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Technical Communication:

Two-Phase Stochastic Dynamic Programming Model for Optimal Operation of Irrigation Reservoir

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Abstract. A two-phase stochastic dynamic programming model is developed for optimal operation of irrigation reservoirs under a multicrop environment. Under a multicrop environment, the crops compete for the available water whenever the water available is less than the irrigation demands. The performance of the reservoir depends on how the deficit is allocated among the competing crops. The proposed model integrates reservoir release decisions with water allocation decisions. The water requirements of crops vary from period to period and are determined from the soil moisture balance equation taking into consideration the contribution of soil moisture and rainfall for the water requirements of the crops. The model is demonstrated over an existing reservoir and the performance of the reservoir under the operating policy derived using the model is evaluated through simulation.

Key words: irrigation reservoirs, operation, water deficit, yield response, multicrop environment, stochastic dynamic programming, soil moisture, water allocation, simulation.

1. Introduction

Application of systems techniques to water management has gained momentum over the years. Many mathematical models have been developed and successfully applied for reservoir planning and operation studies. The use of these models has greatly aided in providing a good insight into the intricacies of various aspects of problems in water management.

In India, irrigation is one of the important purpose for which most reservoirs are operated. Hence, the development of appropriate mathematical models for optimal operation of irrigation reservoirs should receive greater attention. Optimal operation of irrigation reservoirs involves many subtle considerations such as nature and timing of crops being irrigated, their stages of growth, the competition among different crops for the available water and the effect of a deficit water supply on the crop yield. When the available water is less than the aggregate demand, the crops compete for the available water. The available water should be allocated among the various crops, taking into consideration the effect of deficits on the yield of each crop. Thus, the operation policy of an irrigation reservoir should be sufficiently explicit to indicate not only how much water is to be released from the reservoir in a given period but also how much of it should be allocated to a given crop. The stochastic nature of the various variables involved like rainfall, evapotranspiration, soil moisture and the reservoir inflow add to the complexity of reservoir operation.

Mathematical programming, especially stochastic dynamic programming, was extensively used in the development of optimal reservoir operation for irrigation. Many models were developed in the past for optimal operation of irrigation reservoirs with different degrees of complexity. Most of these models treated seasonal crop water demand as deterministic, while inflows into the reservoir were treated as stochastic. Exception to this generalisation are the approaches of Sanford (1969), and Burt and Stauber (1971) which considered intraseasonal variations in crop water requirement. Both stochastic water demand and stochastic inflows were considered by Dudley (1972), Dudley *et al*. (1971a,b, 1972), Dudley and Burt (1973) and Yaqoob (1987). Dudley and Burt (1973) developed an integrated intraseasonal and interseasonal stochastic dynamic programming model to determine an optimal decision rule for intertemporal water application rates and crop acreage decision rule for a single crop. Later Dudley *et al*. (1976) developed a hierarchy of models to aid decisions in multicrop environment with deterministic water requirements. Jones (1983) studied the problem of irrigation scheduling from a limited water supply for a single crop situation. Dudley (1988) advanced this earlier work by simulating the optimal decisions under the assumption of single decision maker. Rao and Sarma (1990) developed a model for optimal multicrop allocation of seasonal and intraseasonal water.

Vedula and Mujumdar (1992) developed a model for optimal operation of a reservoir for irrigation under multiple crop scenario using stochastic dynamic programming. An optimal allocation process is incorporated in the model to determine the allocations to individual crops when a competition for water exists among them.

In the present study a stochastic dynamic programming model for operation of irrigation reservoirs under multicrop environment is developed. The model considers stochastic reservoir inflows with variable irrigation demands and assumes the soil moisture and precipitation to be deterministic. The demands vary from period to period and are determined from a soil moisture balance equation. An optimal allocation process is incorporated into the model to determine the allocations to individual crops during an intraseasonal period whenever competition for available water exists among the crops. Thus the model integrates the reservoir release decisions with the irrigation allocation decisions with respect to each crop in each period. This model is applied to develop the operating policy of an existing reservoir namely Sri Rama Sagar Reservoir on river Godavari in the state of Andhra Pradesh in India.

2. Formulation of the Model

The proposed model is formulated conceptually to operate in two phases. In the first phase the model uses a deterministic dynamic programming technique to allocate the given release from the reservoir among all crops to optimise the impact of allocation within a period. The second phase uses stochastic dynamic programming technique to determine the steady state operating policy of the reservoir so as to optimise the overall impact of the allocation over a full year. The result of this two phase analysis is a set of decisions indicating the reservoir release to be made in each time period and its distribution among various crops.

2.1. FORMULATION OF DETERMINISTIC DYNAMIC PROGRAMMING MODEL (PHASE I)

The performance of an irrigation system under multicrop environment depends on how the available release (*R*) from the reservoir is allocated among the various crops. The actual water available for irrigation can be determined by adjusting for losses in conveyance and application. The water available for irrigation *X* may be written as

$$X = \alpha * R, \tag{1}$$

where α is the field irrigation efficiency.

If the water available *X* is greater than the aggregate irrigation demand of all the crops, then no competition exists among the crops and the available water can be distributed among the crops as per their individual requirements. The crops compete for the available water only when the available water *X* is less than the total water requirements of all the crops. The response of the system for deficit water supply depends on how the available water is distributed among the crops. The available water should be allocated among the crops in such a way that the adverse effect of the water deficit is minimised.

The response of a crop to deficit water supply depends on the nature of the crop and the stage of its growth and can be described by crop production function. The crop production function as proposed by Hall and Dracup (1970) and Doorenbos and Kassam (1979) is given by

$$1 - Y_a/Y_m = ky(1 - E_a/E_m), \tag{2}$$

where Y_a is the actual yield, Y_m is the potential yield, E_a is the actual evapotranspiration needs provided, E_m is the evapotranspiration needs of the crop, and *ky* is the yield response factor which reflects the sensitivity of the crop yield to a given water deficit.

A deterministic dynamic programming (DDP) model is used to allocate the available water among the crops whenever competition for water exists. The objective function of the DDP model is based on the crop production function (Equation (2)) and is given by

$$\text{Min} \sum_{c=1}^{nc} ky_{ct}[1 - E_{\text{act}}/E_{\text{mct}}] = B, \tag{3}$$

where *c* is the index for crop and *nc* is the number of crops in period *t*. The value of *B* is zero, if the water available *X* is greater than or equal to the water requirement of all the crops. The value of *B* is greater than zero if competition among crops exists and is to be evaluated.

Each crop constitutes a stage in this DDP model and the state variable is the volume of water q_r available at a given stage r, for allocation among all stages upto and including that stage. The backward recursion starts from the last crop *c* = *nc* which corresponds to stage *r* = 1 and ends with the first crop *c* = 1 which is the last stage (*r* = *L*).

If $g_r(q_r)$ is the minimum value of the objective function for given q_r and *q*(*r*) is the water allocated to the crop corresponding to *r*th stage then the general recursive equation for the *r*th stage is given by

$$g_r(q_r) = \text{Min}\,[ky_{nc-r+1,t}(1 - E_a/E_m)_{nc-r+1,t} + g_{r-1}(q_r - q(r))], \tag{4}$$

$$0 \leq q(r) \leq q(r) \leq X.$$

E_a is the actual evapotranspiration needs provided to the crops and depends on the volume of water *q*(*r*) allocated to the crop. E_m is the maximum evapotranspirational needs of the crop. Part of the water requirement of the crop is met by the initial moisture in the soil and rainfall. Thus, the irrigation requirement of a crop *c* in a given period *t* (IRR_{ct}) is given by

$$\text{IRR}_{ct} = 0 \quad \text{if} \quad (\theta_t - Z_w)d_{ct} + P_t \geq E_{mct},$$

$$\text{IRR}_{ct} = E_{mct} - [(\theta_t - Z_w)d_{ct} + P_t] \quad \text{otherwise}, \tag{5}$$

where θ_t is the soil moisture at the beginning of period *t*, Z_w is the moisture level at permanent wilting point, P_t is the effective rainfall in time period *t* and d_{ct} is the root depth of crop *c* in period *t*. The actual evapotranspiration needs E_{act} provided to the crop *c* in time period *t* depend on the actual irrigation applied IRA_{ct}. E_{act} is given by

$$E_{act} = E_{mct} \quad \text{if } \text{IRA}_{ct} \geq \text{IRR}_{ct},$$

$$E_{act} = (\theta_t - Z_w)d_{ct} + P_t + \text{IRA}_{ct} \quad \text{otherwise}. \tag{6}$$

The actual irrigation applied IRA_{ct} to a given crop *c* can be obtained by dividing the volume of water *q*(*r*) allocated to that crop by the area in which the crop is grown.

The depth of root zone increases progressively with time and thus depends on the growth stage of the crop. The root is assumed to attain its maximum depth at the end of flowering stage of the crop and remains constant thereafter till the end of the crop season. The root growth is assumed to be linear and the root depth in any period is approximated by the depth of root corresponding to the midpoint of the period. The effective rainfall P_t is assumed to be a fraction of the mean rainfall in period *t* and thus deterministic.

The recursive equation given by Equation (4) can be solved to obtain the optimal allocation of the available quantity of water *X* given the initial soil moisture level. This equation is solved for all possible releases from the reservoir, for all possible initial moisture levels for each time period *t*. The results of this I phase are used in the II phase of the model.

2.2. FORMULATION OF STOCHASTIC DYNAMIC PROGRAMMING MODEL (PHASE II)

A stochastic dynamic programming (SDP) model is formulated to determine the optimal operation policy of an irrigation reservoir under multicrop environment. The optimal releases from the reservoir are related to three state variables, namely the inflow into the reservoir in time period *t*, the storage state of the reservoir at the beginning of the time period *t* and the average initial soil moisture in the command area at the beginning of the period.

Each of the state variable is discritised into several class intervals and the value of the variable in any given period falls into one of these class intervals. Any value within the range of a class interval can be represented by a single value. Let *i* and *j* represent the class intervals for inflows, *m* and *n* for soil moisture and *k* and *l* for storage at the beginning of periods *t* and *t*+l, respectively. Let Q_{it}, Q_{jt+l}, θ_{mt}, $\theta_{n,t+l}$, S_{kt} and $S_{l,t+l}$ be the representative values of inflow, soil moisture and storage in the respective class intervals.

Given the initial storage volume S_{kt}, the inflow Q_{it}, and final storage volume $S_{l,t+l}$ in period *t*, the release R_{kilt} is determined by the continuity equation

$$R_{kilt} = S_{kt} + Q_{it} - S_{l,t+l} - E_{klt}, \tag{7}$$

where E_{klt} is the possible evaporation loss which depnds on the initial and final storage volumes in period *t*. For a given initial storage state *k* and inflow state *i*, some of the final states *l* may not be feasible as they result in a negative value of R_{kilt}.

The objective function of the SDP model is to minimise the expected value of *B*, a measure of the system performance. The system performance *B* is a function of reservoir release, R_{kilt} and the soil moisture θ_t during the time period *t*. The value of *B* is determined from the DDP model of the I phase. The objective function of the SDP model can be written as

$$\underset{\{l\}}{\text{Minimise}}\; E[B(k,\, i,\, l,\, m,\, t)] \quad \forall\, k,\, i,\, m, \tag{8}$$

where $E[]$ denotes the expected value over all periods of the function contained in the brackets and $\{l\}$ is the set if all feasible l.

The solution of the model is obtained by backward recursion, which is initiated at some arbitrarily chosen future year in the last period T. The time interval considered in the present study is a month and thus the last time period corresponds to the last month of the year. Let $f_t{}^n(k,i,m)$ represent the optimum expected value of the system performance with n periods to go, including the current period t, given that in the current period the initial storage, inflow and initial soil moisture are S_{kt}, Q_{it} and θ_{mt}, respectively. The general recursive equation of the model can be written as

$$f_t^n(k,i,m) = \underset{\{l\}}{\text{Min}} \; [B(k,i,l,m,t) + \sum_j p_{ij}^t f_{t+1}^{n-1}(l,j,n)] \quad \forall k,i,m, \tag{9}$$

where $p_{ij}{}^t$ is the transition probability, defined as the probability that the inflow in period t+1 is in state j, given that the inflow in period t is in state i, and n is the soil moisture state at the beginning of period t+1, which can be determined for any given k, i, l and m using soil moisture continuity.

For a given level of irrigation application IRA_{ct} for a given crop c in period t, the final soil moisture, θ_{fct} can be computed by

$$\theta_{fct} d_{ct} = \theta_{mt} d_{ct} + IRA_{ct} + P_t - E_{act}. \tag{10}$$

The value of θ_{fct} should be adjusted to represent the initial soil moisture to be used for the next time period t+1 as the root depth for t+1 is different from that of t. Thus,

$$\theta_{f,c,t+1} = \text{Min}\left[\frac{\theta_{fct} d_{ct} \;+\; \theta_{\delta d}(d_{c,t+1} \;-\; d_{ct})}{d_{c,t+1}},\; Z_f\right], \tag{11}$$

where $\theta_{f,c,t+l}$ is the initial soil moisture at the beginning of time period t+l for crop c and $\theta_{\delta d}$ is the soil moisture in the layer of soil added to the root zone in the previous period, and Z_f is the soil moisture at the field capacity.

The initial soil moisture θ_{mt} is assumed to be same over all crops and using soil moisture balance the value of $\theta_{f,c,t+1}$ is calculated for each crop. The class interval to which the average value of $\theta_{f,c,t,+1}$ over all crops belongs is denoted as n and is taken as the initial soil moisture state for next time period t+l, to be used in Equation (9).

Equation (9) is solved recursively, until a steady state solution is reached, defining the optimal policy l^* (k,i,m,t) for all values of k, i and m and for all periods t.

3. Application

The two-phase stochastic dynamic programming model discussed in the previous section is applied to the Sri Rama Sagar Reservoir on the River Godavari in the state

Table I. Details of cropping patterns CP1 and CP2

Crop name	Area under the crop in hectares	
	CP1	CP2
Sugarcane	10 100	14 880
Rice (K)	231 200	40 548
Maize (K)	103 800	57 320
Groundnut (K)	143 600	147 032
Chillies	28 300	104 580
Cotton	12 200	18 850
Sorghum (K)	105 300	2 738
Maize (R)	–	62 726
Groundnut (R)	–	101 977
Sorghum (R)	–	9 672
Pules	–	28 588
Green gram	–	37 733
Red gram	–	6 123
Rice (R)	4 000	–

K – Kharif Season (June – October)
R – Rabi Season (October – March)

of Andhra Pradesh, India. Though it is a multipurpose reservoir, it is predominantly an irrigation reservoir, with hydropower and municipal supply consuming only a small part of the releases made from the reservoir. The soils in the command area mostly consist of red sandy loam soils. Two alternate cropping patterns are considered for the purpose of demonstrating the proposed model. The first cropping pattern (CP1) is the existing cropping pattern in the command area and the second cropping pattern (CP2) is the alternate cropping pattern recommended by the World Bank for the project. Table I shows the details of the two cropping patterns. The two-phase dynamic programming model is applied to each of these cropping patterns and the performance of the reservoir under these cropping patterns is compared.

Monthly inflows into the reservoir measured over a period of 15 years are available at the site. The average annual flow into the reservoir is 11 707 million cubic metres (MCM) with a standard deviation of 9073 MCM. The monthly flows are modelled using a Periodic Autoregressive Moving Average model of order (1,1) [PARMA(1,1)] with log transformation. Using this model, 100 sequences, each of 15 years duration are generated. The inflows in each month are discretised into eight states. The generated sequences are used to develop the transistional probability matrices required for the stochastic dynamic programming model.

The active capacity of the reservoir is 2320 MCM. The storage is discretised into 20 states in each month. The field capacity of sandy loam soils is 15% and the permanent wilting point is 5%. Thus, the available soil moisture is 10%. This is divided into five states. The growth stages of the crops and their yield response factors for different growth stages are adopted from Doorenbos and Kassam (1979).

Table II. Monthly irrigation requirements of CP1 and CP2

Month	Irrigation demands in MCM	
	CP1	CP2
June	254.95	123.78
July	1086.14	479.32
August	1562.83	664.20
September	1116.82	463.84
October	836.11	797.58
November	576.33	769.39
December	103.97	582.72
January	70.47	354.95
February	72.79	90.21
March	92.63	117.27
April	46.46	56.36
May	31.84	46.90

The water requirements of the crops are estimated using the modified Penman method. The irrigation demands for each of the cropping pattern are shown in Table II. The details of the discretisation of state variables, their respective class intervals, model formulation, and application are reported in Umamahesh (1994).

The stochastic dynamic programming model is applied to derive the optimal operating policy of the reservoir under each cropping pattern. A typical operating policy for the month of August for initial soil moisture state $m = 2$ is given in Table III. Similarly, Table IV shows the operating policy for the month of August for initial soil moisture state $m = 2$ for CP2. The steady-state operating policy so derived can be used for real-time operation of the reservoir under the two cropping patterns.

The operation of the reservoir is simulated using the operating policy derived, over 100 synthetic inflow sequences each of length 15 years to evaluate the performance of the reservoir. Three performance inficators namely, reliability, resilience, and expected annual deficit, are used to evaluate the performance of the reservoir.

Reliability is defined as the probability that the system performance is satisfactory. Resilience is defined as the probability of system recovery from a failure when it occurs. The system output is satisfactory when the release from the reservoir is at least equal to the irrigation demands. The magnitude of failure is measured in terms of expected annual deficit. These three performance indicators are evaluated for each of the cropping patterns and are shown in Table V. In addition to these three performance indicators, the reliability of meeting the demand in a month is computed for each month for both the cropping patterns and these values are given in Table VI.

Table III. Operating policy of the reservoir under CP1 for August for initial moisture level 2. (The table gives the final storage states to be maintained in the reservoir)

Initial storage state	Inflow state							
	1	2	3	4	5	6	7	8
1	1	16	17	17	17	17	17	17
2	1	17	17	17	17	17	17	17
3	2	17	17	17	17	17	17	17
4	2	17	17	17	17	17	17	17
5	3	17	17	17	17	17	17	17
6	3	17	17	17	17	17	17	17
7	6	17	17	17	17	17	17	17
8	6	17	17	17	17	17	17	17
9	8	17	17	17	17	17	17	17
10	9	17	17	17	17	17	17	17
11	9	17	17	17	17	17	17	17
12	10	17	17	17	17	17	17	17
13	12	17	17	17	17	17	17	17
14	13	17	17	17	17	17	17	17
15	14	17	17	17	17	17	17	17
16	15	17	17	17	17	17	17	17
17	16	17	17	17	17	17	17	17
18	17	17	17	17	17	17	17	17
91	17	17	17	17	17	17	17	17
20	17	17	17	17	17	17	17	17

Comparing the performance of the reservoir under the two alternate cropping patterns considered, it is evident that there is a significant improvement in the performance of the system under the second cropping pattern. In the existing cropping pattern CP1, the irrigation activity is mostly concentrated in the rainy season (Kharif season) and a large area is allocated to wet crops. Thus, the irrigation demands are not uniform over different seasons in a year. Both the system reliability and resilience under the alternate cropping pattern are higher than the corresponding values under, the existing cropping pattern. It is observed from Table VI that the reliability of meeting the demand in a given month is higher under CP2 than under CP1, especially during the Kharif season.

This comparative study of the system under two alternate cropping patterns demonstrates the scope of the two-phase stochastic dynamic programming model to evolve an optimal cropping pattern and reservoir operating policy for an irrigation reservoir. The reliability and resilience, even under the second cropping pattern, are low and there is a need to improve the performance of the reservoir further, for which studies are being undertaken by the project authorities.

Table IV. Operating policy of the reservoir under CP2 for August for moisture level 2. (The table gives the final storage states to be maintained in thr reservoir)

Initial storage state	Inflow state 1	2	3	4	5	6	7	8
1	7	14	14	14	14	14	14	14
2	9	14	14	14	14	14	14	14
3	9	14	14	14	14	14	14	14
4	10	14	14	14	14	14	14	14
5	10	14	14	14	14	14	14	14
6	10	14	14	14	14	14	14	14
7	10	14	14	14	14	14	14	14
8	10	14	14	14	14	14	14	14
9	12	14	14	14	14	14	14	14
10	13	14	14	14	14	14	14	14
11	14	14	14	14	14	14	14	14
12	14	14	14	14	14	14	14	14
13	14	14	14	14	14	14	14	14
14	14	14	14	14	14	14	14	14
15	14	14	14	14	14	14	14	14
16	14	14	14	14	14	14	14	14
17	14	14	14	14	14	14	14	14
18	14	14	14	14	14	14	14	14
19	14	14	14	14	14	14	14	14
20	14	14	14	14	14	14	14	14

Table V. Performance of the reservoir under alternate cropping patterns

Performance indicator	Cropping pattern CP1	CP2
Reliability	0.625	0.694
Resilience	0.497	0.565
Average Annual Deficit (in MCM)	971.5	526.9

4. Summary and Conclusions

A three-state variable stochastic dynamic programming model is developed for the operation of irrigation reservoirs. The model integrates the water allocation decisions with reservoir release decisions. The irrigation requirements of crops vary from period to period and are determined from the soil moisture balance equation taking into consideration the soil moisture and rainfall. This model is

Table VI. Reliability of meeting the monthly irrigation demands

Month	Cropping pattern	
	CP1	CP2
June	0.26	0.66
July	0.50	0.72
August	0.62	0.73
September	0.61	0.63
October	0.60	0.63
November	0.50	0.46
December	0.66	0.61
January	0.73	0.63
February	0.67	0.73
March	0.82	0.72
April	0.86	0.86
May	0.69	0.96

applied to Sri Rama Sagar Reservoir on the River Godavari in the state of Andhra Pradesh, India. Two alternate cropping patterns are considered for demonstrating the use of the model. The performance of the reservoir under the operating policy derived using the proposed model is evaluated through a simulation of both the cropping patterns. It is observed that the performance of the reservoir is poor under the existing cropping in comparison with the alternate cropping pattern.

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