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SLOW STEADY ROTATION OF AN APPROXIMATE SPHERE IN AN INCOMPRESSIBLE MICROPOLAR FLUID

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Abstract—The flow generated by the slow steady rotation of an approximate sphere about its axis of symmetry in an incompressible micropolar fluid is studied. Expressions for the velocity and microrotation components are obtained in terms of modified Bessel functions and Gegenbauer's functions. The couple experienced by the approximate sphere is evaluated and the effects of the polarity parameters and deformation parameters on the couple are numerically studied. It is noticed that under the Stokesian assumption, the deformation in the body has no great influence on the couple experienced. The flows generated by a rotating sphere and a rotating oblate spheroid are obtained as special cases.

INTRODUCTION

Payne and Pell, in their classic paper [1] discussed the Stokes flow of a viscous liquid past a class of axially symmetric bodies with uniform streaming at infinity parallel to the axis of symmetry and obtained a general formula for the drag experienced by the body in terms of the stream function. The Stokesian flow of a viscous liquid generated by the slow steady rotation of an axisymmetric body placed in an incompressible viscous liquid which is otherwise at rest was studied by Kanwal [2]. An expression for the couple experienced by the rotating body was also derived by Kanwal in terms of the toroidal velocity component [2]. Ramkissoon and Majumdar [3] and Ramkissoon [4] studied these respective problems in the case of an incompressible micropolar fluid whose study was initiated by Eringen [5, 6] and obtained elegant formulae for the drag and couple experienced by the bodies under consideration. Though Stokes flows are somewhat rare, their mathematical analysis has received considerable attention in view of their occurrence in the important field of small particle dynamics. In some of the fluid mechanical operations such as sedimentation, particles of highly irregular shapes are encountered and it is very difficult to estimate the drag or couple experienced by the submerged particles. In such cases particles are assumed to be regular spheres and the evaluation of the drag or couple is carried out with considerable ease. However a reasonably more realistic formulation is by taking them to be approximate spheres rather than spheres.

Happel and Brenner have studied in detail the Stokes flow of an incompressible viscous liquid past an approximate sphere [7] and Ramkissoon has recently discussed the flow of a viscoelastic fluid of Oldroyd type past a spheroid, treating the spheroid as an approximate sphere [8]. Iyengar and Srinivasa Charya have studied the Stokes flow of an incompressible micropolar fluid past an approximate sphere and obtained expressions for the velocity and microrotation components and the drag experienced by the approximate sphere [9].

In this paper, we study the flow generated by the slow steady rotation of an approximate sphere about its axis of symmetry in an incompressible micropolar fluid. The field equations of micropolar fluids involve the velocity vector $\bar{q}$ and microrotation vector $\bar{v}$ and the theory provides for six material constans. The field equations for an incompressible micropolar fluid flow are

$$\operatorname{div} \bar{q} = 0 \tag{1}$$

$$\rho\, \mathrm{d}\bar{q}/\mathrm{d}t = \rho\bar{f} - \operatorname{grad} p + k \operatorname{curl} \bar{v} - (\mu + k)\operatorname{curl}\operatorname{curl} \bar{q} + (\lambda + 2\mu + k)\operatorname{grad}\operatorname{div} \bar{q} \tag{2}$$

$$\rho j\, \mathrm{d}\bar{v}/\mathrm{d}t = \rho\bar{l} - 2k\bar{v} + k \operatorname{curl} \bar{q} - \gamma \operatorname{curl}\operatorname{curl} \bar{v} + (\alpha + \beta + \gamma)\operatorname{grad}\operatorname{div} \bar{v}. \tag{3}$$

In the above, the scalar quantities ρ and j are respectively the density and gyration parameters and are assumed constant. The vectors $\bar{q}$, $\bar{v}$, $\bar{f}$, $\bar{l}$ are the velocity, microrotation, body force per unit mass and body couple per unit mass. The material constants λ, μ, k and α, β, γ denote the viscosity and gyroviscosity coefficients and these are subject to the inequalities

$$k \geq 0; \qquad 2\mu + k \geq 0; \qquad 3\lambda + 2\mu + k \geq 0;$$

$$\gamma \geq 0; \qquad |\beta| \geq 0; \qquad 3\alpha + \beta + \gamma \geq 0. \tag{4}$$

The stress tensor t_{ij} and the couple stress tensor m_{ij} are given by

$$t_{ij} = (-p + \lambda \operatorname{div} \bar{q})\delta_{ij} + (2\mu + k)d_{ij} + k\varepsilon_{ijm}(\omega_m - v_m) \tag{5}$$

$$m_{ij} = \alpha(\operatorname{div} \bar{v})\delta_{ij} + \beta v_{i,j} + \gamma v_{j,i}. \tag{6}$$

In (5) and (6), v_i and $2\omega_i$ are the components of the microrotation vector and the vorticity vector respectively, d_{ij} are the components of the rate of strain and a comma denotes covariant differentiation.

STATEMENT OF THE PROBLEM

Let (r, θ, ϕ) be a spherical polar coordinate frame with origin at the centre of a sphere $r = a$. Consider the body $r = a(1 + f(\theta))$ where $f(\theta)$ is a function of θ which can be expressed as $f(\theta) = \sum \beta_m \vartheta_m(\zeta)$ where $\vartheta_m(\zeta) = [P_{m-2}(\zeta) - P_m(\zeta)]/(2m-1)$, $\zeta = \cos\theta$ in which $P_m(\zeta)$ is Legendre function of the first kind. In this paper for small β_m's we refer to this body as an approximate sphere. We assume that the approximate sphere is rotating slowly with angular speed Ω about the axis of symmetry $\theta = 0$ in an infinite expanse of an incompressible micropolar fluid which is otherwise at rest. Since the rotation is assumed to be slow, the velocity ($\bar{q}$) has its only component along the vector $\bar{e}_\phi$ and the microrotation vector ($\bar{v}$) lies in the meridian plane. The flow is time independent and all the quantities are independent of ϕ. Thus we choose $\bar{q}$ and $\bar{v}$ in the form

$$\bar{q} = V(r, \theta)\bar{e}_\phi \tag{7}$$

$$\bar{v} = A(r, \theta)\bar{e}_r + B(r, \theta)\bar{e}_\theta. \tag{8}$$

Assuming the flow to be Stokesian, neglecting the inertial and gyroinertial terms, the field equations reduce to the form

$$\operatorname{grad} p = k \operatorname{curl} \bar{v} - (\mu + k)\operatorname{curl}\operatorname{curl} \bar{q} \tag{9}$$

$$2k\bar{v} = k \operatorname{curl} \bar{q} - \gamma \operatorname{curl}\operatorname{curl} \bar{v} + (\alpha + \beta + \gamma)\operatorname{grad}(\operatorname{div} \bar{v}). \tag{10}$$

There is no loss of generality in neglecting the $\operatorname{grad} p$ term and hence the equations governing the flow are equations (10) and (11):

$$k \operatorname{curl} \bar{v} - (\mu + k)\operatorname{curl}\operatorname{curl} \bar{q} = 0. \tag{11}$$

Introducing

$$\operatorname{div} \bar{v} = f(r, \theta); \qquad \operatorname{curl} \bar{v} = g(r, \theta)\bar{e}_\phi \tag{12}$$

we find that the basic equations reduce to

$$kh_3g + (\mu + k)E^2(h_3v) = 0 \tag{13}$$

$$2kA = \frac{k}{h_2h_3}\frac{\partial}{\partial\theta}(h_3v) - \frac{\gamma}{h_2h_3}\frac{\partial}{\partial\theta}(h_3g) + \frac{\alpha+\beta+\gamma}{h_1}\frac{\partial f}{\partial r} \tag{14}$$

$$2kB = \frac{-k}{h_1h_3}\frac{\partial}{\partial r}(h_3v) + \frac{\gamma}{h_1h_3}\frac{\partial}{\partial r}(h_3g) + \frac{\alpha+\beta+\gamma}{h_1}\frac{\partial f}{\partial\theta} \tag{15}$$

where the Stokesian stream function operator E^2 is given by

$$E^2 = \frac{h_3}{h_1h_2}\left[\frac{\partial}{\partial r}\left(\frac{h_2}{h_1h_3}\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{h_1}{h_2h_3}\frac{\partial}{\partial\theta}\right)\right]. \tag{16}$$

Using (14) and (15)

$$\nabla^2 f = \frac{2k}{\alpha+\beta+\gamma}f \tag{17}$$

where

$$\nabla^2 = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial r}\left(\frac{h_2h_3}{h_1}\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{h_3h_1}{h_2}\frac{\partial}{\partial\theta}\right)\right]. \tag{18}$$

Eliminating grad(div $\bar{v}$) from (10) and using the resultant equation with (11) we can eliminate the term involving curl $\bar{v}$. We then get

$$\text{curl curl curl curl } \bar{q} + (\lambda^2/a^2)\text{curl curl } \bar{q} = 0 \tag{19}$$

where

$$\lambda^2/a^2 = k(2\mu + k)/[\gamma(\mu + k)]. \tag{20}$$

Using (7) we see that V can be determined from

$$E^2(E^2 - (\lambda^2/a^2))(r\sin\theta V) = 0. \tag{21}$$

Thus determining V from the above and f from the equation

$$(\nabla^2 - (c^2/a^2))f = 0 \tag{22}$$

where

$$c^2/a^2 = 2k/(\alpha + \beta + \gamma) \tag{23}$$

we can write the expressions for microrotation component A and B using (14) and (15).

The arbitrary constants that arise in solving the equations (21) and (22) are to be determined subject to the hyperstick condition on the rotating body and the regularity condition at infinity. This means that

$$\bar{q}_{\text{boundary}} = \Omega r\sin\theta\bar{e}_\phi \tag{24}$$

$$\bar{v}_{\text{boundary}} = (1/2)\text{curl } \bar{q}_{\text{boundary}} = (1/2)\text{curl}(\Omega r\sin\theta\bar{e}_\phi) \tag{25}$$

on the solid body [10, 11].

SOLUTION OF THE PROBLEM

For simplicity, we first consider the approximate sphere given by the equation $r = a(1 + \beta_m\vartheta_m(\zeta))$, where the coefficient β_m is sufficiently small so that its squares and higher powers can be neglected. Later we can adopt the same procedure to obtain the solution for more general surface $r = a(1 + \sum\beta_m\vartheta(\zeta))$.

The solution of (21) is obtained by superimposing the solutions of

$$E^2(r\sin\theta V)=0 \tag{26}$$

and

$$(E^2-(\lambda^2/a^2))(r\sin\theta V)=0 \tag{27}$$

and using the standard technique of method of separation of variables, the solution of (21) which vanishes at infinity is seen to be

$$r\sin\theta V=[B_2/r+C_2\sqrt{r}K_{3/2}(\lambda r/a)]\vartheta_2(\zeta)+\sum_{n=3}^{\infty}[B_nr^{-n+1}+C_n\sqrt{r}K_{n-1/2}(\lambda r/a)]\vartheta_n(\zeta). \tag{28}$$

The solution of (22) which is regular at infinity is seen to be

$$f(r,\theta)=\sum_{n=2}^{\infty}(1/\sqrt{r})F_nK_{n-1/2}(cr/a)P_{n-1}(\zeta). \tag{29}$$

The function $g(r,\theta)$ can be obtained as

$$g(r,\theta)=\frac{-1}{r\sin\theta}\frac{\lambda^2}{a^2}\frac{\mu+k}{k}\sum_{n=2}^{\infty}C_n\sqrt{r}K_{n-1/2}(\lambda r/a)\vartheta_n(\zeta). \tag{30}$$

Thus, using the expressions for g, f and V in the equations (14) and (15), the expressions for A and B are obtained as

$$A(r,\theta)=\frac{1}{2r^2}\Bigg\{\Bigg[\frac{B_2}{r}+\Lambda^2C_2\sqrt{r}K_{3/2}(\lambda r/a)-\frac{2a^2}{c^2}F_2\sqrt{r}\{2K_{3/2}(cr/a)+(cr/a)K_{1/2}(cr/a)\}\Bigg]P_1(\zeta)+\sum_{n=3}^{\infty}\Bigg[B_nr^{-n+1}+\Lambda^2C_n\sqrt{r}K_{n-1/2}(\lambda r/a)-\frac{2a^2}{c^2}F_2\sqrt{r}\{nK_{n-1/2}(cr/a)+(cr/a)K_{n-3/2}(cr/a)\}\Bigg]P_{n-1}(\zeta)\Bigg\} \tag{31}$$

$$B(r,\theta)=\frac{-1}{2r\sin\theta}[\{-B_2/r^2-\Lambda^2C_2r^{-1/2}(K_{3/2}(\lambda r/a)+(\lambda r/a)K_{1/2}(\lambda r/a))+(4a^2/c^2)F_2r^{-1/2}K_{3/2}(cr/a)\}\vartheta_2(\zeta)+\sum_{n=3}^{\infty}\{(1-n)B_nr^{-n}-\Lambda^2C_n\sqrt{r}[(n-1)K_{n-1/2}(\lambda r/a)+(\lambda r/a)K_{n-3/2}(\lambda r/a)]\}\vartheta_n(\zeta)+(2a^2/c^2)F_n(1/\sqrt{r})K_{n-1/2}(cr/a)\sin^2\theta P'_{n-1}(\zeta)] \tag{32}$$

where $\Lambda^2=2(\mu+k)/k$.

Using the relation

$$(1-\zeta^2)P'_{n-1}(\zeta)=n(n-1)\vartheta_n(\zeta) \tag{33}$$

the expression for $B(r,\theta)$ can be written as

$$B(r,\theta)=\frac{-1}{2r\sin\theta}[\{-B_2/r^2-\Lambda^2C_2r^{-1/2}(K_{3/2}(\lambda r/a)+(\lambda r/a)K_{1/2}(\lambda r/a))+(4a^2/c^2)F_2r^{-1/2}K_{3/2}(cr/a)\}\vartheta_2(\zeta)+\sum\{(1-n)B_nr^{-n}-\Lambda^2C_n\sqrt{r}[(n-1)K_{n-1/2}(\lambda r/a)+(\lambda r/a)K_{n-3/2}(\lambda r/a)]+(2a^2/c^2)n(n-1)F_n(1/\sqrt{r})K_{n-1/2}(cr/a)\}\vartheta_n(\zeta)]. \tag{34}$$

Let us introduce the following non-dimensional scheme

$$r=a\tilde{r};\qquad V=a\tilde{V};\qquad B_n=\Omega a^{n+1}\tilde{B}_n;\qquad C_n=\Omega a^{3/2}\tilde{C}_n;\qquad F_n=\Omega a^{-1/2}\tilde{F}_n$$
$$A=\Omega\tilde{A};\qquad B=\Omega\tilde{B} \tag{35}$$

and later drop the tildes. We notice that the expressions for non-dimensional velocity and microrotation components are given by

$$r\sin\theta V=[B_2/r+C_2\sqrt{r}K_{3/2}(\lambda r)]\vartheta_2(\zeta)+\sum_{n=3}^{\infty}[B_nr^{-n+1}+C_n\sqrt{r}K_{n-1/2}(\lambda r)]\vartheta_n(\zeta) \quad (36)$$

$$A(r,\theta)=\frac{1}{2r^2}\left\{\left[\frac{B_2}{r}+\Lambda^2C_2\sqrt{r}K_{3/2}(\lambda r)-\frac{2}{c^2}F_2\sqrt{r}\{2K_{3/2}(cr)+(cr)K_{1/2}(cr)\}\right]P_1(\xi)\right.$$
$$\left.+\sum_{n=3}^{\infty}\left[B_nr^{-n+1}+\Lambda^2C_n\sqrt{r}K_{n-1/2}(\lambda r)-\frac{2}{c^2}F_n\sqrt{r}\{nK_{n-1/2}(cr)+(cr)K_{n-3/2}(cr)\}\right]P_{n-1}(\zeta)\right\} \quad (37)$$

$$B(r,\theta)=\frac{-1}{2r\sin\theta}[\{-B_2/r^2-\Lambda^2C_2r^{-1/2}(K_{3/2}(\lambda r)+(\lambda r)K_{1/2}(\lambda r))+(4/c^2)F_2r^{-1/2}K_{3/2}(cr)\}\vartheta_2(\zeta)$$
$$+\sum_{n=3}^{\infty}\{(1-n)B_nr^{-n}-\Lambda^2C_n\sqrt{r}[(n-1)K_{n-1/2}(\lambda r)+(\lambda r)K_{n-3/2}(\lambda r)]$$
$$+(2/c^2)n(n-1)F_n(1/\sqrt{r})K_{n-1/2}(cr)\}\vartheta_n(\zeta)]. \quad (38)$$

Let us compare the above solutions with those obtained in the case of slow steady rotation of a sphere rotating in an infinite expanse of micropolar fluid which is otherwise at rest [12]. The expressions in our present problem are obtained from [12] just by adding the expressions involving B_n, C_n and F_n for $n>2$. The body that we are considering now is an approximate sphere and the flow generated is not expected to be far different from the one generated by a rotating sphere. Also the coefficients B_n, C_n, F_n for $n>2$ will be of order β_m. Therefore as in Happel and Brenner [7] and in the case of Stokes flow of a micropolar fluid past an approximate sphere [9], in the terms involving B_n, C_n, F_n for $n>2$, we ignore the departure from the spherical form and set $r=1$ while implementing the boundary conditions.

On the boundary $r=(1+\beta_m\vartheta_m(\zeta))$, the non-dimensional version of the boundary conditions is

$$V=r\sin\theta,\qquad A=\cos\theta,\qquad B=-\sin\theta. \quad (39)$$

These respectively yield

$$(B_2+C_2K_{3/2}(\lambda)-2)\vartheta_2(\zeta)-(B_2+4)\beta_m\vartheta_2(\zeta)\vartheta_m(\zeta)+\sum[B_n+C_nK_{n-1/2}(\lambda)]\vartheta_n(\zeta)=0 \quad (40)$$

$$(B_2+\Lambda^2C_2K_{3/2}(\lambda)-(2/c^2)F_2\{2K_{3/2}(c)+cK_{1/2}(c)\}-2)P_1(\zeta)$$
$$+(-B_2-(2/c^2)F_2cK_{1/2}(c)-4)\beta_m\vartheta_m(\zeta)P_1(\zeta)$$
$$+\sum[B_n+\Lambda^2C_nK_{n-1/2}(\lambda)-(2/c^2)\{nK_{n-1/2}(c)+cK_{n-3/2}(c)\}]P_{n-1}(\zeta)=0 \quad (41)$$

$$(-4-B_2-\Lambda^2C_2\{K_{3/2}(\lambda)+\lambda K_{1/2}(\lambda)\}+(4/c^2)F_2K_{3/2}(c))\vartheta_2(\zeta)$$
$$+(-4+2B_2+\Lambda^2C_2K_{3/2}(\lambda)-(4/c^2)F_2K_{3/2}(c))\beta_m\vartheta_2(\zeta)\vartheta_m(\zeta)$$
$$+\sum((1-n)B_n-\Lambda^2C_n\{(n-1)K_{n-1/2}(\lambda)+\lambda K_{n-3/2}(\lambda)\}$$
$$+(2/c^2)F_nn(n-1)K_{n-1/2}(c))\vartheta_n(\zeta)=0. \quad (42)$$

Equating leading coefficients to zero

$$B_2+C_2K_{3/2}(\lambda)-2=0 \quad (43)$$

$$B_2+\Lambda^2C_2K_{3/2}(\lambda)-(2/c^2)F_2\{2K_{3/2}(c)+cK_{1/2}(c)\}-2=0 \quad (44)$$

$$-4-B_2-\Lambda^2C_2\{K_{3/2}(\lambda)+\lambda K_{1/2}(\lambda)\}+(4/c^2)F_2K_{3/2}(c)=0. \quad (45)$$

Solving these for B_2, C_2, F_2 we have

$$B_2 = 2 + 6(2K_{3/2}(c) + cK_{1/2}(c))K_{3/2}(\lambda)/D(\lambda) \tag{46}$$

$$C_2 = -6(2K_{3/2}(c) + cK_{1/2}(c))/D(\lambda) \tag{47}$$

$$F_2 = -6\Lambda^2 K_{3/2}(\lambda)/D(\lambda) \tag{48}$$

where

$$D(\lambda) = \Lambda^2 \lambda K_{1/2}(\lambda)[2K_{3/2}(c) + cK_{1/2}(c)] + (\Lambda^2 - 1)cK_{3/2}(\lambda)K_{1/2}(c). \tag{49}$$

Substituting these values in (40), (41) and (42), we have

$$\sum [B_n + C_n K_{n-1/2}(\lambda)]\vartheta_n(\zeta) = (B_2 + 4)\beta_m \vartheta_2(\zeta)\vartheta_m(\zeta) \tag{50}$$

$$\sum [B_n + \Lambda^2 C_n K_{n-1/2}(\lambda) - (2/c^2)\{nK_{n-1/2}(c) + cK_{n-3/2}(c)\}]P_{n-1}(\zeta) = (B_2 + (2/c^2)F_2 cK_{1/2}(c) + 4)\beta_m \vartheta_m(\zeta)P_1(\zeta) \tag{51}$$

$$\sum [(1-n)B_n - \Lambda^2 C_n\{(n-1)K_{n-1/2}(\lambda) + \lambda K_{n-3/2}(\lambda)\} + (2/c^2)F_n n(n-1)K_{n-1/2}(c)]\vartheta_n(\zeta) = (4 - 2B_2 - \Lambda^2 C_2 K_{3/2}(\lambda) + (4/c^2)F_2 K_{3/2}(c))\beta_m \vartheta_2(\zeta)\vartheta_m(\zeta). \tag{52}$$

Using the standard identities

$$\vartheta_m(\zeta)\vartheta_2(\zeta) = -\frac{(m-2)(m-3)}{2(2m-1)(2m-3)}\vartheta_{m-2}(\zeta) + \frac{m(m-1)}{(2m+1)(2m-3)}\vartheta_m(\zeta) - \frac{(m+1)(m+2)}{2(2m-1)(2m+1)}\vartheta_{m+2}(\zeta) \tag{53}$$

and

$$P_1(\zeta)\vartheta_m(\zeta) = \frac{(m-2)}{(2m-1)(2m-3)}P_{m-3}(\zeta) + \frac{1}{(2m+1)(2m-3)}P_{m-1}(\zeta) - \frac{(m+1)}{(2m+1)(2m-1)}P_{m+1}(\zeta) \tag{54}$$

in (50), (51) and (52), we notice that

$$B_n = C_n = F_n = 0 \quad \text{for} \quad n \neq m-2, m, m+2$$

and for $n = m-2$, m, $m+2$ we get

$$B_n + C_n K_{n-1/2}(\lambda) = a_n \varepsilon_1 \tag{55}$$

$$B_n + \Lambda^2 C_n K_{n-1/2}(\lambda) - (2/c^2)\{nK_{n-1/2}(c) + cK_{n-3/2}(c)\} = b_n \varepsilon_2 \tag{56}$$

$$(1-n)B_n - \Lambda^2 C_n\{(n-1)K_{n-1/2}(\lambda) + \lambda K_{n-3/2}(\lambda)\} + (2/c^2)F_n n(n-1)K_{n-1/2}(c) = a_n \varepsilon_3 \tag{57}$$

where

$$a_{m-2} = -\frac{(m-2)(m-3)}{2(2m-1)(2m-3)}; \quad a_m = \frac{m(m-1)}{(2m+1)(2m-3)}; \quad a_{m+2} = -\frac{(m+1)(m+2)}{2(2m-1)(2m+1)} \tag{58}$$

$$b_{m-2} = \frac{(m-2)}{(2m-1)(2m-3)}; \quad b_m = \frac{1}{(2m+1)(2m-3)}; \quad b_{m+2} = -\frac{(m+1)}{(2m+1)(2m-1)} \tag{59}$$

and

$$\varepsilon_1 = 6 + 6(2K_{3/2}(c) + cK_{1/2}(c))K_{3/2}(\lambda)/D(\lambda) \tag{60}$$

$$\varepsilon_2 = 6 + C_2K_{3/2}(\lambda)(-2K_{3/2}(c) + (\Lambda^2 - 2)cK_{1/2}(c))/\{2K_{3/2}(c) + cK_{1/2}(c)\} \tag{61}$$

$$\varepsilon_3 = C_2K_{3/2}(\lambda)(2K_{3/2}(c) - (\Lambda^2 - 2)cK_{1/2}(c))/\{2K_{3/2}(c) + cK_{1/2}(c)\}. \tag{62}$$

Solving these equations, we get the expressions for B_n, C_n and F_n.

Thus the velocity component $V(r, \theta)$ and the microrotation components $A(r, \theta)$, $B(r, \theta)$ are determined completely. In case the approximate sphere is $r = a(1 + \sum \beta_m \vartheta_m(\zeta))$, we employ the above technique for each m and obtain the expressions for V, A and B by superimposition of the expressions thus obtained.

DETERMINATION OF THE COUPLE

An elegant formula for the couple N acting on an axisymmetric body, rotating about its axis of symmetry in a micropolar fluid has been derived by Ramkissoon [4] and is given by

$$N = 4\pi(2\mu + k) \operatorname*{Lt}_{r\to\infty} \frac{r^3 V}{r \sin\theta} \tag{63}$$

where V and r are dimensional. After lengthy, but straightforward calculation the couple on the body is seen to be

$$\text{Couple} = 2\pi(2\mu + k)\left(B_2 + \left(\frac{1}{5}\right)B_2''\beta_2 + \left(\frac{2}{35}\right)B_2'\beta_4\right)\Omega a^3 \tag{64}$$

where

$$B_2 = (2(c^2 + 2c + 2)(\Lambda^2\lambda^2 + 3\lambda + 3) + 2(\Lambda^2 - 1)c^2(\lambda + 1))/D'(\lambda) \tag{65}$$

$$\begin{aligned} B_2' = {} & (3(c^2 + 2c + 2)(\lambda + 1 - \lambda^2\Lambda^2) + c^2(\lambda + 1)(\Lambda^2 - 12))/D'(\lambda) \\ & + 3(\lambda + 1)^2\{3[2c + 2 - (\Lambda^2 - 2)c^2](c^2 + 2c + 2) + [-6(c + 1) + (2\Lambda^2 - 5)c^2]\}/[D'(\lambda)]^2 \end{aligned} \tag{66}$$

$$\begin{aligned} B_2'' = {} & ((c^2 + 2c + 2)\{12\Lambda^2\lambda^2 + 18(\lambda + 1)\} + c^2(\lambda + 1)(\Lambda^2 + 3))/D'(\lambda) \\ & - (\lambda + 1)^2[6\{2c + 2 - (\Lambda^2 - 2)c^2\}(c^2 + 2c + 2) - 6\{2(c + 1) + \Lambda^2c^2\}]/[D'(\lambda)]^2 \end{aligned} \tag{67}$$

and

$$D'(\lambda) = \Lambda^2\lambda^2(c^2 + 2c + 2) + (\Lambda^2 - 1)c^2(\lambda + 1). \tag{68}$$

Defining the non-dimensional couple as

$$C_{\mathrm{ND}} = (\text{couple})/4\pi(2\mu + k)\Omega a^3 \tag{69}$$

we see that

$$\begin{aligned} C_{\mathrm{ND}} = \left(\frac{1}{2}\right)\Big\{ & (2(c^2 + 2c + 2)(\Lambda^2\lambda^2 + 3\lambda + 3) + 2(\Lambda^2 - 1)c^2(\lambda + 1))/D'(\lambda) \\ & + (\varepsilon/35)((c^2 + 2c + 2)[132(\lambda + 1) + 78\lambda^2\Lambda^2)] + c^2(\lambda + 1)(9\Lambda^2 - 3))/D'(\lambda) \\ & + \frac{\varepsilon(\lambda + 1)^2}{35[D'(\lambda)]^2}\{24[2(c^2 + c + 1) - \Lambda^2c^2](c^2 + 2c + 2) - 48(c + 1) - 54\Lambda^2c^2 + 30c^2\}\Big\} \end{aligned} \tag{70}$$

where

$$\beta_2 = \beta_4 = \varepsilon.$$

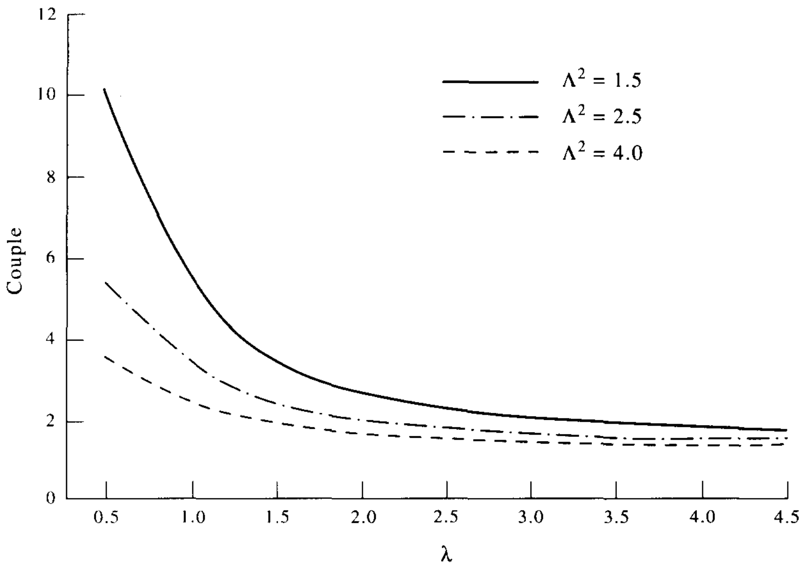

Fig. 1. Variation of couple with λ (approximate sphere) $c = 1.0$, $\varepsilon = 0.1$.

It is interesting to note that though the boundary surface is $r = a(1 + \sum \beta_m \vartheta_m(\zeta))$, the coefficients β_2 and β_4 only contribute to the couple [see equation (65)]. This implies that in Stokes flow the couple on the approximate sphere is relatively insensitive to the details of the surface geometry. This was observed to be true even in the case of drag experienced by an approximate sphere when there is a flow of micropolar fluid past the body with uniform stream at infinity [9].

If $\beta_m = 0$, for $m > 2$, the above couple simplifies to

$$((c^2 + 2c + 2)(\Lambda^2\lambda^2 + 3\lambda + 3) + (\Lambda^2 - 1)c^2(\lambda + 1))/D'(\lambda) \tag{71}$$

which is the same as the couple experienced by a sphere rotating in a micropolar fluid [12]. However, this expression differs from the one given in [12] due to the revised boundary conditions that were employed in our present work.

The variation of the non-dimensional couple for various values of Λ^2, c, λ and $\varepsilon = \beta_2 = \beta_4$ is studied numerically and the results are presented through a representative set of graphs given in Figs 1, 2 and 3. The following observations are worth noticing:

(i) For a fixed c, ε, Λ^2 as the micropolarity parameter λ increases, the couple decreases.

(ii) For fixed c, λ, ε as micropolar viscosity parameter Λ^2 increases, the couple decreases.

These two features are same as those observed in the case of drag experienced by an approximate sphere when there is a flow past the body [9].

(iii) For fixed c, λ, Λ^2 as the deformity parameter ε increases, the couple increases.

However, when there is a uniform flow of micropolar fluid past an approximate sphere, for fixed Λ^2, λ as the deformity parameter ε increases, the drag on the body decreases.

(iv) For fixed ε, λ, Λ^2 as c increases, the couple decreases.

It is to be noticed that c is an extra micropolarity parameter which does not enter into the analysis in [9].

COUPLE ON AN OBLATE SPHEROID

The polar equation of an oblate spheroid is given by

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{a^2(1 - \varepsilon)^2} = 1 \tag{72}$$

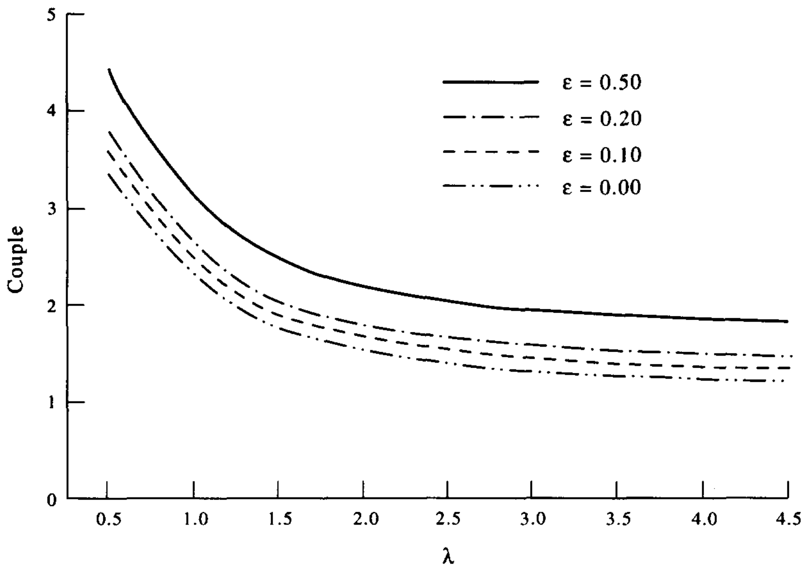

Fig. 2. Variation of couple with λ (approximate sphere) $c = 1.0$, $\Lambda^2 = 4.0$.

whose equatorial radius is 'a' in which ε is so small that ε^2 and higher powers may be neglected, can be put in the form $r = c(1 + 2\varepsilon\vartheta_2(\zeta))$ where $c = a(1 - \varepsilon)$ (see [7, p. 144]). This is like

$$r = c(1 + \beta_2\vartheta_2(\zeta)) \tag{73}$$

where

$$a = c \quad \text{and} \quad \beta_2 = 2\varepsilon. \tag{74}$$

Using (36), (37) and (38), the expressions for $V(r, \theta)$, $A(r, \theta)$ and $B(r, \theta)$ can be determined. Using the formula (63), the non-dimensional couple is seen to be

$$C_{0s} = \left(\frac{1}{2}\right)[B_2 + (-2B_2 + (2/5)B_2'')\varepsilon] \tag{75}$$

where B_2, B_2'' are given by (65) and (67).

The couple on the spheroid is less than that would be exerted on the sphere of radius equal

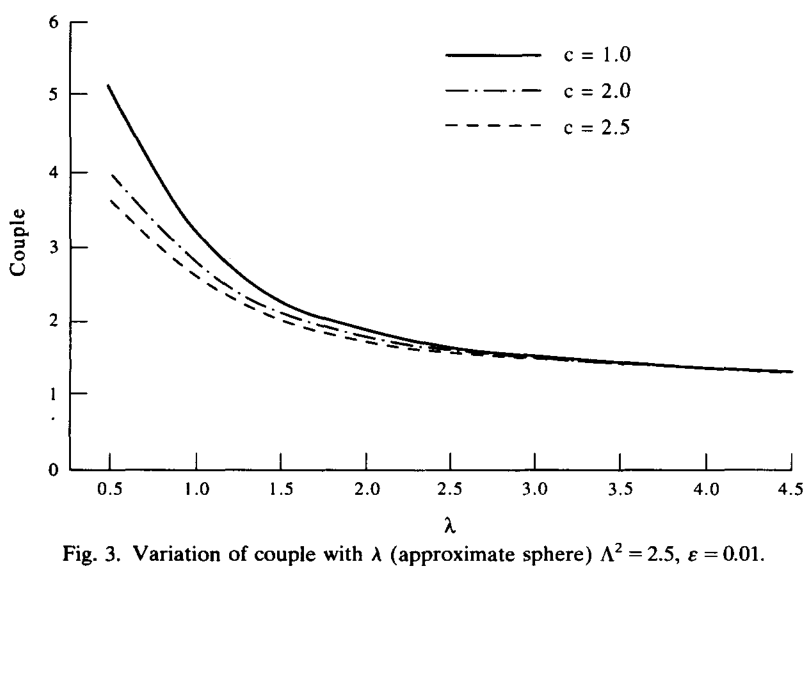

Fig. 3. Variation of couple with λ (approximate sphere) $\Lambda^2 = 2.5$, $\varepsilon = 0.01$.

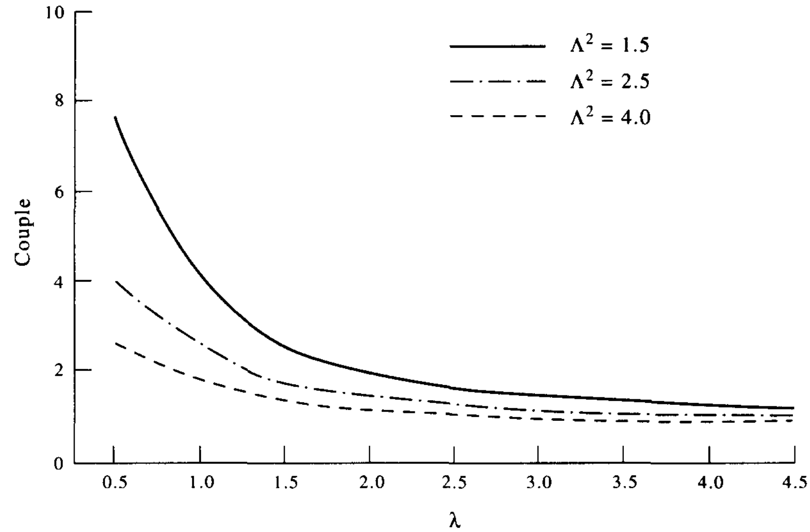

Fig. 4. Variation of couple with λ (oblate spheroid) $c = 1.0$, $\varepsilon = 0.1$.

to the equatorial radius of the spheroid. The authors obtained a similar result in the case of the drag experienced by a spheroid when there is a uniform flow of micropolar fluid past the body [9].

We notice that the volume of the spheroid is $(4/3)\pi a^3(1-\varepsilon)$ and a sphere of equal volume can be obtained by choosing its radius equal to $a(1-\varepsilon/3)$ (with ε^2 and higher powers neglected). The non-dimensional couple on such a sphere is

$$((c^2+2c+2)(\Lambda^2\lambda^2+3\lambda+3)+(\Lambda^2-1)c^2(\lambda+1))(1-\varepsilon/3)/D'(\lambda) \tag{76}$$

and this is greater than the couple on the spheroid. Similar comment holds good concerning the couple on the sphere of equal surface area as that of the spheroid. This couple is numerically evaluated for sets of values of λ, Λ^2, ε and c and the variation is peresented in Figs 4, 5 and 6.

The evaluation of the couple on the oblate spheroid here is based on the neglect of ε^2 and higher terms, while the calculation of the couple in [13] is based on a truncation of an infinite system of simultaneous equations.

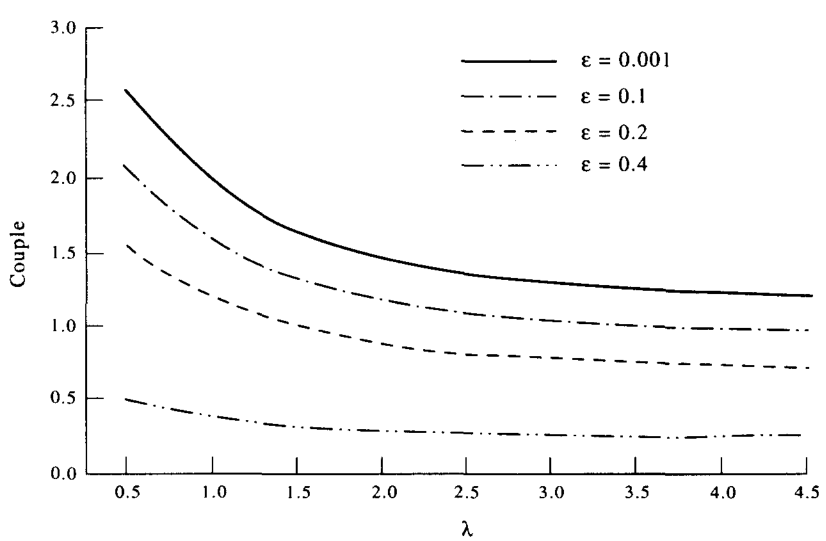

Fig. 5. Variation of couple with λ (oblate spheroid) $c = 2.0$, $\Lambda^2 = 40$.

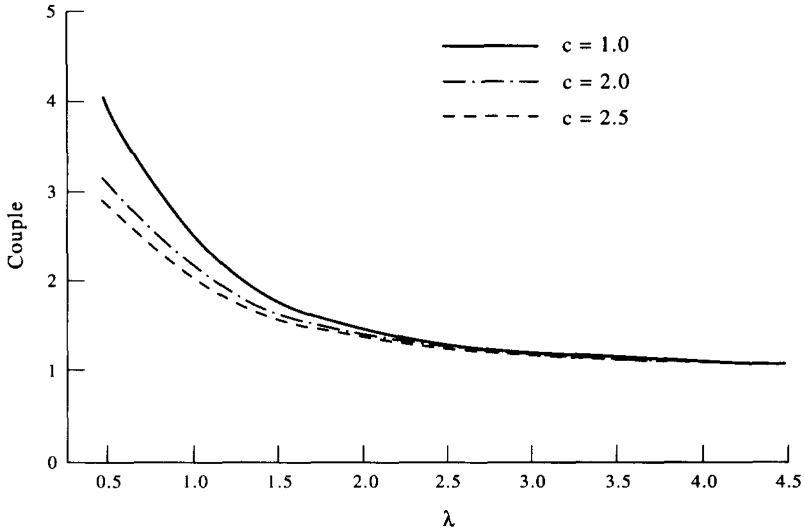

Fig. 6. Variation of couple with λ (oblate spheroid) $\Lambda^2 = 2.5$, $\varepsilon = 0.1$.

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