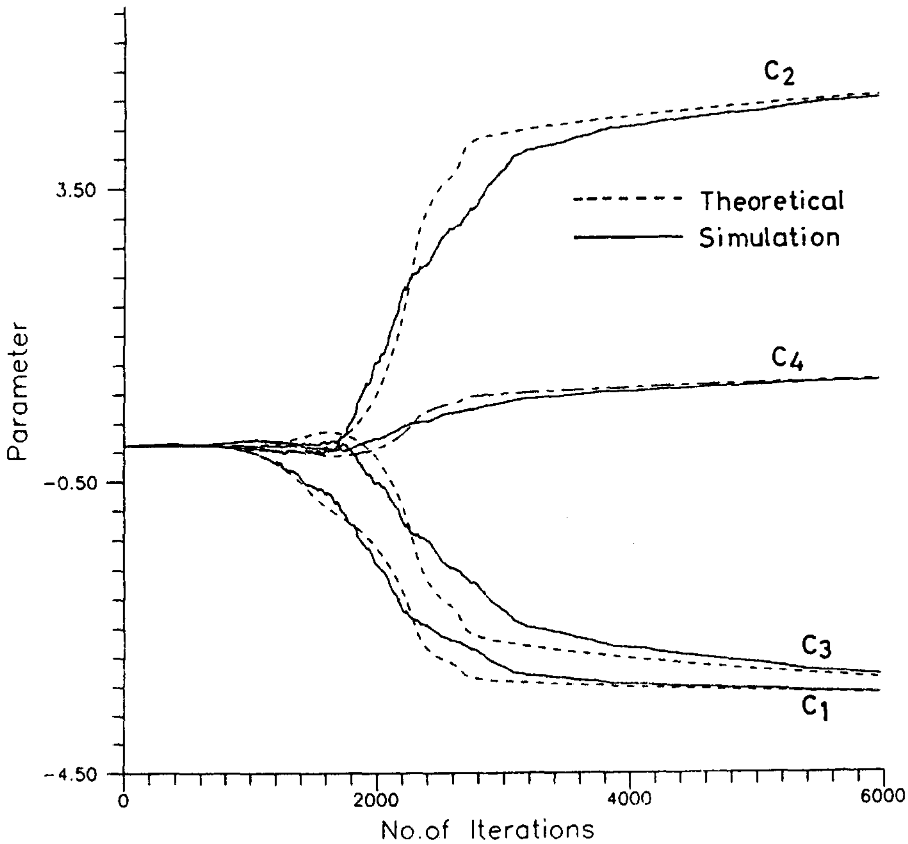

Fig. 6. The pole parameter trajectories of the adaptive algorithm for the two sinusoidal input (input SNR = 0 dB, $\mu = 0.05$ and $\lambda = 0.995$).

convergence. This kind of behaviour is as a result of the nonquadratic performance function. Usually, the slope of the criterion function is relatively small, far away from the optimum. The slope increases sharply as the parameter vector comes close to the optimum. The parameter covariance also decreases sharply as the optimum is approached which yields a very fast intermediate convergences. The nature and time of convergence is specific for the problem and the input psd. However, the analytical results are general enough to represent the mean characteristics.

5. Conclusions

In this paper, a method of solving the ODEs, which represent the ensemble behaviour of the IIR adaptive filtering algorithms is derived to analyse the mean convergence behaviour of these filters. This method uses the mean description of the input in the form of the psd. Further in this work, this procedure is applied to study the convergence behaviour of an IIR predictor. Effectiveness of this method is illustrated through several analytical and simulation results obtained from two examples of adaptive prediction.

It is shown that the analytical method presented here can determine the ensemble convergence behaviour of IIR adaptive filters, very accurately. The speed of convergence is nearly proportional to $\mu(1-\lambda)$ and increases slightly with the input SNR. IIR adaptive filters exhibit slow initial convergence and high intermediate convergence speeds when initialised by values away from the optimum.

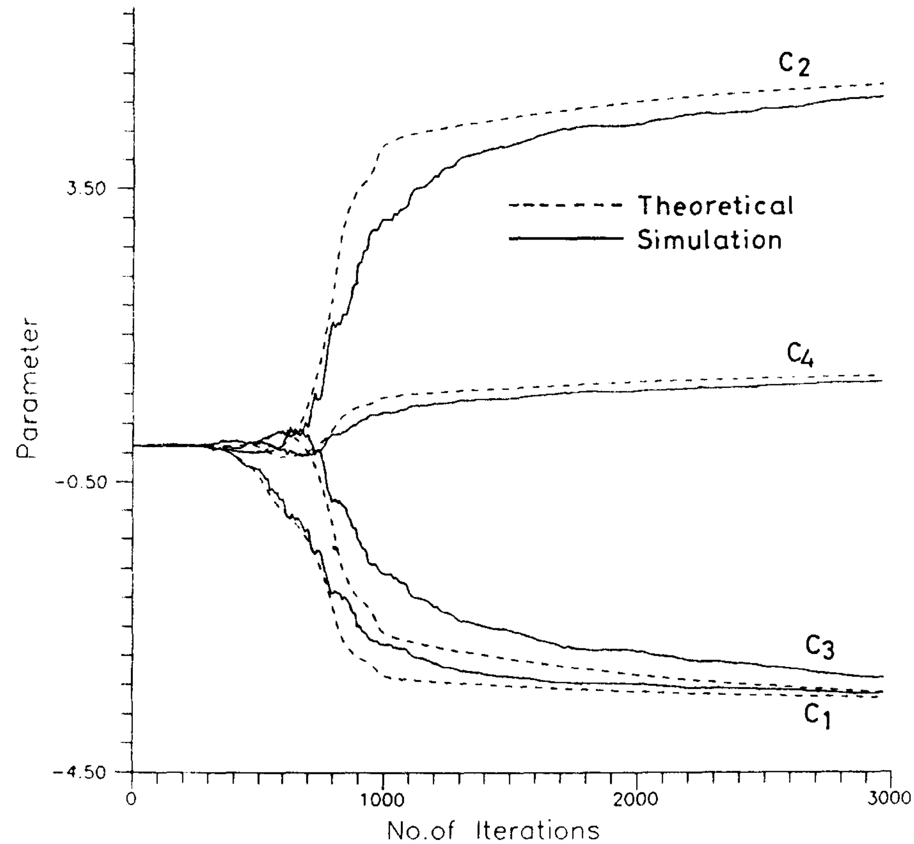

Fig. 7. The pole parameter trajectories of the adaptive algorithm for the two sinusoidal input (input SNR = 0 dB, $\mu = 0.05$ and $\lambda = 0.985$).

The present method of analysis yields convergence results which are specific for the problem and input. However, these are general enough to represent the ensemble behaviour which gives a good idea regarding the nature and time of convergence.

References

[1] C.M. Anderson, E.H. Satorius, J.R. Zeidler, Adaptive enhancement of finite bandwidth signals in white Gaussian noise, IEEE Trans. Acoust. Speech Signal Process. ASSP-31 (February 1983) 17–28.

[2] A. Benveniste, M. Goursat, G. Ruget, Analysis of stochastic approximation schemes with discontinuous and dependent forcing terms with applications to data communication algorithms, IEEE Trans. Automat. Control AC-25 (6) (December 1980) 1042–1058.

[3] C.F.N. Cowan, P.M. Grant, Adaptive Filters, Prentice-Hall, Englewood Cliffs, NJ, 1985.

[4] H. Fan, Application of Benveniste's convergence results in the study of adaptive IIR filtering algorithms, IEEE Trans. Inform. Theory IT-34 (4) (July 1988) 692–709.

[5] B. Friedlander, System identification techniques for adaptive signal processing, IEEE Trans. Acoust. Speech Signal Process. ASSP-30 (April 1982) 240–246.

[6] B. Friedlander, A recursive maximum likelihood algorithm for ARMA line enhancement, IEEE Trans. Acoust. Speech Signal Process. ASSP-30 (April 1982) 651–657.

[7] G.C. Goodwin, K.S. Sin, Adaptive Filtering, Prediction and Control, Prentice-Hall, Englewood Cliffs, NJ, 1984.

[8] C.R. Johnson, Jr., Adaptive IIR filtering: Current results and open issues, IEEE Trans. Inform. Theory IT-30 (2) (March 1984) 237–250.

[9] C.R. Johnson, Jr., M.G. Larimore, J.R. Treichler, B.D.O. Anderson, SHARF convergence properties, IEEE Trans. Acoust. Speech Signal Process. ASSP-29 (June 1981) 659–670.

[10] L. Ljung, Analysis of recursive stochastic algorithms, IEEE Trans. Automat. Control AC-22 (4) (August 1977) 551–575.

[11] L. Ljung, T. Soderstrom, Theory and Practice of Recursive Identification, MIT Press, Cambridge, MA, 1983.

[12] S.M. Kay, Modern Spectral Estimation: Theory and Application, Prentice-Hall, Englewood Cliffs, NJ, 1988.
[13] H.J. Kushner, A. Shwartz, Weak convergence and asymptotic properties of adaptive filters with constant gains, IEEE Trans. Inform. Theory IT-30 (2) (March 1984) 177–182.
[14] R.V. Raja Kumar, Recursive center-frequency adaptive filters for the enhancement and tracking of bandpass signals, Ph.D. Thesis, Department of E&ECE, IIT, Kharagpur, India, May 1986.
[15] R.V. Raja Kumar, R.N. Pal, Performance analysis of the recursive center-frequency adaptive bandpass filters, Signal Processing 17 (2) (June 1989) 105–118.
[16] R.V. Raja Kumar, R.N. Pal, Tracking of bandpass signals using center-frequency adaptive filters, IEEE Trans. Acoust. Speech Signal Process. ASSP-38 (October 1990) 1710–1721.
[17] C.B. Rama Rao, Convergence and tracking behavioural analysis of some IIR adaptive algorithms, Ph.D. Thesis, Department of E & ECE, IIT, Kharagpur, India, February 1992.
[18] D.B. Rao, R. Peng, Tracking characteristics of the constrained IIR adaptive notch filter, IEEE Trans. Acoust. Speech Signal Process. ASSP-36 (September 1988) 1466–1479.
[19] J.J. Shynk, Adaptive IIR filtering, IEEE ASSP Mag. 6 (2) (April 1989) 4–21.
[20] B. Widrow, S.D. Stearns, Adaptive Signal Processing, Prentice-Hall, Englewood Cliffs, NJ, 1984.